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Exponential Stability for Delayed Stochastic Bidirectional Associative Memory Neural Networks with Markovian Jumping and Impulses

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Abstract In this paper, the problem of stability analysis for a class of delayed stochastic bidirectional associative memory neural network with Markovian jumping parameters and impulses are being investigated. The jumping parameters assumed here are continuous-time, discrete-state homogenous Markov chain and the delays are time-variant. Some novel criteria for exponential stability in the mean square are obtained by using a Lyapunov function, Ito's formula and linear matrix inequality optimization approach. The derived conditions are presented in terms of linear matrix inequalities. The estimate of the exponential convergence rate is also given, which depends on the system parameters and impulsive disturbed intension. In addition, a numerical example is given to show that the obtained result significantly improve the allowable upper bounds of delays over some existing results.

Keywords Global exponential stability · Lyapunov-Krasovskii function · Time varying delay · Markovian jumping parameters · Linear matrix inequality optimization approach · Impulses

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1 Introduction

The neural networks have received a lot of attention for the past few decades. In particular, the bidirectional associative memory (BAM) is a type of recurrent neural network. BAM was introduced by Kosko in 1988 [1]. There are two types of associative memory, auto-associative and hetero-associative. It contains two layers of neurons, we shall call u and v. Layer u and v are fully connected with each other. Once the weights have been established, input into layer u presents the pattern in layer v, and vice versa. BAM is hetero-associative, meaning given a pattern it can return another pattern which is potentially of a different size. However, regular networks return patterns of the same size. Moreover, it has been shown that the BAM neural networks structure leads to better results than the regular neural networks structures [2]. For example, the basic transmitting unit in the nervous system in brain cells is so called neurons. The neuron is not one homogenous integrative unit but is divided in many sub-integrative units, each one with the ability of mediating a local synaptic output to another cell or a local electro-tonic output to another part of the same cell. Because of this, the synapse can be viewed as having bidirectional networks. The BAM neural networks have wide applications in many fields such as pattern recognition, image and signal processing, automatic control and artificial intelligence etc. Such applications of neural networks heavily depend on the dynamical behaviors of the networks. Therefore, a large number of criteria on the stability of bidirectional associative memory neural networks with time delays have been reported in the literature [3–10]. Moreover, there are only a few papers which have taken the random fluctuations of external circumstance into account in the stability analysis of neural networks. Practically, a real system is usually affected by complex external perturbations which can be treated as stochastic inputs to the system. Markovian jump system can be employed to model abrupt phenomena such as random failures, changes in the interconnections of sub systems and sudden environment changes, etc. Recently, the stability of stochastic neural networks with Markovian switching has received much attention [11, 12]. Zhang and Wang [13] have investigated the stability analysis of Markovian jumping stochastic Cohen-Grossberg neural networks with mixed time delays via linear matrix inequality approach. Mao [14] discussed the exponential stability of stochastic delay interval systems with Markovian switching. Wang et al. [15] studied the exponential stability for stochastic neural network with mixed time-delays and Markovian jumping parameters.

In the literature, there are several works on the exponential stability for stochastic neural network with mixed time-delays and Markovian jumping parameters. On the other hand, in real world problems many physical systems also undergo abrupt changes at certain moments due to instantaneous perturbations, which lead to impulsive effects. For more details about impulsive theory and its applications one can refer [16]. So far, several interesting results have been reported that have focused on the impulsive effect of neural networks without Markovian switching [17–21]. Zhou [22] derived a set of sufficient conditions for obtaining the existence and global exponential stability of a unique equilibrium for BAM neural networks without assuming the differentiability and the monotonicity of the activation functions. Recently, Samidurai et al. [23] introduce a class of BAM neural networks with distributed delays and impulses, provided some sufficient conditions for the global exponential stability of unique equilibrium point for BAM neural networks. More recently, stability criterion is obtained by LMI optimization algorithms to guarantee the exponential stability of uncertain fuzzy BAM neural networks with time-varying delays in [24–26]. More recently, Song et al. [27] investigated the global exponential stability of BAM neural networks with reaction-diffusion terms and distributed delays by using the theory of topological degree, properties of M-matrix and Lyapunov functional.

Moreover, the neural network models are often subject to impulsive perturbations that in turn affect dynamical behaviors of the systems. Therefore, it is significant to consider the impulsive effect when investigating the stability of neural networks. Further, the state of electronic networks is often subject to instantaneous changes, and will experience abrupt changes at certain instants, which can be caused by frequency change, switching phenomenon, or by the effect of some noise. Hence, the importance of formulating neural network models with impulses is straightforward. The incorporation of impulses in neural network models is also motivated by the biological reality. Impulsive phenomena can be found in a wide variety of biological systems, such as biological neural networks and bursting rhythm models in pathology, in which many sudden and sharp changes occur instantaneously, in the form of impulses. Therefore, the analysis of neural network models with impulses also leads to a better understanding of the complex phenomena occurring in biological neural networks. To the best of the authors knowledge, no work has been reported for the problem of global exponential stability of stochastic BAM neural networks with impulsive effects and Markovian jumping parameters and this motivates the present work. The presence of impulses requires some modifications and the imposing of additional conditions on the systems. Therefore, it is important to study the stability issue of BAM neural networks with impulses and Markovian jumping parameters. The main purpose of this paper is to construct a suitable Lyapunov functional to investigate the exponential stability of the equilibrium of stochastic delayed BAM neural networks with Markovian switching and impulsive effects. A new set of sufficient conditions are derived for obtaining the global exponential stability for the time varying delayed BAM neural networks with impulses and Markovian switching. Finally, an example is provided to illustrate the effectiveness of the obtained result.

2 Problem Formulation and Preliminaries

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_{t\geq 0}\}, \mathrm{P})$ be a complete probability space with a filtration $\{\mathcal{F}_{t\geq 0}\}$ satisfying the usual conditions and $\mathrm{E}\{\cdot\}$ represents the mathematical expectation. Let R^n and $\mathrm{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n \times m$ matrices respectively. $|\cdot|$ is the Euclidean norm in R^n. I_n is the $n \times n$ identity matrix and for a matrix A, $\lambda_{max}(A)$ and $\lambda_{min}(A)$ represent the largest and the smallest eigenvalue of A, respectively. The superscript "T" denotes the matrix transposition. For a real matrix S, S^T denotes its transpose, and $S > 0$ ($S < 0$) means that S is positive-definite (negative-definite). Let $PC^b_{\mathcal{F}_0}([-\bar{\tau}, 0], \mathrm{R}^n)$ be the family of all bounded and $\mathcal{F}_0$-measurable.

Consider the following time delayed BAM neural networks with impulses

$$\begin{aligned}
\dot{u}(t) &= -Cu(t) + A\hat{f}(v(t-\delta(t))) + I, \quad t \neq t_k,\\
u(t_k) &= E_k u(t_k^-), \quad t = t_k,\\
\dot{v}(t) &= -Dv(t) + B\hat{g}(u(t-\tau(t))) + J, \quad t \neq t_k,\\
v(t_k) &= G_k v(t_k^-), \quad t = t_k,
\end{aligned} \tag{1}$$

for $t > 0$ and $k = 1, 2, \ldots,$ where $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T$, $v(t) = (v_1(t), v_2(t), \ldots, v_n(t))^T$ are the state vectors associated with n neurons; $C = \mathrm{diag}(c_1, c_2, \ldots, c_n) > 0$, $D = \mathrm{diag}(d_1, d_2, \ldots, d_n) > 0$ denotes the decay rates of the neurons; $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$ denotes the connection weight matrices; $\hat{f}(v) = (\hat{f}_1(v_1), \hat{f}_2(v_2), \ldots, \hat{f}_n(v_n))$, $\hat{g}(u) = (\hat{g}_1(u_1), \hat{g}_2(u_2), \ldots, \hat{g}_n(u_n))$ denotes the neuron activation functions; $I = (I_1, I_2, \ldots, I_n)^T$, $J = (J_1, J_2, \ldots, J_n)^T$ are the constant external input vectors from outside the system; $\tau(t), \delta(t)$ represents the time-varying delays satisfying $0 \le \tau(t) \le \bar{\tau}$ and $0 \le \delta(t) \le \bar{\delta}$, here $\bar{\tau}$ and $\bar{\delta}$ are known positive integers; $u(t_k) = E_k u(t_k^-)$ and $v(t_k) = G_k v(t_k^-)$ are the impulse at moment of time t_k satisfying $t_1 < t_2 <, \ldots, \lim_{k\to\infty} t_k = +\infty$ and $u(t^-) = \lim_{s\to t^-} u(s)$, $v(t^-) = \lim_{s\to t^-} v(s)$; E_k and G_k are the constant matrices at the moments of time t_k. The initial conditions of system (1) are described as $u(t) = \hat{\phi}(t), \quad t \in [-\bar{\tau}, 0] \quad v(t) = \hat{\psi}(t), \quad t \in [-\bar{\delta}, 0]$, here $\hat{\phi} : [-\bar{\tau}, 0] \to \mathrm{R}^n$ and $\hat{\psi} : [-\bar{\delta}, 0] \to \mathrm{R}^n$ with the sup-norms $|\hat{\phi}| = \sup_{-\bar{\tau}\le s\le 0} \|\hat{\phi}(s)\|$ and $|\hat{\psi}| = \sup_{-\bar{\delta}\le s\le 0} \|\hat{\psi}(s)\|$.

(I) The neuron activation functions $\hat{f}(\cdot)$ and $\hat{g}(\cdot)$ are bounded on R^n and satisfies the Lipschitz condition, i.e., there exist constants $L_1, L_2 \in \mathrm{R}^{n\times n}$ such that

$$|\hat{f}(u) - \hat{f}(v)| \le L_1|u - v|, \qquad |\hat{g}(u) - \hat{g}(v)| \le L_2|u - v|, \quad u, v \in \mathrm{R}^n.$$

Assume $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$, $y^* = (y_1^*, y_2^*, \ldots, y_n^*)^T$ be the equilibrium point of (1) and for convenience, we shift the equilibrium point to the origin by translation $x(t) = u(t) - u^*$, $y(t) = v(t) - v^*$, then (1) can be transformed into

$$\begin{aligned}
\dot{x}(t) &= -Cx(t) + Af(y(t-\delta(t))), \quad t \neq t_k,\\
x(t_k) &= E_k x(t_k^-), \quad t = t_k,\\
\dot{y}(t) &= -Dy(t) + Bg(x(t-\tau(t))), \quad t \neq t_k,\\
y(t_k) &= G_k y(t_k^-), \quad t = t_k,
\end{aligned} \tag{2}$$

where $f(y(t)) = \hat{f}(y(t) + v^*) - \hat{f}(v^*)$, $g(x(t)) = \hat{g}(x(t) + u^*) - \hat{g}(u^*)$, $x(t_k) = u(t_k) - u^*(t_k) = E_k x(t_k^-)$, $y(t_k) = v(t_k) - v^*(t_k) = G_k y(t_k^-)$. Obviously, the activation function can be rewritten as $|f(y)| \le L_1|y|, \quad |g(x)| \le L_2|x|, \ \forall\, x, y \in \mathrm{R}^n$.

(II) The time-varying delays $\delta(t)$ and $\tau(t)$ satisfy the following conditions

$$\begin{aligned}
0 \le h_1 \le \delta(t) \le \bar{\delta} \le h_2, &\quad \dot{\delta}(t) \le \mu < 1,\\
0 \le r_1 \le \tau(t) \le \bar{\tau} \le r_2, &\quad \dot{\tau}(t) \le \lambda < 1,
\end{aligned}$$

where $h_1, h_2, r_1, r_2, \mu, \lambda$ are constants.

Let $\{r(t), t \geq 0\}$ be a right-continuous Markov process on the probability space which takes values in the finite space $S = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\gamma_{ij}), (i, j \in S)$ given by

$$P(r(t+\delta) = j | (r(t) = i)) = \begin{cases} \gamma_{ij}\Pi + o(\Pi), & \text{if } i \neq j \\ 1 + \gamma_{ij}\Pi + o(\Pi), & \text{if } i = j \end{cases} \tag{3}$$

where $\gamma_{ij} \geq 0 (i \neq j)$ is the transition rate from i to j and $\gamma_{ii} = -\sum_{i \neq j} \gamma_{ij}$, $\Pi > 0$ and $\lim_{\delta \to 0} o(\Pi)/\Pi = 0$. In this paper, we consider the stochastic BAM neural networks with Markovian jumping parameters and impulses

$$\begin{aligned} dx(t) &= [-C(r(t))x(t) + A(r(t)) f(y(t - \delta(t)))]dt \\ &\quad + \sigma(t, x(t), y(t - \delta(t)), r(t)) dw_1(t), \quad t \neq t_k, \\ x(t_k) &= E_k(r(t))x(t_k^-), \quad t = t_k, \\ dy(t) &= [-D(r(t))y(t) + B(r(t)) g(x(t - \tau(t)))]dt \\ &\quad + \rho(t, y(t), x(t - \tau(t)), r(t)) dw_2(t), \quad t \neq t_k, \\ y(t_k) &= G_k(r(t))y(t_k^-), \quad t = t_k, \end{aligned} \tag{4}$$

where $r(t)$ is the Markovian chain defined in (3), $w_i(t) = \{w_{i1}(t), w_{i2}(t), \ldots, w_{in}(t)\}$, $(i = 1, 2)$ are the n-dimensional Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_{t \geq 0}\}, \mathrm{P})$ and $\sigma : \mathrm{R}^+ \times \mathrm{R}^n \times \mathrm{R}^n \times \mathrm{S} \to \mathrm{R}^{n \times n}$, $\rho : \mathrm{R}^+ \times \mathrm{R}^n \times \mathrm{R}^n \times \mathrm{S} \to \mathrm{R}^{n \times n}$. Based on the discussion in the introduction, in particular, (4) is considered with the impulsive term $x(t_k) = E_k(r(t))x(t_k^-), \quad t = t_k$, $y(t_k) = G_k(r(t))y(t_k^-), \quad t = t_k$. Therefore, our proposed model is different from the existing results.

For the sake of simplicity, we take $r(t) = i \in S$, therefore $A(i) = A_i$, $B(i) = B_i$, $C(i) = C_i$, $D(i) = D_i$. Now the system (4) can be transformed into

$$\begin{aligned} dx(t) &= [-C_i x(t) + A_i f(y(t - \delta(t)))]dt \\ &\quad + \sigma_i(t, x(t), y(t - \delta(t)), i) dw_1(t), \quad t \neq t_k, \\ x(t_k) &= E_{ik} x(t_k^-), \quad t = t_k, \\ dy(t) &= [-D_i y(t) + B_i g(x(t - \tau(t)))]dt \\ &\quad + \rho_i(t, y(t), x(t - \tau(t)), i) dw_2(t), \quad t \neq t_k, \\ y(t_k) &= G_{ik} y(t_k^-), \quad t = t_k. \end{aligned} \tag{5}$$

Further, in order to get the required result, we assume the following conditions:

(III) There exist matrices $R_1 \geq 0$, $R_2 \geq 0$, $Q_1 \geq 0$, $Q_2 \geq 0$ such that for all $i \in \mathrm{S}$

$$\begin{aligned} &\operatorname{trace}[\sigma^T(t,x(t),y(t-\delta(t)),i)\sigma(t,x(t),y(t-\delta(t)),i)] \\ &\quad \leq x^T(t)R_1x(t)+y^T(t-\delta(t))R_2y(t-\delta(t)), \\ &\operatorname{trace}[\rho^T(t,y(t),x(t-\tau(t)),i)\rho(t,y(t),x(t-\tau(t)),i)] \\ &\quad \leq y^T(t)Q_1y(t)+x^T(t-\tau(t))Q_2x(t-\tau(t)). \end{aligned}$$

(IV) $\sigma(t,0,0,r(t)) \equiv 0; \quad \rho(t,0,0,r(t)) \equiv 0.$

Definition 2.1 For the stochastic impulsive BAM neural network (5) and every initial condition $\phi \in PC^b_{\mathcal{F}_0}([-\bar{\tau},0],\mathrm{R}^n)$, $\psi \in PC^b_{\mathcal{F}_0}([-\bar{\delta},0],\mathrm{R}^n)$, the equilibrium point is globally exponentially stable in the mean square, if for every network mode, there exist scalars $\alpha > 0$ and $\mathcal{K}_1 > 0$, $\mathcal{K}_2 > 0$ such that

$$\mathrm{E}[|x(t,\phi)|^2+|y(t,\psi)|^2] \leq e^{-\alpha t}\Big(\mathcal{K}_1\mathrm{E}\|\phi\|^2+\mathcal{K}_2\mathrm{E}\|\psi\|^2\Big),$$

here $x(t,\phi)$ and $y(t,\psi)$ are state trajectories.

Before closing this section, let us give some lemmas that we will use in the rest of the article.

Lemma 2.1 ([28]) *The function* $V:[t_0,\infty)\times\mathrm{R}^n \to \mathrm{R}^+$ *belongs to class* v_0 *if*

(1) *the function* V *is continuous on each of the sets* $[t_{k-1},t_k)\times\mathrm{R}^n$ *and for all* $t \geq t_0$, $V(0,t)\equiv 0$;
(2) $V(x,t)$ *is locally Lipschitzian in* $x \in \mathrm{R}^n$;
(3) *for each* $k=1,2,\ldots,$ *there exist finite limits*

$$\begin{aligned} &\lim_{(q,t)\to(x,t_k^-)} V(q,t)=V(x,t_k^-) \\ &\lim_{(q,t)\to(x,t_k^+)} V(q,t)=V(x,t_k^+) \quad \textit{with} \\ &V(x,t_k^+)=V(x,t_k) \ \textit{satisfied}. \end{aligned}$$

Lemma 2.2 ([29]) *Given any real matrices* $X, Y, P > 0$ *of appropriate dimensions, for any* $\varepsilon > 0$, *the following inequality holds*:

$$X^TY+Y^TX \leq \frac{1}{\varepsilon}X^TPX+\varepsilon Y^TP^{-1}Y.$$

Lemma 2.3 ([29]) *The LMI*

$$S=\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} > 0,$$

with $S_{11} = S_{11}^T$, $S_{22} = S_{22}^T$, *is equivalent to*

$$S_{22} < 0, \qquad S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0.$$

3 Main Result

In this section, we will discuss the exponential stability of the BAM neural networks with impulses, and give their proofs. Recently, the generalized Ito's formula has played an important role in the analysis of stochastic systems [14]. Before discussing the stability analysis of the considered problem, we first introduce the Ito's formula for a general stochastic system with Markovian switching in the following:

Consider a general stochastic system $dx(t) = f(t, x(t), r(t)) + g(t, x(t), r(t))dw(t)$ on $t \geq 0$ with initial value $x(0) = x_0 \in \mathrm{R}^n$, where $f : \mathrm{R}^+ \times \mathrm{R}^n \times \mathrm{S} \to \mathrm{R}^n$, $g : \mathrm{R}^+ \times \mathrm{R}^n \times \mathrm{S} \to \mathrm{R}^n$ and $r(t)$ is the Markov chain defined in the previous section. Let $C^{2,1}(\mathrm{R}^+ \times \mathrm{R}^n \times \mathrm{S}; \mathrm{R}^+)$ be the family of all nonnegative function $V(t, x, i)$ on $\mathrm{R}^+ \times \mathrm{R}^n \times \mathrm{S}$ which are continuously twice differentiable in x and once differentiable in t. If $V \in C^{2,1}(\mathrm{R}^+ \times \mathrm{R}^n \times \mathrm{S}; \mathrm{R}^+)$ an operator $\mathcal{L}V$ is defined from $\mathrm{R}^+ \times \mathrm{R}^n \times \mathrm{S}$ to R by

$$\begin{aligned}\mathcal{L}V(t, x, i) &= V_t(t, x, i) + V_x(t, x, i)f(t, x, i)\\ &\quad + \frac{1}{2}trace[g^T(t, x, i)V_{xx}(t, x, i)g(t, x, i)] + \sum_{j=1}^{N}\gamma_{ij}V(t, x, j),\end{aligned}$$

where $V_t(t, x, i) = \frac{\partial V(t,x,i)}{\partial t}$, $V_x(t, x, i) = (\frac{\partial V(t,x,i)}{\partial x_1}, \ldots, \frac{\partial V(t,x,i)}{\partial x_n})$, $V_{xx}(t, x, i) = \frac{\partial^2 V(t,x,i)}{\partial x_j \partial x_r}$. By the generalized Ito's formula, one can get

$$\mathrm{E}V(t, x(t), r(t)) = \mathrm{E}V(0, x(0), r(0)) + \mathrm{E}\int_0^t \mathcal{L}V(s, x(s), r(s))ds.$$

Now, we are in a position to establish the conditions under which the BAM network model (5) is globally exponentially stable in the mean square. The stability criteria can be expressed in terms of linear matrix inequalities.

Theorem 3.1 *Assume that* (I)–(IV) *hold. For given positive scalars* $h_2 > h_1 \geq 0$, $r_2 > r_1 \geq 0$, $\bar{\tau} > 0$, $\bar{\delta} > 0$, μ *and* λ, *the system* (5) *is said to be globally exponentially stable in the mean square if there exist positive definite matrices* $P_{1i} > 0$, $P_{2i} > 0$, $Q_{1i}, Q_{2i}, Q_{3i}, Q_{4i} > 0$, $R_{1i}, R_{2i}, R_{3i}, R_{4i} > 0$, $Y_1, Y_2 > 0$ *and* $Z_1, Z_2 > 0$ *and constants* $\theta_1, \theta_2, \mu_1, \mu_2, \mu_3, \omega_1, \omega_2, \omega_3, \varepsilon_1, \varepsilon_2, \eta_1, \eta_2$ *such that the following inequalities hold for all* $i \in \mathrm{S}, k \in N$:

$$P_{1i} \leq \theta_1 I, \qquad P_{2i} \leq \theta_2 I, \tag{6}$$

$$\begin{bmatrix} \Omega_{1i} & P_{1i}A_i \\ A_i^T P_{1i} & -\varepsilon_i \end{bmatrix} < 0, \qquad \begin{bmatrix} \Omega_{2i} & P_{2i}B_i \\ B_i^T P_{2i} & -\eta_i \end{bmatrix} < 0, \tag{7}$$

$$\Xi_1 = \begin{bmatrix} \Phi_1 & 0 & 0 & 0 & 0 \\ * & -R_{2i} & 0 & 0 & 0 \\ * & * & -R_{3i} & 0 & 0 \\ * & * & * & -\frac{e^{-\alpha r_2}}{r_2} Z_1 & 0 \\ * & * & * & * & -\frac{e^{-\alpha(r_2+r_1)}}{r_2-r_1} Z_2 \end{bmatrix} \le 0, \tag{8}$$

$$\Xi_2 = \begin{bmatrix} \Psi_1 & 0 & 0 & 0 & 0 \\ * & -Q_{2i} & 0 & 0 & 0 \\ * & * & -Q_{3i} & 0 & 0 \\ * & * & * & -\frac{e^{-\alpha h_2}}{h_2} Y_1 & 0 \\ * & * & * & * & -\frac{e^{-\alpha(h_2+h_1)}}{h_2-h_1} Y_2 \end{bmatrix} \le 0, \tag{9}$$

$$\bar{\tau}\lambda_{max}\left[\sum_{j=1}^{N}\gamma_{ij}(Q_{1j} + L_2^T Q_{4j} L_2)\right] \le \mu_1,$$

$$\bar{\delta}\lambda_{max}\left[\sum_{j=1}^{N}\gamma_{ij}(R_{1j} + L_1^T R_{4j} L_1)\right] \le \omega_1, \tag{10}$$

$$h_1\lambda_{max}\left(\sum_{j=1}^{N}\gamma_{ij} Q_{2j}\right) \le \mu_2, \qquad r_1\lambda_{max}\left(\sum_{j=1}^{N}\gamma_{ij} R_{2j}\right) \le \omega_2, \tag{11}$$

$$h_2\lambda_{max}\left(\sum_{j=1}^{N}\gamma_{ij} Q_{3j}\right) \le \mu_3, \qquad r_2\lambda_{max}\left(\sum_{j=1}^{N}\gamma_{ij} R_{3j}\right) \le \omega_3, \tag{12}$$

$$E_{ik}^T P_{1j} E_{ik} - P_{1i} < 0, \qquad G_{ik}^T P_{2j} G_{ik} - P_{2i} < 0, \tag{13}$$

where

$$\begin{aligned} \Omega_{1i} = {} & -P_{1i}C_i - C_i P_{1i} + \theta_1 R_1 + Q_{1i} + Q_{2i} + Q_{3i} + L_2^T Q_{4i} L_2 \\ & + \sum_{j=1}^{N}\gamma_{ij} P_{1j} + h_2 Y_1 + (h_2 - h_1) Y_2 - (\mu_1 + \mu_2 + \mu_3) I, \\ \Omega_{2i} = {} & -P_{2i}D_i - D_i P_{2i} + \theta_2 Q_1 + R_{1i} + R_{2i} + R_{3i} + L_1^T R_{4i} L_1 \\ & + \sum_{j=1}^{N}\gamma_{ij} P_{2j} + r_2 Z_1 + (r_2 - r_1) Z_2 - (\omega_1 + \omega_2 + \omega_3) I, \\ \Phi_1 = {} & \theta_1 R_2 - (1-\lambda)[R_{1i} + L_1^T R_{4i} L_1] + \varepsilon_i L_1^T L_1, \\ \Psi_1 = {} & \theta_2 Q_2 - (1-\mu)[Q_{1i} + L_2^T Q_{4i} L_2] + \eta_i L_2^T L_2. \end{aligned}$$

Proof In order to prove the exponential stability result, we consider the following Lyapunov function

$$\begin{aligned}V(t,x,y,i) &= V_1(t,x,y,i)+V_2(t,x,y,i)+V_3(t,x,y,i)+V_4(t,x,y,i)\\&\quad+V_5(t,x,y,i)+V_6(t,x,y,i),\end{aligned}$$

where

$$\begin{aligned}
V_1(t,x,y,i) &= e^{\alpha t}x^T(t)P_{1i}x(t),\quad V_2(t,x,y,i)=e^{\alpha t}y^T(t)P_{2i}y(t),\\
V_3(t,x,y,i) &= \int_{t-\bar{\tau}}^{t} e^{\alpha(s+\bar{\tau})}x^T(s)Q_{1i}x(s)ds+\int_{t-h_1}^{t} e^{\alpha(s+h_1)}x^T(s)Q_{2i}x(s)ds\\
&\quad+\int_{t-h_2}^{t} e^{\alpha(s+h_2)}x^T(s)Q_{3i}x(s)ds\\
&\quad+\int_{t-\bar{\tau}}^{t} e^{\alpha(s+\bar{\tau})}g^T(x(s))Q_{4i}g(x(s))ds,\\
V_4(t,x,y,i) &= \int_{t-\bar{\delta}}^{t} e^{\alpha(s+\bar{\delta})}y^T(s)R_{1i}y(s)ds+\int_{t-r_1}^{t} e^{\alpha(s+r_1)}y^T(s)R_{2i}y(s)ds \qquad (14)\\
&\quad+\int_{t-r_2}^{t} e^{\alpha(s+r_2)}y^T(s)R_{3i}y(s)ds\\
&\quad+\int_{t-\bar{\delta}}^{t} e^{\alpha(s+\bar{\delta})}f^T(y(s))R_{4i}f(y(s))ds,\\
V_5(t,x,y,i) &= \int_{-h_2}^{0}\int_{t+\theta}^{t} e^{\alpha s}x^T(s)Y_1x(s)dsd\theta+\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} e^{\alpha s}x^T(s)Y_2x(s)dsd\theta,\\
V_6(t,x,y,i) &= \int_{-r_2}^{0}\int_{t+\theta}^{t} e^{\alpha s}y^T(s)Z_1y(s)dsd\theta+\int_{-r_2}^{-r_1}\int_{t+\theta}^{t} e^{\alpha s}y^T(s)Z_2y(s)dsd\theta.
\end{aligned}$$

When $t=t_k$, we have

$$\begin{aligned}
&V(t_k,x,y,j)-V(t_k^-,x,y,i)\\
&\quad= e^{\alpha t_k}\Big\{x^T(t_k)P_{1j}x(t_k)-x^T(t_k^-)P_{1i}x(t_k^-)+y^T(t_k)P_{2j}y(t_k)-y^T(t_k^-)P_{2i}y(t_k^-)\Big\}\\
&\qquad+\int_{t_k-\bar{\tau}}^{t_k} e^{\alpha(s+\bar{\tau})}x^T(s)Q_{1j}x(s)ds-\int_{t_k^--\bar{\tau}}^{t_k^-} e^{\alpha(s+\bar{\tau})}x^T(s)Q_{1i}x(s)ds\\
&\qquad+\int_{t_k-h_1}^{t_k} e^{\alpha(s+h_1)}x^T(s)Q_{2j}x(s)ds-\int_{t_k^--h_1}^{t_k^-} e^{\alpha(s+h_1)}x^T(s)Q_{2i}x(s)ds\\
&\qquad+\int_{t_k-h_2}^{t_k} e^{\alpha(s+h_2)}x^T(s)Q_{3j}x(s)ds-\int_{t_k^--h_2}^{t_k^-} e^{\alpha(s+h_2)}x^T(s)Q_{3i}x(s)ds
\end{aligned}$$

$$+\int_{t_k-\bar\tau}^{t_k} e^{\alpha(s+\bar\tau)} g^T(x(s))Q_{4j}g(x(s))ds - \int_{t_k^- -\bar\tau}^{t_k^-} e^{\alpha(s+\bar\tau)} g^T(x(s))Q_{4i}g(x(s))ds$$

$$+\int_{t_k-\bar\delta}^{t_k} e^{\alpha(s+\bar\delta)} y^T(s)R_{1j}y(s)ds - \int_{t_k^- -\bar\delta}^{t_k^-} e^{\alpha(s+\bar\delta)} y^T(s)R_{1i}y(s)ds$$

$$+\int_{t_k-r_1}^{t_k} e^{\alpha(s+r_1)} y^T(s)R_{2j}y(s)ds - \int_{t_k^- -r_1}^{t_k^-} e^{\alpha(s+r_1)} y^T(s)R_{2i}y(s)ds$$

$$+\int_{t_k-r_2}^{t_k} e^{\alpha(s+r_2)} y^T(s)R_{3j}y(s)ds - \int_{t_k^- -r_2}^{t_k^-} e^{\alpha(s+r_2)} y^T(s)R_{3i}y(s)ds$$

$$+\int_{t_k-\bar\delta}^{t_k} e^{\alpha(s+\bar\delta)} f^T(y(s))R_{4j}f(y(s))ds - \int_{t_k^- -\bar\delta}^{t_k^-} e^{\alpha(s+\bar\delta)} f^T(y(s))R_{4i}f(y(s))ds$$

$$+\int_{-h_2}^{0}\int_{t_k+\theta}^{t_k} e^{\alpha s} x^T(s)Y_1x(s)dsd\theta - \int_{-h_2}^{0}\int_{t_k^-+\theta}^{t_k^-} e^{\alpha s} x^T(s)Y_1x(s)dsd\theta$$

$$+\int_{-h_2}^{-h_1}\int_{t_k+\theta}^{t_k} e^{\alpha s} x^T(s)Y_2x(s)dsd\theta - \int_{-h_2}^{-h_1}\int_{t_k^-+\theta}^{t_k^-} e^{\alpha s} x^T(s)Y_2x(s)dsd\theta$$

$$+\int_{-r_2}^{0}\int_{t_k+\theta}^{t_k} e^{\alpha s} y^T(s)Z_1y(s)dsd\theta - \int_{-r_2}^{0}\int_{t_k^-+\theta}^{t_k^-} e^{\alpha s} y^T(s)Z_1y(s)dsd\theta$$

$$+\int_{-r_2}^{-r_1}\int_{t_k+\theta}^{t_k} e^{\alpha s} y^T(s)Z_2y(s)dsd\theta - \int_{-r_2}^{-r_1}\int_{t_k^-+\theta}^{t_k^-} e^{\alpha s} y^T(s)Z_2y(s)dsd\theta.$$

By simple calculation, it can be verified that

$$\begin{aligned}
&V(t_k,x,y,j)-V(t_k^-,x,y,i)\\
&= e^{\alpha t_k}\Big\{x^T(t_k^-)[E_{ik}^TP_{1j}E_{ik}-P_{1i}]x(t_k^-)+y^T(t_k^-)[G_{ik}^TP_{2j}G_{ik}-P_{2i}]y(t_k^-)\Big\}\\
&\quad+\int_{t_k^-}^{t_k} e^{\alpha(s+\bar\tau)}x^T(s)Q_{1j}x(s)ds+\int_{t_k^-}^{t_k} e^{\alpha(s+h_1)}x^T(s)Q_{2j}x(s)ds\\
&\quad+\int_{t_k^-}^{t_k} e^{\alpha(s+h_2)}x^T(s)Q_{3j}x(s)ds+\int_{t_k^-}^{t_k} e^{\alpha(s+\bar\tau)}g^T(x(s))Q_{4j}g(x(s))ds\\
&\quad+\int_{t_k^-}^{t_k} e^{\alpha(s+\bar\delta)}y^T(s)R_{1j}y(s)ds+\int_{t_k^-}^{t_k} e^{\alpha(s+r_1)}y^T(s)R_{2j}y(s)ds\\
&\quad+\int_{t_k^-}^{t_k} e^{\alpha(s+r_2)}y^T(s)R_{3j}y(s)ds+\int_{t_k^-}^{t_k} e^{\alpha(s+\bar\delta)}f^T(y(s))R_{4j}f(y(s))ds\\
&\quad+\int_{-h_2}^{0}\int_{t_k^-}^{t_k} e^{\alpha s}x^T(s)Y_1x(s)dsd\theta+\int_{-h_2}^{-h_1}\int_{t_k^-}^{t_k} e^{\alpha s}x^T(s)Y_2x(s)dsd\theta
\end{aligned}$$

$$+\int_{-r_2}^{0}\int_{t_k^-}^{t_k} e^{\alpha s} y^T(s) Z_1 y(s) ds d\theta + \int_{-r_2}^{-r_1}\int_{t_k^-}^{t_k} e^{\alpha s} y^T(s) Z_2 y(s) ds d\theta.$$

Since E_{ik} and G_{ik} are constant matrices at the moment t_k and in the mode i for $i \in \mathrm{S}, k \in N$. The terms involving positive-definite constant matrices Q_{1j}, Q_{2j}, Q_{3j}, Q_{4j}, R_{1j}, R_{2j}, R_{3j}, R_{4j}, Y_1, Y_2, Z_1, Z_2 are equal to zero and hence $V(t_k, x, y, j) - V(t_k^-, x, y, i) < 0$.

Before proving the stability part, it is necessary to show that $\mu_1 \geq 0$, $\mu_2 \geq 0$, $\mu_3 \geq 0$, $\omega_1 \geq 0$, $\omega_2 \geq 0$, $\omega_3 \geq 0$. First, we prove that $\mu_1 \geq 0$. By choosing $\lambda_{min}(Q_{1i} + L_2^T Q_{4i} L_2)$ to be the smallest of $\lambda_{min}(Q_{1j} + L_2^T Q_{4j} L_2)$, $(1 \leq j \leq N)$, i.e., $\lambda_{min}(Q_{1i} + L_2^T Q_{4i} L_2) = \min_{1\leq j\leq N} \lambda_{min}(Q_{1i} + L_2^T Q_{4i} L_2)$ and let $x \neq 0$ be the corresponding eigenvector of Q_{1i} and $L_2^T Q_{4i} L_2$, then $x^T(Q_{1i} + L_2^T Q_{4i} L_2)x = \lambda_{min}(Q_{1i} + L_2^T Q_{4i} L_2)|x|^2$. Furthermore

$$\begin{aligned}
&x^T\left(\sum_{j=1}^{N} \gamma_{ij}(Q_{1j} + L_2^T Q_{4j} L_2)\right)x \\
&= \sum_{j\neq i}^{N} \gamma_{ij} x^T(Q_{1j} + L_2^T Q_{4j} L_2)x + \gamma_{ii} x^T(Q_{1j} + L_2^T Q_{4j} L_2)x \\
&\geq \sum_{j\neq i}^{N} \gamma_{ij} \lambda_{min}(Q_{1j} + L_2^T Q_{4j} L_2)|x|^2 + \gamma_{ii} \lambda_{min}(Q_{1j} + L_2^T Q_{4j} L_2)|x|^2 \\
&\geq \lambda_{min}(Q_{1j} + L_2^T Q_{4j} L_2)|x|^2 \sum_{j=1}^{N} \gamma_{ij} \ = \ 0.
\end{aligned}$$

Thus we have, $\lambda_{max}(\sum_{j=1}^N \gamma_{ij}(Q_{1j} + L_2^T Q_{4j} L_2))|x|^2 \geq x^T(\sum_{j=1}^N \gamma_{ij}(Q_{1j} + L_2^T Q_{4j} L_2))x \geq 0$. Since, $|x| > 0$, we get $\lambda_{max}(\sum_{j=1}^N \gamma_{ij}(Q_{1j} + L_2^T Q_{4j} L_2)) \geq 0$. This implies that $\mu_1 \geq 0$ and by following the similar argument, one can prove that $\mu_2 \geq 0, \mu_3 \geq 0, \omega_1 \geq 0, \omega_2 \geq 0, \omega_3 \geq 0$.

Let $\rho_1 = \max_{i\in \mathrm{S}} \lambda_{max}(Q_{1i} + L_2^T Q_{4i} L_2)$, $\beta_1 = \max_{i\in \mathrm{S}} \lambda_{max}(R_{1i} + L_1^T R_{4i} L_1)$, $\rho_2 = \max_{i\in \mathrm{S}} \lambda_{max}(Q_{2i})$, $\beta_2 = \max_{i\in \mathrm{S}} \lambda_{max}(R_{2i})$, $\rho_3 = \max_{i\in \mathrm{S}} \lambda_{max}(Q_{3i})$, $\beta_3 = \max_{i\in \mathrm{S}} \lambda_{max}(R_{3i})$, $v_1 = -\max_{i\in \mathrm{S}} \lambda_{max}(\Omega_{1i})$, $v_2 = -\max_{i\in \mathrm{S}} \lambda_{max}(\Omega_{2i})$. It is easy to see that $\alpha\theta_1 + (e^{\alpha\bar{\tau}} - 1)\rho_1 + (e^{\alpha h_1} - 1)\rho_2 + (e^{\alpha h_2} - 1)\rho_3 + \mu_1 e^{2\alpha\bar{\tau}} + \mu_2 e^{2\alpha h_1} + \mu_3 e^{2\alpha h_2} - v_1 = 0$ and $\alpha\theta_2 + (e^{\alpha\bar{\delta}} - 1)\beta_1 + (e^{\alpha r_1} - 1)\beta_2 + (e^{\alpha r_2} - 1)\beta_3 + \omega_1 e^{2\alpha\bar{\delta}} + \omega_2 e^{2\alpha r_1} + \omega_3 e^{2\alpha r_2} - v_2 = 0$ both have unique positive roots and we denote them by α_1 and α_2, respectively. Let $\alpha = \min\{\alpha_1, \alpha_2\}$, we can obtain

$$\begin{aligned}
&\alpha\theta_1 + (e^{\alpha\bar{\tau}} - 1)\rho_1 + (e^{\alpha h_1} - 1)\rho_2 + (e^{\alpha h_2} - 1)\rho_3 + \mu_1 e^{\alpha\bar{\tau}} \\
&\quad + \mu_2 e^{\alpha h_1} + \mu_3 e^{\alpha h_2} - v_1 \leq 0, \qquad (15)\\
&\alpha\theta_2 + (e^{\alpha\bar{\delta}} - 1)\beta_1 + (e^{\alpha r_1} - 1)\beta_2 + (e^{\alpha r_2} - 1)\beta_3 + \omega_1 e^{\alpha\bar{\delta}} \\
&\quad + \omega_2 e^{\alpha r_1} + \omega_3 e^{\alpha r_2} - v_2 \leq 0. \qquad (16)
\end{aligned}$$

When $t \neq t_k$, the infinitesimal operator of $\mathcal{L}V$ can be computed as follows

$$\begin{aligned}\mathcal{L}V_1(t,x,i) &= e^{\alpha t}\Big\{\alpha x^T(t)P_{1i}x(t) + 2x^T(t)P_{1i}[-C_i x(t) + A_i f(y(t-\delta(t)))]\\ &\quad + trace[\sigma^T(t,x(t),y(t-\delta(t)),i)P_{1i}\sigma(t,x(t),y(t-\delta(t)),i)]\\ &\quad + \sum_{j=1}^N \gamma_{ij}x^T(t)P_{1j}x(t)\Big\}\\ &\le e^{\alpha t}\Big\{x^T(t)\Big[\alpha P_{1i} - P_{1i}C_i - C_i P_{1i} + \theta_1 R_1 + \sum_{j=1}^N \gamma_{ij}P_{1j}\\ &\quad + \varepsilon_i^{-1}P_{1i}A_i A_i^T P_{1i}\Big]x(t) + \varepsilon_i y^T(t-\bar{\delta})L_1^T L_1 y(t-\bar{\delta})\\ &\quad + y^T(t-\bar{\delta})\theta_1 R_2 y(t-\bar{\delta})\Big\},\end{aligned} \tag{17}$$

$$\begin{aligned}\mathcal{L}V_2(t,y,i) &= e^{\alpha t}\Big\{\alpha y^T(t)P_{2i}y(t) + 2y^T(t)P_{2i}[-D_i y(t) + B_i g(x(t-\tau(t)))]\\ &\quad + trace[\sigma^T(t,y(t),x(t-\tau(t)),i)P_{2i}\sigma(t,y(t),x(t-\tau(t)),i)]\\ &\quad + \sum_{j=1}^N \gamma_{ij}y^T(t)P_{2j}y(t)\Big\}\\ &\le e^{\alpha t}\Big\{y^T(t)\Big[\alpha P_{2i} - P_{2i}D_i - D_i P_{2i} + \theta_2 Q_1 + \sum_{j=1}^N \gamma_{ij}P_{2j}\\ &\quad + \eta_i^{-1}P_{2i}B_i B_i^T P_{2i}\Big]y(t) + \eta_i x^T(t-\bar{\tau})L_2^T L_2 x(t-\bar{\tau})\\ &\quad + x^T(t-\bar{\tau})\theta_2 Q_2 x(t-\bar{\tau})\Big\},\end{aligned} \tag{18}$$

$$\begin{aligned}\mathcal{L}V_3(t,x,i) &= e^{\alpha(t+\bar{\tau})}x^T(t)Q_{1i}x(t) - (1-\dot{\tau}(t))e^{\alpha t}x^T(t-\bar{\tau})Q_{1i}x(t-\bar{\tau})\\ &\quad + \sum_{j=1}^N \gamma_{ij}\int_{t-\bar{\tau}}^t e^{\alpha s}x^T(s)Q_{1j}x(s)ds + e^{\alpha(t+h_1)}x^T(t)Q_{2i}x(t)\\ &\quad - e^{\alpha t}x^T(t-h_1)Q_{2i}x(t-h_1) + \sum_{j=1}^N \gamma_{ij}\int_{t-h_1}^t e^{\alpha s}x^T(s)Q_{2j}x(s)ds\\ &\quad + e^{\alpha(t+h_2)}x^T(t)Q_{3i}x(t) - e^{\alpha t}x^T(t-h_2)Q_{3i}x(t-h_2)\end{aligned}$$

$$+\sum_{j=1}^{N}\gamma_{ij}\int_{t-h_2}^{t}e^{\alpha s}x^T(s)Q_{3j}x(s)ds+e^{\alpha(t+\bar\tau)}g^T(x(t))Q_{4i}g(x(t))$$
$$-(1-\dot\tau(t))e^{\alpha t}g^T(x(t-\bar\tau))Q_{4i}g(x(t-\bar\tau))$$
$$+\sum_{j=1}^{N}\gamma_{ij}\int_{t-\bar\tau}^{t}[1pt]e^{\alpha s}g^T(x(s))Q_{4j}g(x(s))ds$$
$$\le e^{\alpha(t+\bar\tau)}x^T(t)Q_{1i}x(t)-(1-\mu)e^{\alpha t}x^T(t-\bar\tau)Q_{1i}x(t-\bar\tau)$$
$$+\mu_1\bar\tau^{-1}\int_{t-\bar\tau}^{t}|x(s)|^2ds+e^{\alpha(t+h_1)}x^T(t)Q_{2i}x(t)$$
$$-e^{\alpha t}x^T(t-h_1)Q_{2i}x(t-h_1)+\mu_2h_1^{-1}\int_{t-h_1}^{t}|x(s)|^2ds$$
$$+e^{\alpha(t+h_2)}x^T(t)Q_{3i}x(t)-e^{\alpha t}x^T(t-h_2)Q_{3i}x(t-h_2)$$
$$+\mu_3h_2^{-1}\int_{t-h_2}^{t}|x(s)|^2ds+e^{\alpha(t+\bar\tau)}x^T(t)L_2^TQ_{4i}L_2x(t)$$
$$-(1-\mu)e^{\alpha t}x^T(t-\bar\tau)L_2^TQ_{4i}L_2x(t-\bar\tau), \tag{19}$$

$$\mathcal{L}V_4(t,y,i)=e^{\alpha(t+\bar\delta)}y^T(t)R_{1i}y(t)-(1-\dot\delta(t))e^{\alpha t}y^T(t-\bar\delta)R_{1i}y(t-\bar\delta)$$
$$+\sum_{j=1}^{N}\gamma_{ij}\int_{t-\bar\delta}^{t}e^{\alpha s}y^T(s)R_{1j}y(s)ds+e^{\alpha(t+r_1)}y^T(t)R_{2i}y(t)$$
$$-e^{\alpha t}y^T(t-r_1)R_{2i}y(t-r_1)+\sum_{j=1}^{N}\gamma_{ij}\int_{t-r_1}^{t}e^{\alpha s}y^T(s)R_{2j}y(s)ds$$
$$+e^{\alpha(t+r_2)}y^T(t)R_{3i}y(t)-e^{\alpha t}y^T(t-r_2)R_{3i}y(t-r_2)$$
$$+\sum_{j=1}^{N}\gamma_{ij}\int_{t-r_2}^{t}e^{\alpha s}y^T(s)R_{3j}y(s)ds+e^{\alpha(t+\bar\delta)}f^T(y(t))R_{4i}f(y(t))$$
$$-(1-\dot\delta(t))e^{\alpha t}f^T(y(t-\bar\delta))R_{4i}f(y(t-\bar\delta))$$
$$+\sum_{j=1}^{N}\gamma_{ij}\int_{t-\bar\delta}^{t}e^{\alpha s}f^T(y(s))R_{4j}f(y(s))ds$$
$$\le e^{\alpha(t+\bar\delta)}y^T(t)R_{1i}y(t)-(1-\lambda)e^{\alpha t}y^T(t-\bar\delta)R_{1i}y(t-\bar\delta)$$
$$+\omega_1\bar\delta^{-1}\int_{t-\bar\delta}^{t}|y(s)|^2ds$$
$$+e^{\alpha(t+r_1)}y^T(t)R_{2i}y(t)-e^{\alpha t}y^T(t-r_1)R_{2i}y(t-r_1)$$

$$
\begin{aligned}
&+\omega_2 r_1^{-1}\int_{t-r_1}^{t}|y(s)|^2ds+e^{\alpha(t+r_2)}y^T(t)R_{3i}y(t)\\
&-e^{\alpha t}y^T(t-r_2)R_{3i}y(t-r_2)+\omega_3 r_2^{-1}\int_{t-r_2}^{t}|y(s)|^2ds\\
&+e^{\alpha(t+\bar{\delta})}y^T(t)L_1^TR_{4i}L_1y(t)\\
&-(1-\lambda)e^{\alpha t}y^T(t-\bar{\delta})L_1^TR_{4i}L_1y(t-\bar{\delta}),
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
\mathcal{L}V_5(t,x,i)={}&h_2e^{\alpha t}x^T(t)Y_1x(t)-e^{\alpha(t-h_2)}\int_{t-h_2}^{t}x^T(s)Y_1x(s)ds\\
&+(h_2-h_1)e^{\alpha t}x^T(t)Y_2x(t)-e^{\alpha(t-h_2)}\int_{t-h_2}^{t-h_1}x^T(s)Y_2x(s)ds\\
\le{}&h_2e^{\alpha t}x^T(t)Y_1x(t)\\
&-e^{\alpha(t-h_2)}\left(\int_{t-h_2}^{t}x(s)ds\right)^T\left[\frac{1}{h_2}Y_1\right]\left(\int_{t-h_2}^{t}x(s)ds\right)\\
&+(h_2-h_1)e^{\alpha t}x^T(t)Y_2x(t)-e^{\alpha(t-h_2-h_1)}\\
&\times\left(\int_{t-h_2}^{t-h_1}x(s)ds\right)^T\left[\frac{1}{h_2-h_1}Y_2\right]\left(\int_{t-h_2}^{t-h_1}x(s)ds\right),
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\mathcal{L}V_6(t,y,i)={}&r_2e^{\alpha t}y^T(t)Z_1y(t)-e^{\alpha(t-r_2)}\int_{t-r_2}^{t}y^T(s)Z_1y(s)ds\\
&+(r_2-r_1)e^{\alpha t}y^T(t)Z_2y(t)-e^{\alpha(t-r_2)}\int_{t-r_2}^{t-r_1}y^T(s)Z_2y(s)ds\\
\le{}&r_2e^{\alpha t}y^T(t)Z_1y(t)-e^{\alpha(t-r_2)}\left(\int_{t-r_2}^{t}y(s)ds\right)^T\left[\frac{1}{r_2}Z_1\right]\left(\int_{t-r_2}^{t}y(s)ds\right)\\
&+(r_2-r_1)e^{\alpha t}y^T(t)Z_2y(t)\\
&-e^{\alpha(t-r_2-r_1)}\left(\int_{t-r_2}^{t-r_1}y(s)ds\right)^T\left[\frac{1}{r_2-r_1}Z_2\right]\left(\int_{t-r_2}^{t-r_1}y(s)ds\right).
\end{aligned}
\tag{22}
$$

From (17)–(22), we have

$$
\begin{aligned}
\mathcal{L}V(t,x,y,i)={}&\mathcal{L}V_1(t,x,y,i)+\mathcal{L}V_2(t,x,y,i)+\mathcal{L}V_3(t,x,y,i)+\mathcal{L}V_4(t,x,y,i)\\
&+\mathcal{L}V_5(t,x,y,i)+\mathcal{L}V_6(t,x,y,i)\\
\le{}&e^{\alpha t}\big\{x^T(t)[\alpha P_{1i}+(e^{\alpha\bar{\tau}}-1)(Q_{1i}+L_2^TQ_{4i}L_2)+(e^{\alpha h_1}-1)(Q_{2i})\\
&+(e^{\alpha h_2}-1)(Q_{3i})+\Omega_{1i}]x(t)\\
&+y^T(t)[\alpha P_{2i}+(e^{\alpha\bar{\delta}}-1)(R_{1i}+L_1^TR_{4i}L_1)\\
&+(e^{\alpha r_1}-1)(R_{2i})+(e^{\alpha r_2}-1)(R_{3i})+\Omega_{2i}]y(t)
\end{aligned}
$$

$$+\xi^T(t)\ \Xi_1\ \xi(t)+\zeta^T(t)\ \Xi_2\ \zeta(t)\big\}+e^{\alpha t}\mu_1\bar{\tau}^{-1}\int_{t-\bar{\tau}}^{t}|x(s)|^2ds$$
$$+e^{\alpha t}\mu_2 h_1^{-1}\int_{t-h_1}^{t}|x(s)|^2ds+e^{\alpha t}\mu_3 h_2^{-1}\int_{t-h_2}^{t}|x(s)|^2ds$$
$$+e^{\alpha t}\omega_1\bar{\delta}^{-1}\int_{t-\bar{\delta}}^{t}|y(s)|^2ds+e^{\alpha t}\omega_2 r_1^{-1}\int_{t-r_1}^{t}|y(s)|^2ds$$
$$+e^{\alpha t}\omega_3 r_2^{-1}\int_{t-r_2}^{t}|y(s)|^2ds, \tag{23}$$

where

$$\xi(t)=\left[y^T(t-\bar{\delta})\ y^T(t-r_1)\ y^T(t-r_2)\left(\int_{t-r_2}^{t}y(s)ds\right)^T\left(\int_{t-r_2}^{t-r_1}y(s)ds\right)^T\right]^T,$$
$$\zeta(t)=\left[x^T(t-\bar{\tau})\ x^T(t-h_1)\ x^T(t-h_2)\left(\int_{t-h_2}^{t}x(s)ds\right)^T\left(\int_{t-h_2}^{t-h_1}x(s)ds\right)^T\right]^T.$$

By integrating the last six parts from 0 to t in (23) and by changing the order of integration, we have

$$\int_0^t e^{\alpha s}\left(\int_{s-\bar{\tau}}^{s}|x(\theta)|^2d\theta\right)ds\le\int_{-\bar{\tau}}^{t}\left(\int_{\theta}^{\theta+\bar{\tau}}e^{\alpha s}|x(\theta)|^2dsd\theta\right)$$
$$\le\bar{\tau}e^{\alpha\bar{\tau}}\int_{-\bar{\tau}}^{t}e^{\alpha s}|x(s)|^2ds.$$

Similarly, we obtain

$$\int_0^t e^{\alpha s}\left(\int_{s-h_1}^{s}|x(\theta)|^2d\theta\right)ds\le\int_{-h_1}^{t}\left(\int_{\theta}^{\theta+h_1}e^{\alpha s}|x(\theta)|^2ds\right)d\theta$$
$$\le h_1e^{\alpha h_1}\int_{-h_1}^{t}e^{\alpha s}|x(s)|^2ds$$
$$\int_0^t e^{\alpha s}\left(\int_{s-h_2}^{s}|x(\theta)|^2d\theta\right)ds\le\int_{-h_2}^{t}\left(\int_{\theta}^{\theta+h_2}e^{\alpha s}|x(\theta)|^2ds\right)d\theta$$
$$\le h_2e^{\alpha h_2}\int_{-h_2}^{t}e^{\alpha s}|x(s)|^2ds$$
$$\int_0^t e^{\alpha s}\left(\int_{s-\bar{\delta}}^{s}|y(\theta)|^2d\theta\right)ds\le\int_{-\bar{\delta}}^{t}\left(\int_{\theta}^{\theta+\bar{\delta}}e^{\alpha s}|y(\theta)|^2ds\right)d\theta$$
$$\le\bar{\delta}e^{\alpha\bar{\delta}}\int_{-\bar{\delta}}^{t}e^{\alpha s}|y(s)|^2ds$$
$$\int_0^t e^{\alpha s}\left(\int_{s-r_1}^{s}|y(\theta)|^2d\theta\right)ds\le\int_{-r_1}^{t}\left(\int_{\theta}^{\theta+r_1}e^{\alpha s}|y(\theta)|^2ds\right)d\theta$$

$$\leq r_1 e^{\alpha r_1} \int_{-r_1}^{t} e^{\alpha s} |y(s)|^2 ds$$

$$\int_0^t e^{\alpha s} \left(\int_{s-r_2}^{s} |y(\theta)|^2 d\theta \right) ds \leq \int_{-r_2}^{t} \left(\int_{\theta}^{\theta+r_2} e^{\alpha s} |y(\theta)|^2 ds \right) d\theta$$
$$\leq r_2 e^{\alpha r_2} \int_{-r_2}^{t} e^{\alpha s} |y(s)|^2 ds.$$

Therefore, by the generalized Ito's formula and Eqn. (14), we have

$$\begin{aligned}
&\mathrm{E}V(t, x, y, i) \\
&= \mathrm{E}V(0, x(0), y(0), r(0)) + \int_0^t \mathrm{E}\, \mathcal{L}V(s, x(s), y(s), r(s)) ds \\
&\leq V_0 + [\alpha\theta_1 + (e^{\alpha\bar{\tau}} - 1)\rho_1 \\
&\quad + (e^{\alpha h_1} - 1)\rho_2 + (e^{\alpha h_2} - 1)\rho_3 - v_1]\mathrm{E} \int_0^t e^{\alpha s} |x(s)|^2 ds \\
&\quad + [\alpha\theta_2 + (e^{\alpha\bar{\delta}} - 1)\beta_1 + (e^{\alpha r_1} - 1)\beta_2 \\
&\quad + (e^{\alpha r_2} - 1)\beta_3 - v_2]\mathrm{E} \int_0^t e^{\alpha s} |y(s)|^2 ds + \mu_1 \bar{\tau}^{-1} \int_0^t e^{\alpha s} \left(\int_{s-\bar{\tau}}^{s} |x(\theta)|^2 d\theta \right) ds \\
&\quad + \mu_2 h_1^{-1} \int_0^t e^{\alpha s} \left(\int_{s-h_1}^{s} |x(\theta)|^2 d\theta \right) ds \\
&\quad + \mu_3 h_2^{-1} \int_0^t e^{\alpha s} \left(\int_{s-h_2}^{s} |x(\theta)|^2 d\theta \right) ds + \omega_1 \bar{\delta}^{-1} \int_0^t e^{\alpha s} \left(\int_{s-\bar{\delta}}^{s} |y(\theta)|^2 d\theta \right) ds \\
&\quad + \omega_2 r_1^{-1} \int_0^t e^{\alpha s} \left(\int_{s-r_1}^{s} |y(\theta)|^2 d\theta \right) ds + \omega_3 r_2^{-1} \int_0^t e^{\alpha s} \left(\int_{s-r_2}^{s} |y(\theta)|^2 d\theta \right) ds \\
&\leq V_0 + \left(\mu_1 \bar{\tau} e^{\alpha\bar{\tau}} + \mu_2 h_1 e^{\alpha h_1} + \mu_3 h_2 e^{\alpha h_2} \right) \mathrm{E}\|\phi\|^2 \\
&\quad + \left(\omega_1 \bar{\delta} e^{\alpha\bar{\delta}} + \omega_2 r_1 e^{\alpha r_1} + \omega_3 r_2 e^{\alpha r_2} \right) \mathrm{E}\|\psi\|^2,
\end{aligned} \tag{24}$$

where $V_0 = \mathrm{E}V(0, x(0), y(0), r(0))$. On the other hand, from the definition of $V(t, x, y, i)$

$$\begin{aligned}
\mathrm{E}V(0, x(0), y(0), r(0)) &\leq [\theta_1 + (\bar{\tau} e^{\alpha\bar{\tau}} + h_1 e^{\alpha h_1} + h_2 e^{\alpha h_2}) v_1]\mathrm{E}\|\phi\|^2 \\
&\quad + [\theta_2 + (\bar{\delta} e^{\alpha\bar{\delta}} + r_1 e^{\alpha r_1} + r_2 e^{\alpha r_2}) v_2]\mathrm{E}\|\psi\|^2,
\end{aligned} \tag{25}$$

$$\mathrm{E}V(t, x(t), y(t), i) \geq e^{\alpha t} \lambda_{min} \{\mathrm{E}|x(t)|^2 + \mathrm{E}|y(t)|^2\}, \tag{26}$$

where $\lambda_{min} = \min\{\min_{i\in \mathrm{S}} \lambda_{min}(P_{1i}), \min_{i\in \mathrm{S}} \lambda_{min}(P_{2i})\}$.

From (23) and (25), we obtain

$$\mathrm{E}\{|x(t)|^2 + |y(t)|^2\} \leq e^{-\alpha t} \{\mathcal{K}_1 \mathrm{E}\|\phi\|^2 + \mathcal{K}_2 \mathrm{E}\|\psi\|^2\}, \tag{27}$$

where

$$\mathcal{K}_1 = \frac{\theta_1 + (v_1+\mu_1)\bar{\tau}e^{\alpha\bar{\tau}} + (v_1+\mu_2)h_1e^{\alpha h_1} + (v_1+\mu_3)h_2e^{\alpha h_2})}{\lambda_{min}},$$

$$\mathcal{K}_2 = \frac{\theta_2 + (v_2+\omega_1)\bar{\delta}e^{\alpha\bar{\delta}} + (v_2+\omega_2)r_1e^{\alpha r_1} + (v_2+\omega_3)r_2e^{\alpha r_2})}{\lambda_{min}}.$$

Therefore by Definition 2.1, the stochastic impulsive BAM neural network (5) is globally exponentially stable in the mean square. This completes the proof. □

Further, consider a deterministic BAM neural network with Markovian jumping parameters and impulsive effects

$$\begin{aligned}
\dot{x}(t) &= [-C(r(t))x(t) + A(r(t))f(y(t-\delta(t)))], \quad t \neq t_k\\
x(t_k) &= E_k(r(t))x(t_k^-), \quad t = t_k,\\
\dot{y}(t) &= [-D(r(t))y(t) + B(r(t))g(x(t-\tau(t)))], \quad t \neq t_k\\
y(t_k) &= G_k(r(t))y(t_k^-), \quad t = t_k.
\end{aligned} \tag{28}$$

By using Theorem 3.1, the stability criterion for the above system can be derived as follows:

Corollary 3.1 *Assume that (I)–(II) hold. For given positive scalars $h_2 > h_1 \geq 0$ and $r_2 > r_1 \geq 0, \mu$ and λ, the system (28) is globally exponentially stable in the mean square if there exist positive definite matrices $P_{1i} > 0, P_{2i} > 0$, $Q_{1i}, Q_{2i}, Q_{3i}, Q_{4i} > 0$, $R_{1i}, R_{2i}, R_{3i}, R_{4i} > 0$, $Y_1, Y_2 > 0$, $Z_1, Z_2 > 0$ and constants $\mu_1, \mu_2, \mu_3, \omega_1, \omega_2, \omega_3, \varepsilon_1, \varepsilon_2, \eta_1, \eta_2$ such that the following inequalities hold for all $i \in S, k \in N$:*

$$\begin{bmatrix} \Theta_{1i} & P_{1i}A_i \\ A_i^T P_{1i} & -\varepsilon_i \end{bmatrix} < 0, \qquad \begin{bmatrix} \Theta_{2i} & P_{2i}B_i \\ B_i^T P_{2i} & -\eta_i \end{bmatrix} < 0, \tag{29}$$

$$\Pi_1 = \begin{bmatrix} \Phi_3 & 0 & 0 & 0 & 0 \\ * & -R_{2i} & 0 & 0 & 0 \\ * & * & -R_{3i} & 0 & 0 \\ * & * & * & -\frac{e^{-\alpha r_2}}{r_2}Z_1 & 0 \\ * & * & * & * & -\frac{e^{-\alpha(r_2+r_1)}}{r_2-r_1}Z_2 \end{bmatrix} \leq 0, \tag{30}$$

$$\Pi_2 = \begin{bmatrix} \Psi_3 & 0 & 0 & 0 & 0 \\ * & -Q_{2i} & 0 & 0 & 0 \\ * & * & -Q_{3i} & 0 & 0 \\ * & * & * & -\frac{e^{-\alpha h_2}}{h_2}Y_1 & 0 \\ * & * & * & * & -\frac{e^{-\alpha(h_2+h_1)}}{h_2-h_1}Y_2 \end{bmatrix} \leq 0, \tag{31}$$

$$\tau\lambda_{max}\left[\sum_{j=1}^{N}\gamma_{ij}(Q_{1j} + L_2^T Q_{4j} L_2)\right] \leq \mu_1,$$

$$\delta\lambda_{max}\left[\sum_{j=1}^{N}\gamma_{ij}(R_{1j}+L_1^TR_{4j}L_1)\right]\le\omega_1, \tag{32}$$

$$h_1\lambda_{max}\left(\sum_{j=1}^{N}\gamma_{ij}Q_{2j}\right)\le\mu_2,\qquad r_1\lambda_{max}\left(\sum_{j=1}^{N}\gamma_{ij}R_{2j}\right)\le\omega_2, \tag{33}$$

$$h_2\lambda_{max}\left(\sum_{j=1}^{N}\gamma_{ij}Q_{3j}\right)\le\mu_3,\qquad r_2\lambda_{max}\left(\sum_{j=1}^{N}\gamma_{ij}R_{3j}\right)\le\omega_3, \tag{34}$$

$$E_{ik}^TP_{1j}E_{ik}-P_{1i}<0,\qquad G_{ik}^TP_{2j}G_{ik}-P_{2i}<0, \tag{35}$$

where

$$\begin{aligned}\Theta_{1i}&=-P_{1i}C_i-C_iP_{1i}+Q_{1i}+Q_{2i}+Q_{3i}+L_2^TQ_{4i}L_2\\&\quad+\sum_{j=1}^{N}\gamma_{ij}P_{1j}+h_2Y_1+(h_2-h_1)Y_2-(\mu_1+\mu_2+\mu_3)I,\\\Theta_{2i}&=-P_{2i}D_i-D_iP_{2i}+R_{1i}+R_{2i}+R_{3i}+L_1^TR_{4i}L_1\\&\quad+\sum_{j=1}^{N}\gamma_{ij}P_{2j}+r_2Z_1+(r_2-r_1)Z_2-(\omega_1+\omega_2+\omega_3)I,\\\Phi_3&=\varepsilon_iL_1^TL_1-(1-\lambda)[R_{1i}+L_1^TR_{4i}L_1],\\\Psi_3&=\eta_iL_2^TL_2-(1-\mu)[Q_{1i}+L_2^TQ_{4i}L_2].\end{aligned}$$

Proof The proof of this corollary is similar to that of Theorem 3.1 and hence it is omitted. □

4 Illustrative Example

In this section, a numerical example is given to show the validity and effectiveness of our developed theoretical results.

Example Consider a stochastic BAM neural network with Markovian jumping parameters and impulses

$$\begin{aligned}dx(t)&=[-C_ix(t)+A_if(y(t-\delta(t)))]dt+\sigma_i(t,x(t),y(t-\delta(t)))dw_1(t),\quad t\ne t_k\\x(t_k)&=E_{ik}x(t_k^-),\quad t=t_k,\ k=1,2.\\dy(t)&=[-D_iy(t)+B_ig(x(t-\tau(t)))]dt+\rho_i(t,y(t),x(t-\tau(t)))dw_2(t),\quad t\ne t_k\\y(t_k)&=G_{ik}y(t_k^-),\quad t=t_k,\ k=1,2.\end{aligned} \tag{36}$$

The parameters of the BAM neural network (36) are given as

$$\Gamma = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1.3 & 0 \\ 0.1 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \\ 0.1 & 1.5 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 1.1 & -0.3 \\ 0 & 0.9 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1.2 & 0.3 \\ 0 & 1.6 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 2 & 2 \\ 0 & 2.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.7 & -0.4 \\ 0 & 0.6 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1.2 & 0.3 \\ 0 & 1.6 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & -0.2 \\ 0.1 & 1.2 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.6 & 0.6 \\ 0.6 & 0.6 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.2 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.15 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

By using the Matlab LMI toolbox and solving the LMIs (6)–(13) in Theorem 3.1, we obtain the following feasible solutions:

$$P_{11} = \begin{bmatrix} 6.5906 & 0.0587 \\ 0.0587 & 6.8545 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 5.6572 & -0.6164 \\ -0.6164 & 5.5986 \end{bmatrix},$$

$$P_{21} = \begin{bmatrix} 5.3411 & -0.3526 \\ -0.3526 & 4.1858 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 5.2837 & -0.2308 \\ -0.2308 & 5.3583 \end{bmatrix},$$

$$Q_{11} = \begin{bmatrix} 4.6767 & 3.4742 \\ 3.4742 & 4.6788 \end{bmatrix}, \quad Q_{12} = \begin{bmatrix} 4.6767 & 3.4742 \\ 3.4742 & 4.6788 \end{bmatrix},$$

$$Q_{21} = \begin{bmatrix} 9.4758 & -1.2178 \\ -1.2178 & 9.5503 \end{bmatrix}, \quad Q_{22} = \begin{bmatrix} 9.4758 & -1.2178 \\ -1.2178 & 9.5503 \end{bmatrix},$$

$$Q_{31} = \begin{bmatrix} 4.4434 & 3.8510 \\ 3.8510 & 4.4447 \end{bmatrix}, \quad Q_{32} = \begin{bmatrix} 4.4434 & 3.8510 \\ 3.8510 & 4.4447 \end{bmatrix},$$

$$Q_{41} = \begin{bmatrix} 9.8666 & 4.3472 \\ 4.3472 & 9.8691 \end{bmatrix}, \quad Q_{42} = \begin{bmatrix} 9.8666 & 4.3472 \\ 4.3472 & 9.8691 \end{bmatrix},$$

$$R_{11} = \begin{bmatrix} 5.0084 & 3.5592 \\ 3.5592 & 5.0048 \end{bmatrix}, \quad R_{12} = \begin{bmatrix} 5.0084 & 3.5592 \\ 3.5592 & 5.0048 \end{bmatrix},$$

$$R_{21} = \begin{bmatrix} 9.4473 & -1.1139 \\ -1.1139 & 9.3174 \end{bmatrix}, \quad R_{22} = \begin{bmatrix} 9.4473 & -1.1139 \\ -1.1139 & 9.3174 \end{bmatrix},$$

$$R_{31} = \begin{bmatrix} 4.4307 & 3.8387 \\ 3.8387 & 4.4281 \end{bmatrix}, \quad R_{32} = \begin{bmatrix} 4.4307 & 3.8387 \\ 3.8387 & 4.4281 \end{bmatrix},$$

$$R_{41} = \begin{bmatrix} 12.8428 & 2.8069 \\ 2.8069 & 12.8399 \end{bmatrix}, \qquad R_{42} = \begin{bmatrix} 12.8428 & 2.8069 \\ 2.8069 & 12.8399 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 2.6757 & -1.3471 \\ -1.3471 & 2.7549 \end{bmatrix}, \qquad Y_2 = \begin{bmatrix} 4.3293 & -2.0300 \\ -2.0300 & 4.4537 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 2.5195 & -1.1044 \\ -1.1044 & 2.3988 \end{bmatrix}, \qquad Z_2 = \begin{bmatrix} 4.1648 & -1.7183 \\ -1.7183 & 3.9508 \end{bmatrix},$$

$$\mu_1 = 20.4017, \quad \mu_2 = 46.5795, \quad \mu_3 = 22.2400, \quad \omega_1 = 21.2281,$$

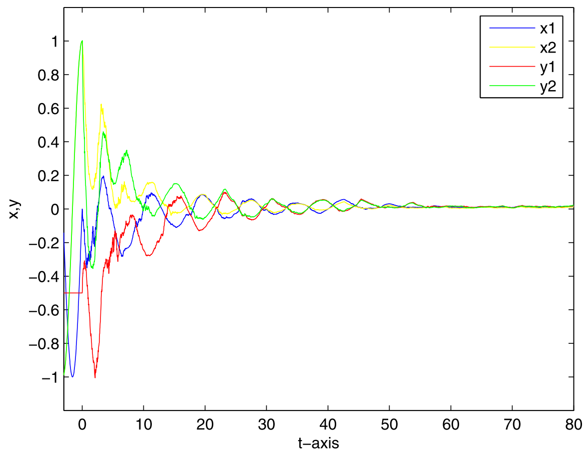
$$\omega_2 = 51.2658, \quad \omega_3 = 23.1889, \quad \theta_1 = 8.5989, \quad \theta_2 = 8.3374,$$

$$\epsilon_1 = 8.7287, \quad \epsilon_2 = 8.7287, \quad \eta_1 = 6.3527, \quad \eta_2 = 6.3527$$

For $\mu = \lambda = h_1 = r_1 = 0$, by solving the LMIs (6)–(13) in Theorem 3.1, we not only obtain that the delayed stochastic BAM neural networks is globally exponentially stable in the mean square but also get the convergence rate. By solving an optimization problem, we get the maximum allowable upper bound time delay $\bar{\delta} = \bar{\tau} = h_2 = r_2 = 58.9874$ and the convergence rate is $\alpha = 1.1894$. This shows that the established results in this paper are new and finer when compared to the existing results [23].

Suppose, if we choose the initial values of system (36) as $[x_1(s), x_2(s)] = [\sin(s), \cos(s)]$, $s \in [-2, 0]$, $[y_1(s), y_2(s)] = [-0.5, \cos(s)]$, $s \in [-2, 0]$, then we can plot the trajectories of the BAM neural network (36) as shown in Fig. 1. The simulation results reveal that the system (36) is convergent to the equilibrium point quickly, and is stable at the equilibrium point. By using LMI Control toolbox in MATLAB, it is easy to check the feasibility of the solution within few seconds. The computational time is about minutes on a regular PC.

Fig. 1 Trajectories of the state variables $x_1(t), x_2(t), y_1(t), y_2(t)$ of the system (36)



5 Conclusion

The exponential stability problem for time delayed BAM neural networks with Markovian switching and impulses are being discussed in this paper. By making use of a novel Lyapunov functional and some new techniques, stability conditions are established to ensure the BAM neural network system stochastically stable. In particular, the stability conditions are presented in terms of linear matrix inequalities, which can be easily solved by some standard numerical packages. Finally, a numerical example has been provided to demonstrate the effectiveness of the proposed criteria.

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