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R&D Incentives and Market Structure: Dynamic Analysis[1]

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Abstract. We investigate dynamic R&D for process innovation in an oligopoly where firms invest in cost-reducing activities. We focus on the relationship between R&D intensity and market structure, proving that the industry R&D investment increases monotonically with the number of firms. This Arrowian result contradicts the established wisdom acquired from static games on the same topic.

Key Words. Differential games, optimal control, research and development.

1. Introduction

We propose a dynamic analysis of the relationship between market power and R&D efforts, in order to reassess a well-known issue in the theory of industrial organization, that can be traced back to the debate between Schumpeter (Ref. 1) and Arrow (Ref. 2). The so-called Schumpeterian hypothesis maintains that there exists an inverse relationship between the intensity of competition and the pace of technical progress. That is, according to Schumpeter, monopoly is the market structure that should ensure the fastest and largest technical progress. This relies upon the idea that monopoly ensures the highest profit level and therefore the larger internal sources for funding R&D activities. Exactly the

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opposite view is expressed by Arrow, since he focuses upon the replacement effect, according to which a monopolist is induced to rest on his laurels, while a firm operating in a competitive environment strives for new technologies or new products, in order to throw its rivals out of business. This debate generated a large body of literature, both theoretical and applied, seeking to discriminate between the Schumpeterian and the Arrowian views. As it could be expected from the outset, this contest has not yet found its ultimate winner.[4]

In order to assess this issue, we take a differential game perspective, proposing a dynamic version of a model first introduced in a static framework by d'Aspremont and Jacquemin (Ref. 6). We consider an oligopoly where n firms sell a homogeneous product and compete in quantities. Moreover, they invest also at each point in time in R&D for process innovation, i.e., reducing the marginal cost of production of the final good. The R&D activity is characterized by positive externalities, entailing that each firm receives a positive spillover from the investments carried out by all other firms in the industry.

Our model has the desirable property of being state-redundant or perfect, so that the open-loop solution is a Markovian equilibrium. We proceed in two steps. First, we characterize the individually optimal path of R&D investment for a given level of marginal production cost. Second, we obtain the steady-state levels of investment and marginal cost. With respect to both the optimal path and the steady-state level of R&D investment, the following conclusions hold. The individual effort is always decreasing in the number of firms, while the opposite holds for the aggregate R&D investments. This result has an Arrowian flavor, since as the degree of competition becomes tougher, the aggregate investment becomes larger. This is in sharp contrast with the conclusions drawn from the static version of the same model (Hinloopen, Ref. 7) where a nonmonotone relationship exists between aggregate R&D investment and market structure. Under this perspective, our model highlights the value added of a properly dynamic analysis over the static approach based upon a multistage game.

The remainder of the paper is structured as follows. Section 2 illustrates the basic setup. The solution of the open-loop game is investigated in Section 3, while the industry R&D performance is assessed in Section 4. Section 5 contains concluding remarks.

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[4] For an exhaustive overview of the related literature, see Tirole (Ref. 3), Reinganum (Ref. 4), and Martin (Ref. 5).

2. Setup

We consider an oligopoly with n firms selling homogeneous goods over a continuous time $t \in [0, \infty)$. At every instant, the market demand function is written as follows:

$$p(t) = A - q_i(t) - Q_{-i}(t), \tag{1}$$

where

$$Q_{-i}(t) \equiv \sum_{j \neq i} q_j(t)$$

is the output supplied by all firms other than i. Each firm supplies the market through a technology characterized by a constant marginal cost c_i. Accordingly, its instantaneous cost function for the production of the final good is

$$C_i(c_i(t), q_i(t)) = c_i(t) q_i(t).$$

The marginal cost borne by firm i evolves over time according to the following equation:

$$dc_i(t)/dt \equiv \dot{c}_i(t) = c_i(t)[-k_i(t) - \beta K_{-i}(t) + \delta], \tag{2}$$

where $k_i(t)$ is the R&D effort exerted by firm i at time t, while

$$K_{-i}(t) \equiv \sum_{j \neq i} k_j(t)$$

is the aggregate R&D effort of all other firms and the parameter $\beta \in [0, 1]$ measures the positive technological spillover that firm i receives from the R&D activity of the rivals. The parameter $\delta \in [0, 1]$ is a constant depreciation rate measuring the instantaneous decrease in productive efficiency due to the aging of technology. Equation (2) is indeed a dynamic version of the linear R&D technology employed by d'Aspremont and Jacquemin (Ref. 6).[5]

The instantaneous cost of running R&D activity is

$$\Gamma(k_i(t)) = b[k_i(t)]^2, \tag{3}$$

where b is a positive parameter. Throughout the game, the firms discount future profits at the common and constant discount rate $\rho > 0$.

The firms adopt a strictly noncooperative behavior in choosing both the output levels and the R&D efforts, each firm operating its own R&D

[5]This paper has generated a large body of literature. See Kamien et al. (Ref. 8), Suzumura (Ref. 9), and Amir (Ref. 10) inter alia.

division.[6] Define by $k(t), q(t), c(t)$ the vectors of controls and states. Then, the objective of firm i consists in maximizing the discounted profits

$$\pi_i(k_i(t),q(t),c_i(t)) \equiv \int_0^\infty \left\{[A-q_i(t)-Q_{-i}(t)-c_i(t)]q_i(t)-b[k_i(t)]^2\right\}e^{-\rho t}dt, \tag{4}$$

subject to the set of dynamic constraints (2), initial conditions $c(0)=\{c_{0i}\}$ and the appropriate transversality conditions, which are specified below. The corresponding Hamiltonian function is

$$\begin{aligned}\mathcal{H}_i(k(t), q(t), c(t)) = e^{-\rho t} \Bigg\{ & [A - q_i(t) - Q_{-i}(t) - c_i(t)]q_i(t) \\ & - b[k_i(t)]^2 - \lambda_{ii}(t)c_i(t)[k_i(t) + \beta K_{-i}(t) - \delta] \\ & - \sum_{j\neq i} \lambda_{ij}(t)c_j(t)\left[k_j(t)+\beta\left(k_i(t)+\sum_{l\neq i,j} k_l(t)\right)-\delta\right]\Bigg\},\end{aligned} \tag{5}$$

where

$$\lambda_{ij}(t) = \mu_{ij}(t)e^{\rho t}$$

is the costate variable (evaluated at time t) associated with the state variable $c_j(t)$.

3. Open-Loop Solution

Here, we characterize the Nash equilibrium under the open-loop information structure. As a first step, we prove the following result.

Lemma 3.1. The open-loop Nash equilibrium of the game is subgame (or Markov) perfect.

Proof. We are going to show that the present setup is a perfect game in the sense of Leitmann and Schmitendorf (Ref. 12) and Feichtinger (Ref. 13). In summary, a differential game is perfect whenever the closed-loop equilibrium collapses into the open-loop one, the latter being thus strongly

[6] For a discussion of R&D cooperation in the same model, see Cellini and Lambertini (Ref. 11).

time consistent, i.e., subgame perfect.[7] In building the proof, we use the same procedure as in Mehlmann (Ref. 17, Chapter 4), while the method adopted by Leitmann and Schmitendorf (Ref. 12) consists in verifying that the optimal control, of each player obtained from the first-order conditions, is indeed independent of the rival states. Consider the closed-loop information structure. The relevant first-order conditions (FOCs) are[8]

$$\partial \mathcal{H}_i / \partial q_i = A - 2q_i - Q_{-i} - c_i = 0, \tag{6}$$

$$\partial \mathcal{H}_i / \partial k_i = -2bk_i - \lambda_{ii} c_i - \beta \sum_{j \neq i} \lambda_{ij} c_j = 0. \tag{7}$$

As a first step, observe that (6) contains only the state variable of firm i; hence, in choosing the optimal output at any time during the game, firm i may disregard the current efficiency of the rival. That is, there is no feedback effect in the output choice. Conversely, at first sight, there seems to be a feedback between the R&D decisions, as (7) indeed contains all the state variables, at least for any positive spillover effect.[9] The core of the proof consists in showing that no feedback effect is actually present, even for positive spillover levels.

Taking the above considerations into account, the adjoint or costate equations are

$$-\partial \mathcal{H}_i / \partial c_i - \sum_{j \neq i} (\partial \mathcal{H}_i / \partial k_j) \cdot (\partial k_j^* / \partial c_i) = \partial \lambda_{ii} / \partial t - \rho \lambda_{ii}$$

$$\Leftrightarrow \partial \lambda_{ii} / \partial t = q_i + \lambda_{ii} [k_i + \beta K_{-i} + \rho - \delta] \tag{8}$$

$$-(\beta / 2b) \sum_{j \neq i} \lambda_{ji} \left[\beta \lambda_{ii} c_i + \lambda_{ij} c_j + \beta \sum_{l \neq i,j} \lambda_{il} c_l \right], \tag{9}$$

[7]The label "perfect game" is due to Fershtman (Ref. 14), where one can find a general technique to identify any such games. Other classes of games where open-loop equilibria are subgame perfect are investigated in Clemhout and Wan (Ref. 15) and Reinganum (Ref. 16). For further details, see Mehlmann (Ref. 17, Chapter 4) and Dockner et al. (Ref. 18, Chapter 7).

[8]Henceforth, we omit for brevity the indication of time and exponential discounting, except for the transversality conditions (14).

[9]Intuitively, if $\beta = 0$, then the investment plans are completely independent; therefore, it is apparent that no feedback effect operates.

$$
\begin{aligned}
&-\partial \mathcal{H}_i/\partial c_j - (\partial \mathcal{H}_i/\partial k_i)\cdot(\partial k_i^*/\partial c_j)\\
&-\sum_{l\neq i,j}(\partial \mathcal{H}_i/\partial k_l)\cdot(\partial k_l^*/\partial c_j) = \partial\lambda_{ij}/\partial t - \rho\lambda_{ij}\\
&\Leftrightarrow \partial\lambda_{ij}/\partial t = \lambda_{ij}\left[k_j + \beta k_i + \beta\sum_{l\neq i,j} k_l + \rho - \delta\right.\\
&\qquad\qquad\left. -(\beta/2b)\left(2bk_i + \lambda_{ii}c_i + \beta\sum_{j\neq i}\lambda_{ij}c_j\right)\right], \qquad (10)
\end{aligned}
$$

$$
-(\beta/2b)\sum_{l\neq i,j}\lambda_{lj}\left[\beta\lambda_{ii}c_i + \lambda_{il}c_l + \beta\sum_{j\neq i,l}\lambda_{ij}c_j\right], \qquad (11)
$$

where each term

$$
(\partial \mathcal{H}_i/\partial k_j)\cdot(\partial k_j^*/\partial c_i) \qquad (12)
$$

captures the feedback effect from j to i and the partial derivatives $\partial k_j^*/\partial c_i$ are calculated using the optimal values of the investments as from the FOC (7),

$$
k_j^* = -(\lambda_{jj}c_j + \beta\lambda_{ji}c_i)/2b. \qquad (13)
$$

These conditions must be evaluated along with the initial conditions $c(0) = \{c_{0i}\}$ and the transversality conditions

$$
\lim_{t\to\infty} e^{-\rho t}\lambda_{ij}\cdot c_j = 0, \quad \forall i, j. \qquad (14)
$$

Note that, on the basis of ex ante symmetry across firms,

$$
\lambda_{ij} = \lambda_{ij}, \quad \text{for all } l.
$$

Accordingly, from (11), we have

$$
\partial\lambda_{ij}/\partial t = 0
$$

in $\lambda_{ij} = 0$. Then, using this piece of information, we may rewrite the expression for the optimal investment of firm i as follows:

$$
k_i^* = -\lambda_{ii}c_i/2b, \qquad (15)
$$

which entails that

$$
\partial k_i^*/\partial c_j = 0, \quad \text{for all } j \neq i;
$$

i.e., feedback effects (cross effects) are nil along the equilibrium path. Accordingly, the open-loop equilibrium is strongly time consistent, or equivalently, subgame perfect. It is also worth observing that this procedure shows that the FOCs are indeed unaffected by the initial conditions as well. The property whereby the FOCs on the controls are independent of the states and initial conditions after replacing the optimal values of the costate variables is known as state-redundancy and the game itself as state redundant or perfect. □

On the basis of Lemma 3.1, we can proceed with the characterization of the open-loop solution. The FOCs on the controls as well as the transversality conditions are the same as above, while the costate equations simplify as follows:

$$-\partial\mathcal{H}_i/\partial c_i = \partial\lambda_{ii}/\partial t - \rho\lambda_{ii}$$

$$\Leftrightarrow \partial\lambda_{ii}/\partial t = q_i + \lambda_{ii}[k_i + \beta K_{-i} + \rho - \delta], \tag{16}$$

$$-\partial\mathcal{H}_i/\partial c_j = \partial\lambda_{ij}/\partial t - \rho\lambda_{ij}$$

$$\Leftrightarrow \partial\lambda_{ij}/\partial t = \lambda_{ij}[k_j + \beta K_{-j} + \rho - \delta]. \tag{17}$$

From the FOCs (6)–(7), we have, respectively,

$$q_i^* = (A - Q_{-i} - c_i)/2, \tag{18}$$

$$k_i = -\lambda_{ii} c_i/2b, \tag{19}$$

since

$$\lambda_{ij} = 0, \qquad \text{for all } j \neq i, \text{ at any } t \in [0, \infty).$$

While (18) has the usual appearance of a standard Cournot best reply function, the optimal R&D effort in (19) depends upon the ith costate variable. Such expression can be differentiated w.r.t. time to get the dynamic equation of k_i,

$$dk_i/dt \equiv \dot{k}_i = -(1/2b)\left[c_i\dot{\lambda}_{ii} + \lambda_{ii}\,\dot{c}_i\right], \tag{20}$$

with $\dot{\lambda}_{ii}$ resulting from (9). Then, (20) can be further simplified by using

$$\lambda_{ii} = -2bk_i/c_i, \tag{21}$$

which results from (7). This yields

$$\dot{k}_i = -(1/2b)[c_i q_i - 2bk_i]. \tag{22}$$

The next step consists in imposing the symmetry conditions

$$c_j = c_i,\ \ k_j = k_i,\ \ q_j = q_i,\ \ \text{for all } j,$$

and solve the system of the best reply functions (18), yielding the Cournot-Nash output level of each firm,

$$q^N = (A - c)/(n + 1), \tag{23}$$

which can be inserted into (22). Accordingly, we may simplify the dynamics of the R&D effort of any single firm as follows:

$$\dot{k} = -(1/2b)\,[c(A - c)/(n + 1) - 2b\rho k]. \tag{24}$$

Imposing the stationarity condition $\dot{k} = 0$, we obtain

$$k^N = c[A - c]/2b\rho(n + 1) \geq 0, \qquad \text{for all } c \in [0, A], \tag{25}$$

where the superscript N stands for Nash equilibrium.

The steady-state level of the marginal cost $c(t)$ can be found by solving:

$$\dot{c} = -c\left[k^N(1 + \beta(n - 1)) - \delta c\right] = 0, \tag{26}$$

which yields $c = 0$ and

$$c = \frac{A(1 + \beta(n - 1)) \pm \sqrt{(1 + \beta(n - 1))[A^2(1 + \beta(n - 1)) - 8b\delta\rho(n + 1)]}}{2(1 + \beta(n - 1))}. \tag{27}$$

All solutions in (27) are real if and only if

$$\delta\rho \leq A^2(1 + \beta(n - 1))/[8b(n + 1)].$$

If so, they satisfy also the requirement $c \in [0, A]$. By checking the stability conditions, we can prove the following proposition.

Proposition 3.1. Provided that $\delta\rho \leq A^2(1 + \beta(n - 1))/[8b(n + 1)]$, the steady-state point

$$c^{ss} = \frac{A(1 + \beta(n - 1)) - \sqrt{(1 + \beta(n - 1))[A^2(1 + \beta(n - 1)) - 8b\delta\rho(n + 1)]}}{2(1 + \beta(n - 1))},$$

$$k^{ss} = \delta/[1 + \beta(n - 1)],$$

is the unique saddle-point equilibrium.

Proof. Under symmetry, the dynamics of the control and state variables is written as in (24) and (26). Accordingly, the relevant Jacobian matrix is

$$J = \begin{bmatrix} \partial\dot{c}/\partial c & \partial\dot{c}/\partial k \\ \partial\dot{k}/\partial c & \partial\dot{k}/\partial k \end{bmatrix}, \tag{28}$$

whose trace and determinant are

$$\mathrm{Tr}(J) = \delta + \rho - k\Omega, \tag{29}$$

$$\Delta(J) = \rho(\delta - k\Omega) - c(A - 2c)\Omega/2b(n+1), \tag{30}$$

where

$$\Omega \equiv 1 + \beta(n-1),$$

$$\Phi \equiv A^2\Omega - 8b\delta\rho(n+1).$$

Then, it can be checked easily that the pair

$$c = \left[A\Omega - \sqrt{\Omega\Phi}\right]/2\Omega, \quad k = \delta/\Omega$$

is the only solution yielding $\Delta(J) < 0$ always, while the other two steady state points are both unstable. □

4. Comparative Statics

Now, we focus on the interplay between the market structure (as measured by the number of firms) and the industry incentive to invest in process R&D. To this aim, we examine the effect of a change in n on the individual and aggregate R&D efforts, both along the equilibrium path [expression (25)] and in steady state.

This discussion revisits the debate between Schumpeter (Ref. 1) and Arrow (Ref. 2). Their respective views can be summarized as follows. According to the Schumpeterian hypothesis, R&D investments and technical progress are positively related to the flow of profits; therefore, we should expect to observe higher R&D efforts and a faster innovation process under monopoly than under any other market form. Conversely, Arrow claims that the incentive to generate technical progress is negatively affected by the market power, being then maximized under perfect competition. The Arrowian position relies upon the idea that innovation is more attractive for a competitive firm than for a monopolist who, by definition, cannot improve his market power.

In order to assess this issue in the present model, we proceed as follows. The aggregate R&D investments along the equilibrium path and in steady state are respectively

$$K^N = c[A-c]n/2b\rho(n+1),$$

$$K^{ss} = \delta n/[1+\beta(n-1)]. \tag{31}$$

It is immediate to verify that

$$\partial K^N/\partial n = 2b\rho c[A-c]/4[b\rho(n+1)]^2 > 0, \tag{32a}$$

$$\partial K^{ss}/\partial n = \delta(1-\beta)/[1+\beta(n-1)]^2 \geq 0. \tag{32b}$$

The above properties prove the main result of our model.

Proposition 4.1. The optimal R&D investment of the whole industry is monotonically increasing in the number of firms. This holds both along the equilibrium path and in steady state.

This entails that the behavior of the industry is clearly Arrowian. If instead we examine the individual investment, we obtain

$$\partial k^N(t)/\partial n < 0, \partial k^{ss}/\partial n < 0, \quad \text{everywhere.}$$

This entails that any increase in the number of firms brings about a decrease in individual R&D effort. This is caused by two facts: on the one hand, tougher market competition reduces profits and therefore the funds available to any given firm for conducting R&D activity; on the other hand, a larger population of firms means a larger amount of positive externality that any firm receives from the rivals. On the aggregate, the scale effect prevails, so that the overall expenditure of the industry is monotonically increasing in n.[10]

Hinloopen (Ref. 7) has solved the oligopoly equilibrium with n firms in the static case, finding that both aggregate and individual R&D efforts are nonmonotone (first increasing and then decreasing) w.r.t. n. Under this scenario, the static approach proves to fall short of appropriately accounting for the inherently dynamic nature of research and development, which is not captured by multistage game modeling.

[10]For a similar result concerning the incentives toward R&D for product innovation, see Cellini and Lambertini (Refs. 19–20).

5. Concluding Remarks

We have analyzed dynamic R&D investments for cost-reducing innovation in a Cournot oligopoly in order to evaluate the influence of the market structure on R&D incentives.

The set up employed in the present paper is a dynamic version of a well-known static game examined in d'Aspremont and Jacquemin (Ref. 6), which has generated many follow-ups. Two features of our analysis are worth stressing. First, the game is perfect or state redundant, so that the open-loop solution is Markovian or subgame perfect. Second, a clearcut Arrowian conclusion obtains, since the aggregate R&D effort is everywhere increasing in the number of firms, this being true along the equilibrium path as well as in steady state. The drastic difference between our results and the ambiguous conclusions drawn from the static model relies upon smoothing the investment efforts over a long time horizon, a perspective which is ruled out by definition in a static setting.

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