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CORRECTION

Correction to: Trace Functions with Applications in Quantum Physics

Frank Hansen[1]

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The original version of this article unfortunately contained an error and it has been corrected with this erratum. We consider in [5, p. 810 and Theorem 2.2] the following results which we here put together.

Theorem 1 *The functions*

$$f(t) = (t^p + 1)^{1/p} \qquad t > 0$$

are operator monotone (thus operator concave) for $0 < p \leq 1$ *and operator convex for* $1 \leq p \leq 2$.

The first part ($0 < p \leq 1$) is proved on page 810. Note that the statement is different from Ando [1, Corollary 4.3]. The second part ($1 \leq p \leq 2$) is our [5, Theorem 2.2]. The perspective of an operator concave (convex) function is again an operator concave (convex) function of two variables, cf. [3, Theorem 1.1]. By applying [4, Theorem 1.1] these results entail that for arbitrary K the trace functions

$$(A, B) \to \operatorname{Tr} K^*\bigl(L_A^p + R_B^p\bigr)^{1/p}(K), \tag{1}$$

are concave for $0 < p \leq 1$ and convex for $1 \leq p \leq 2$. We then try to use these results to recover Carlen-Lieb's theorems in [2].

Theorem 2 (Carlen-Lieb) *The trace functions*

$$(A, B) \to \operatorname{Tr}(A^p + B^p)^{1/p} \tag{2}$$

are concave for $0 < p \leq 1$ *and convex for* $1 \leq p \leq 2$.

The original article can be found online at https://doi.org/10.1007/s10955-013-0890-x.

✉ Frank Hansen
frank.hansen@math.ku.dk

1 Institute for International Education, Tohoku University, Sendai, Japan

We do this by claiming the identity

$$\mathrm{Tr}\left(L_A^p + R_B^p\right)^{1/p}(I) = \mathrm{Tr}\,(A^p + B^p)^{1/p}, \tag{3}$$

where the left hand side is obtained by setting $K = I$ (the identity operator) in (1). By replacing A with $A^{1/p}$ and B with $B^{1/p}$ this would then entail that

$$\mathrm{Tr}\left(L_A + R_B\right)^{1/p}(I) = \mathrm{Tr}\,(A + B)^{1/p}$$

for positive definite matrices. This however is wrong. Victoria Chayes informed me in a private communication that for $p = 1/4$ the difference

$$\mathrm{Tr}\left(L_A^p + R_B^p\right)^{1/p}(I) - \mathrm{Tr}\,(A^p + B^p)^{1/p} = \|AB - BA\|_{\mathrm{HS}}^2 \geq 0,$$

and the difference therefore vanishes if and only if A and B commute.

The failed identity in (3) is of no consequence here since we already know that Carlen-Lieb's theorem is true.

Later in the paper [5, Theorem 3.1] we prove that the functions of two variables

$$g(t,s) = \frac{t-s}{t^p - s^p} \qquad t, s > 0$$

are operator concave for $0 < p \leq 1$, and this implies that the trace functions

$$(A, B) \to \mathrm{Tr}\, K^* \frac{L_A - R_B}{L_A^p - R_B^p}(K) \qquad 0 \leq p \leq 1$$

are concave for arbitrary K. We then put $K = I$ and try to use an identity similar to (3) to obtain [5, Theorem 3.2] which claims that the trace functions

$$(A, B) \to \mathrm{Tr}\, \frac{A - B}{A^p - B^p} \qquad 0 < p \leq 1 \tag{4}$$

are concave in positive definite matrices. This claim is no longer verified. In fact, numerical calculations indicate that the trace functions in (4) are concave only for $p = 1/2$ and not concave for $0 < p < 1/2$ and $1/2 < p < 1$.

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