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CORRECTION

Correction to: Non-commutative Calculus, Optimal Transport and Functional Inequalities in Dissipative Quantum Systems

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The claimed bound $\mathrm{Ric}(\mathcal{A}, \nabla, \tau) \geq \gamma$ in Theorem 10.6 in our paper [1] is unfortunately incorrect, as pointed out in [2]. A small modification of the proof shows that the weaker estimate $\mathrm{Ric}(\mathcal{A}, \nabla, \tau) \geq \frac{\gamma}{2}$ holds.

This bound can be obtained by replacing (10.5) by the following computation, using the scalar inequalities $\partial_1\Lambda(a, b), \partial_2\Lambda(a, b) \geq 0$ for $a, b > 0$:

$$\begin{aligned}
\mathrm{Hess}_{\mathscr{K}}\, \mathrm{Ent}(\rho)[A, A] &= -\tau[(\nabla \mathscr{L} A)^* \widehat{\rho} \# \nabla A] + \tau\big[(\nabla A)^* \mathcal{N}^{(\eta)}_{\rho, \mathscr{L}^\dagger \rho} \# (\nabla A)\big] \\
&= \frac{\gamma}{2} \tau\big[(\nabla A)^* (\Lambda + \partial_1 \Lambda + \partial_2 \Lambda)(\rho, \rho) \# (\nabla A)\big] \\
&\geq \frac{\gamma}{2} \tau\big[(\nabla A)^* \Lambda(\rho, \rho) \# (\nabla A)\big] \\
&= \frac{\gamma}{2} \tau[(\nabla A)^* \widehat{\rho} \# \nabla A] = \frac{\gamma}{2} \langle \mathscr{K}_\rho A, A \rangle_{L^2(\tau)} .
\end{aligned}$$

On the matrix algebra $\mathbb{M}_n(\mathbb{C})$, it is possible to improve the curvature bound $\frac{\gamma}{2}$ to $\frac{\gamma}{2}(1 + \frac{1}{n})$, using the scalar inequality $\partial_1\Lambda(a, b) + \partial_2\Lambda(a, b) \geq 1 \geq \frac{1}{n}\Lambda(a, b)$ for $0 < a, b \leq n$.

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