Journal of Statistical Physics, Vol. 126, No. 6, March 2007 (© 2007)
DOI: 10.1007/s10955-007-9294-0

Book Review

Fractional Calculus in Bioengineering. Richard L. Magin, Begell House Publ., Conn. (2006)

The fractional calculus began with Leibniz in 1695 with his answer to a question concerning the 0.5-power derivative of a monomial raised by L'Hospital in a letter. Given this long history it is curious that its development has remained largely in the shadows compared with that of its first cousin the ordinary calculus co-invented with Newton. It has been argued that the fractional calculus remains relatively invisible because of its lack of scientific application. It is only within the last couple of decades that the scientific community as a whole has begun to investigate phenomena that are sufficiently complex that the ordinary calculus and analytic functions are no longer adequate for their description. Examples of such complex phenomena discussed by Magin in his delightful book are biological membranes, viscoelastic materials, certain electrochemical processes and biomedical phenomena.

Historically research engineers did not have the freedom enjoyed by physicists to choose their research topics because in addition to scientific rigor they had to satisfy the additional constraint of practicality. How to determine the response of viscoelastic materials to applied stress was one such area for which constitutive equations were constructed in the absence of a microscopic theory. In the 19th Century it was found that the stress-strain relations in such materials are given by certain integro-differential constitutive equations. Somewhat later it was determined that these integrals were often representations of fractional differentials and the integro-differential equations describing the stress-strain relation were actually fractional differential equations; a direct generalization of the traditional stress-relaxation equations. The solution to such fractional stress-relaxation equations were shown to be Mitag-Leffler functions that are stretched exponential at early times and inverse power law at late times. This remarkable function and how it generalizes the notion of the exponential in a variety of complex phenomena is discussed at length in Magin's book.

One of the key aspects of this book is its readability. For example, the major tool of analysis is the Laplace transform, used to solve the fractional equations of motion by taking the inverse Laplace transform of series with non-integer exponents. Not only does the author explain the formalism of the fractional calculus in a manner accessible to undergraduate students, but also continuously presents data to exemplify the mathematical features of the calculus being discussed. In this way the arguments presented not only make contact with the mathematical literature on modeling, but also with the research literature addressing the applications, as well.

Among the data presented and discussed are stress-relaxation and strain-creep time series for a variety of materials; capacitor-charging current; voltage pulses in electrophysiology; frequency response of motoneurons; neurodynamic response of the semi-circular canal to impulse excitation; and stress response of a porcine aortic valve cusp to applied period strain. In each of these cases and more, Magin discusses that aspect of the fractional calculus necessary to explain the data; whether it is an inverse power law in time and/or frequency, a new kind of transfer function, or a non-local effect in space. The fractional time derivative leads to long-term memory in the process being consider; the fractional spatial derivative leads to a non-local effect in space and the two together describe the distributed effects of such phenomena as heat transfer in heterogeneous materials, electrical conduction along an axon, and anomalous diffusion.

This book is clearly designed for the student, with each chapter ending with from ten to thirty problems, along with answers. The problems range in difficulty from the strictly pedagogical to bordering on research topics, but they are fun to work on. A student that masters the problem sets will have a modern tool kit with which to address new research areas in bioengineering and biophysics. Among the new topics discussed are models of fractional impedance, fractional dielectrics, fractional kinetics and fractional-order control of biomedical phenomena.

I strongly recommend this book not only for students in bioengineering and biophysics, but also to the researcher who has been attempting to understand the mystifying results of many experiments involving complex biomedical and biophysical phenomena. Reading this book may be more useful to the researcher than reading a number of articles that use outmoded methods of analysis to understand complexity in the physical and life sciences.

Bruce J. West
Mathematical & Information Science
US Army Research Office
Research Triangle Park, NC 27709