

# **The impact of overtime as a time-based proactive scheduling and reactive allocation strategy on the robustness of a personnel shift roster**

**Jonas Ingels1 · Broos Maenhout<sup>1</sup>**

Published online: 13 March 2017 © Springer Science+Business Media New York 2017

**Abstract** The workforce size and the overtime budget have an important impact on the total personnel costs of an organisation. Since the personnel costs represent a significant fraction of the operating costs, it is important to define an appropriate hiring and overtime policy. Overtime is defined as an extension of the daily working time or the total working time over the planning period. In this paper, we make the distinction between scheduled and unscheduled overtime when we define the overtime policy. Scheduled overtime is proactively assigned in the baseline personnel roster whereas unscheduled overtime is allocated as a reactive strategy to overcome operational disruptions. The hiring and overtime policy undoubtedly influence the robustness included in a personnel roster, i.e., the capability of an organisation to deal with roster disruptions at an acceptable cost. In this paper, we investigate the trade-off between the hiring budget and the overtime budget and the way overtime should be allocated in the personnel management process. The latter comprehends a trade-off between the proactive scheduling of overtime and the reservation of overtime to balance supply and demand in response to operational variability. Insights are obtained by exploring three different strategies to compose a personnel shift baseline roster. We verify the robustness of each of these strategies by applying a three-step methodology that thrives on optimisation and simulation and we formulate some managerial guidelines to define an appropriate hiring and overtime policy.

 $\boxtimes$  Broos Maenhout Broos.Maenhout@UGent.be

> Jonas Ingels Jonas.Ingels@UGent.be

**Keywords** Personnel scheduling · Robustness · Hiring policy · Proactive and reactive overtime policy

# **1 Introduction**

In organisations, the personnel cost is typically one of the largest operating costs [\(Ernst et al. 2004;](#page-22-0) [Van den Bergh et al.](#page-22-1) [2013](#page-22-1)). An appropriate personnel planning process is therefore indispensable to manage these costs. The use of overtime is one strategy that is frequently applied in different phases of the personnel planning process to reduce the costs. In the higher-level hierarchical phases of staff budgeting and personnel scheduling, there is a clear interrelationship between the hiring strategy and the overtime strategy as the number of hired employees defines both the degree in which overtime is required and the ability to include overtime in the personnel roster [\(Li and Li 2000](#page-22-2)). A lower number of hired employees may lead to a higher number of overtime duties to satisfy the staffing requirements. This practice impacts the flexibility and ability to make decisions in the lower-level operational planning phase as the structure of a line-of-work becomes more rigid. In the latter phase, operational variability arises and the (deterministic) assumptions made in higher-level phases are not able to perfectly represent the operating environment. As a result, the personnel planner should cope with these schedule disruptions on a day-by-day basis by changing the timing and duration of working duties, i.e., by reassigning employees and by assigning overtime ad hoc. [Van den Bergh et al.](#page-22-1) [\(2013\)](#page-22-1) identified three different sources of variability, i.e., uncertainty of demand, uncertainty of arrival and uncertainty of capacity. Uncertainty of demand reflects the variability in the actual demand for employees while the uncertainty of arrival entails the variability in the

<sup>&</sup>lt;sup>1</sup> Faculty of economics and business administration, Ghent University, Tweekerkenstraat 2, 9000 Gent, Belgium

timing of the actual demand. Unexpected absenteeism and sick leave represent the uncertainty of capacity.

Coping with uncertainty is of major importance in the personnel planning process, which is typically decomposed into the strategic staffing phase, the tactical scheduling phase and the operational allocation phase [\(Abernathy et al. 1973](#page-21-0); [Burke et al. 2004\)](#page-22-3). These phases are usually treated separately and the decisions taken in each phase constrain the decision freedom in the subsequent phase(s).

The *strategic staffing phase* represents long-term decisions about the personnel mix and budget, which entail a trade-off between the number of hired employees, the level of cross-training of the employees and overtime. This trade-off has a considerable impact on the organisation's capability to cope with the uncertainty clouding the future operating environment, i.e., the employee availability and personnel demand. This uncertainty is especially large given the long-term nature of the decisions. In this perspective, [Koutsopoulos and Wilson](#page-22-4) [\(1987](#page-22-4)) and [Tan](#page-22-5) [\(2003\)](#page-22-5) define an optimal size of additional staff to respond to uncertainty. A large workforce size results in decreased productivity, while a small size leads to overtime at high marginal cost. According to [Li and Li](#page-22-2) [\(2000](#page-22-2)), the personnel budget should consider the personnel mix and employee cross-training as the availability of cross-trained employees offers a certain leeway to respond to uncertainty.

In the *tactical scheduling phase*, the personnel planner acquires more up-to-date but yet uncertain information on the future operating environment. As such, assumptions are made about the employee availability and personnel demand. Given these assumptions and the strategic personnel budgeting decisions, a duty roster is constructed for a medium-term period. In this respect, the number and mix of employees not only determine the extent to which the demand can be covered but also the extent to which proactive scheduling strategies can be included. These strategies improve the robustness by anticipating uncertainty through the introduction of methods that enable the personnel planner to cope with uncertainty. Common strategies include the definition of resource buffers such as capacity and time buffers or the application of unit crewing.

Capacity buffers exist in terms of preferred requirements and reserve duties. Preferred requirements introduce extra requirements on top of the minimal staffing requirements [\(Dowsland and Thompson 2000](#page-22-6)[;](#page-22-7) De Causmaecker and Vanden Berghe [2003](#page-22-7); [Topaloglu and Selim 2010\)](#page-22-8). Similarly, reserve duties can be introduced by defining specific reserve duty staffing requirements and time-related constraints [\(Ingels and Maenhout 2015](#page-22-9)). Alternatively, capacity buffers can be created through a maximisation of resource substitution possibilities [\(Shebalov and Klabjan](#page-22-10) [2006;](#page-22-10) [Abdelghany et al. 2008](#page-21-1)).

Time buffers increase the time between two consecutive tasks and are typically applied in project management and personnel task scheduling problems to avoid delay propagation. The size of the time buffer between tasks is a general robustness indicator [\(Tam et al. 2011\)](#page-22-11) and the value of a time buffer depends [on](#page-22-12) [the](#page-22-12) [expected](#page-22-12) [delay](#page-22-12) [of](#page-22-12) [the](#page-22-12) [previous](#page-22-12) [task](#page-22-12) [\(](#page-22-12)Ehrgott and Ryan [2002](#page-22-12)). A third option combines capacity buffers with time buffers in terms of overtime. Overtime can pertain to a complete shift or an extension of a shift. The introduction of overtime shifts results in a higher capacity during those shifts and augments the total assignment time for employees. In general, it is possible to distinguish between prescheduled fixed overtime and prescheduled on-call overtime in the tactical scheduling phase [\(Campbell 2012\)](#page-22-13). A last proactive scheduling strategy focuses on the teams assigned to a sequence of tasks. Unit crewing ensures that employee teams stay together as long as possible over a sequence of tasks [\(Tam et al. 2014\)](#page-22-14). This way unit crewing ensures that after the conclusion of a task, the dismantlement of the assigned team is avoided. Team changes certainly contribute to delay propagation by the dependency they create for multiple tasks waiting for employees assigned to an earlier task.

The outcome of the tactical scheduling phase represents the baseline roster in the *operational allocation phase*. In the latter phase, the personnel planner monitors the execution of the baseline roster while accounting for the most recent information, which is subject to a small or no level of uncertainty. It may be possible that the actual employee availability and demand differ from the previously postulated assumptions. As a result of these disruptions, adjustments are required to ensure the feasibility and workability of the personnel roster. Organisations have a number of more or less expensive options at their disposal to deal reactively with disruptions, e.g., the conversion of reserve duties or days off to working duties, resource substitution and overtime. Note that the availability of less expensive options can be facilitated through the application of proactive scheduling strategies during the tactical scheduling phase.

A general recovery action in the airline industry is the conversi[on](#page-21-2) [of](#page-21-2) [reserve](#page-21-2) [duties](#page-21-2) [into](#page-21-2) [working](#page-21-2) [duties](#page-21-2) [\(](#page-21-2)Abdelghany et al. [2004](#page-21-2); [Thiel 2005](#page-22-15); [Abdelghany et al. 2008](#page-21-1)). The study of [Ingels and Maenhout](#page-22-9) [\(2015\)](#page-22-9) indicates that this type of conversion significantly contributes to the resolution of disruptions for shift scheduling problems. The exchange of a day off into a working duty, in contrast, is not equally efficient because of its negative impact on the personal lives of emplo[yees](#page-22-17) [\(Bard and Purnomo 2005](#page-22-16)[;](#page-22-17) Schalk and Van Rijckevorsel [2007;](#page-22-17) [Camden et al. 2011\)](#page-22-18).

A low-cost solution to overcome disruptions is the substitution of resources by means of a swapping mechanism [\(Shebalov and Klabjan 2006;](#page-22-10) [Abdelghany et al. 2008](#page-21-1)) or a reassignment. A swap is defined as a duty exchange between employees and a reassignment is a change in the shift and/or skill an employee is originally assigned to.

The assignment of unscheduled overtime also enables the personnel plan[ner](#page-22-19) [to](#page-22-19) [respond](#page-22-19) [to](#page-22-19) [operational](#page-22-19) [variability](#page-22-19) [\(](#page-22-19)Easton and Rossin [1997\)](#page-22-19). This reactive strategy augments the work duration for employees on a single day and/or over all days. In this perspective, overtime offers some limited flexibility through a change in the shift duration by extending a shift with an overtime period [\(Bard and Purnomo 2005\)](#page-22-16).

In this paper, we define different time-based proactive and reactive strategies, which include overtime, in order to improve the robustness of a personnel shift roster. For all strategies, we study their impact on both the absorption and adjustment capability of the personnel shift roster in the operational allocation phase. The proactive scheduling strategy consists of the possibility of assigning employees to overtime during a prior shift extension and a subsequent shift extension or to a complete shift during the construction of the deterministic baseline roster. Since these shift extensions may increase the daily working time, the employees may receive a smaller number of daily assignments. In this perspective, we investigate the trade-off between the workforce size and overtime included in the deterministic baseline personnel roster, in relation to the quality of the personnel roster that is subject to operational variability. As it is not possible to ensure a perfect absorption of the operational variability at a reasonable cost based on the proactively scheduled overtime, a reactive allocation decision model is formulated to make adaptations to the baseline personnel roster in the short-term operational allocation phase. In this reactive decision model, we include the strategy to assign employees to unscheduled overtime as an extension of the daily and total working time. However, the ability to reactively introduce unscheduled overtime strongly depends on the total overtime that has been proactively scheduled. As more overtime is scheduled before the start of the operational allocation phase, the opportunity to use overtime as a reactive allocation strategy decreases. In this context, we investigate the trade-off between scheduling overtime in the baseline personnel roster and allocating overtime in response to operational variability in the operational allocation phase.

We evaluate the trade-offs and the performance of the different strategies using a three-step methodology, which is similar to the approaches of [Bard and Purnomo](#page-22-16) [\(2005](#page-22-16)), [Abdelghany et al.](#page-21-1) [\(2008\)](#page-21-1) and [Ingels and Maenhout](#page-22-9) [\(2015](#page-22-9)). Based on the performance evaluation, we formulate managerial guidelines about the required workforce size and the different overtime strategies and highlight their impact on the personnel roster robustness.

The remainder of this paper is organised as follows. In Sect. [2,](#page-2-0) we describe the context of the problem under study. In Sect. [3,](#page-4-0) we discuss the research methodology by formulating the different optimisation and simulation models and by defining the time-based proactive and reactive strategies. In Sect. [4,](#page-11-0) we define the test design and discuss the computational experiments and results. Conclusions are given in Sect. [5.](#page-21-3)

# <span id="page-2-0"></span>**2 Problem description**

In this paper, we focus on different time-based strategies to include overtime in the personnel shift roster and investigate their impact on the personnel roster robustness. A personnel roster is robust if it is both stable and flexible when disruptions occur [\(Ionescu and Kliewer 2011\)](#page-22-20). Roster stability describes the degree to which personnel rosters can absorb disruptions. Roster flexibility refers to the capability of a roster to react efficiently to disruptions. Both these aspects of robustness are embedded in the way the personnel budget and a personnel roster are constructed. Roster flexibility is further supported by the way a personnel planner can respond to imbalances in supply and demand. In order to improve the roster robustness, specific proactive scheduling and reactive allocation strategies may need to be adopted. A general overview of the problem context is provided in figure [1](#page-3-0) which is explained below.

# *The baseline personnel roster*

For the problem under study, we first determine the personnel budget and a baseline personnel shift roster simultaneously. The integration of the strategic staffing phase and the tactical scheduling phase allows the realisation of a tradeoff between the hiring budget and the overtime budget. The baseline roster assigns the personnel members on each day to a working shift or to a day off. The standard shift duties have a fixed start time and a fixed duration and they are defined based upon non-overlapping demand periods of four hours. As such, each day is comprised of six demand periods in total. In Fig. [1,](#page-3-0) we define three standard shift duties with a fixed duration of eight hours that cover two consecutive demand periods, i.e., shift 1 (start time: 12 p.m.), shift 2 (start time: 8 a.m.) and shift 3 (start time: 4 p.m.). In order to investigate the impact of time-based strategies on the robustness of a personnel shift roster, we distinguish different types of overtime that extend the working time per employee as follows

– *Daily working time extension*: The extension of a standard shift duty increases the daily working time, which is defined as overtime. This type of overtime is the result of a *prior shift extension* or a *subsequent shift extension*. A prior shift extension and a subsequent shift extension, respectively, include the demand period immediately before and after the demand periods corresponding to a standard shift duty assigned to an employee. Figure [2](#page-3-1) displays the resulting duty types for a particular day *d*. Each of the standard shift duties can be extended



**Fig. 1** Problem description

<span id="page-3-0"></span>

<span id="page-3-1"></span>**Fig. 2** Illustration of the daily working time extensions

with a prior shift extension and a subsequent shift extension. These prior and subsequent shift extensions of a standard shift comprehend an overlap with the corresponding previous and next standard shift duties. The prior shift extension of shift 1 and the subsequent shift extension of shift 3 overlap with day  $d-1$  and day  $d+1$ , respectively.

– *Total working time extension*: The assignment of a worker to a standard shift duty may extend the total working time on top of the maximum number of regular working hours a worker may be allocated to. This type of overtime extends the total working time of a single employee. Note that a daily working time extension may also comprehend an extension of the total working time.

The baseline personnel roster is constructed based on a set of inputs and parameters concerning the personnel budget policy, the performance measures and the constraints (cf. Fig. [1\)](#page-3-0), i.e.,

- The personnel budget policy is characterised by the employee hiring policy and the overtime policy. These policies determine the characteristics of the personnel mix hired, the budget for overtime hours versus the number of regular hours and the amount of scheduled overtime hours to compose the baseline personnel roster. Hence, these inputs determine the balance between the workforce size employed and the number of overtime hours included in the baseline personnel roster.
- The objective function of this integrated staffing and scheduling problem involves the minimisation of different personnel costs, i.e., a wage cost for regular and overtime duties and an understaffing cost.
- In order to construct the personnel roster, staffing requirements and time-related constraints are imposed. The staffing requirements stipulate the number of required workers per demand period. On top of these regular staffing requirements, extra staffing requirements may be imposed that indicate the number of workers that need to be assigned to a prior shift extension or a subsequent shift extension. As such, these extra staffing requirements enable the inclusion of a capacity buffer to proactively anticipate operational variability.

The output of this integrated staffing and scheduling problem is a baseline personnel roster that stipulates the set of employees hired with a number of scheduled regular and overtime duties. The total scheduled overtime determines the overtime budget that has already been consumed and that is no longer available in the operational allocation phase.

# *Reactive balancing of personnel demand and supply*

The baseline personnel roster is then input to the operational allocation phase (cf. Fig. [1\)](#page-3-0). In this phase, we reconsider the baseline roster on a day-by-day basis. Each day, the organisation obtains more accurate information about the employee availability and the staffing requirements per demand period. This information may differ from the assumptions and predictions made in the staffing and scheduling phase. Adjustments may be necessary in the operational allocation phase to restore the workability and/or the feasibility of the baseline roster. In order to enable a good balance between the demand and supply of employees during each demand period, we can apply different reactive allocation strategies, i.e.,

- The *reassignment* of the regular and overtime working duties scheduled in the baseline personnel shift roster is the common reactive allocation strategy that is applied for schedule recovery. Adjustments to the baseline roster are allowed as long as all time-related constraints remain satisfied.
- The *conversion* of a day off to a working duty that is required to cover the demand for staff.
- The allocation of *unscheduled overtime* is a time-based reactive allocation strategy, which improves the reactive flexibility through assigning overtime duties.
- The *cancellation* of regular or overtime duties that are superfluous on top of the actual demand for staff.

# <span id="page-4-0"></span>**3 Methodology**

The objective in this paper is to determine the impact of timebased proactive and reactive strategies on the robustness of personnel shift rosters. For this purpose, we use a methodology that consists of three steps:

- In the first step, a baseline personnel roster and the staffing budget are determined by integrating *the strategic staffing phase and the tactical scheduling phase*. In this step, different proactive time-based strategies are introduced in the baseline personnel roster to hedge against operational variability (Sect. [3.1\)](#page-5-0).
- In the second step, we start from the baseline personnel shift roster and imitate the *operational allocation phase* (Sect. [3.2\)](#page-8-0), which includes a day-by-day simulation of the operational variability and an adjustment decision model to balance the supply and demand for staff (cf. Fig. [1\)](#page-3-0). The adjustment component includes the application of reactive allocation strategies to complement the proactive scheduling strategies. This step is executed multiple times to obtain an accurate idea of the impact of the applied time-based proactive and reactive strategies.

– In the third step, we *evaluate the robustness* of the baseline personnel shift roster by assessing its planned and actual performance (Sect. [3.3\)](#page-11-1).

This methodology of validating robustness through an imitation of the operational phase is similar to the approach of [Bard and Purnomo](#page-22-16) [\(2005\)](#page-22-16), [Abdelghany et al.](#page-21-1) [\(2008](#page-21-1)) and [Ingels and Maenhout](#page-22-9) [\(2015](#page-22-9)) and is discussed in detail in the following sections.

Note that we repeat this methodology for multiple baseline personnel shift rosters, which differ based on the applied proactive and reactive strategies. As such, we can determine the baseline personnel shift roster and the corresponding proactive and reactive strategies that provide the highest level of robustness.

# <span id="page-5-0"></span>**3.1 The integrated strategic staffing and tactical scheduling phase**

We study a personnel shift scheduling problem, which entails the assignment of employees to working shifts over a planning period of multiple days subject to general personnel information, objectives and constraints [\(Burke et al. 2004](#page-22-3); [Van den Bergh et al. 2013](#page-22-1)[\).](#page-22-9) [In](#page-22-9) [contrast](#page-22-9) [to](#page-22-9) Ingels and Maenhout [\(2015\)](#page-22-9), we integrate the strategic staffing decision in the tactical scheduling phase in order to simultaneously determine the personnel budget and a baseline personnel roster [\(Maenhout and Vanhoucke 2013\)](#page-22-21). The overall problem under investigation can be categorised as *AS*1|*RV* |*S*||*LXRG* according t[o](#page-22-22) [the](#page-22-22) [classification](#page-22-22) [of](#page-22-22) De Causmaecker and Vanden Berghe [\(2011](#page-22-22)). Given the high level of uncertainty, we assume that this problem is stochastic in terms of the personnel demand and employee availability.

In the integrated staffing and scheduling step, the workforce size and the allowance of overtime are interrelated by the stipulated time-based robustness strategy. In principle, the number of hired employees is a proactive scheduling strategy since a buff[er](#page-22-4) [against](#page-22-4) [uncertainty](#page-22-4) [is](#page-22-4) [created](#page-22-4) [\(](#page-22-4)Koutsopoulos and Wilson [1987](#page-22-4)). However, since personnel costs significantly contribute to the total operating costs of an organisation [\(Ernst et al. 2004;](#page-22-0) [Van den Bergh et al. 2013](#page-22-1)), organisations may opt to limit the workforce size in favour of overtime [\(Lobo et al. 2013\)](#page-22-23). Hence, there is a trade-off between the workforce size and the budget for overtime expenses in the integrated strategic staffing and tactical scheduling phase. In this perspective, the number of overtime hours employees can work has a significant impact on the number of employees that need to be hired and the applicability of time-based robustness strategies. A lower number of allowed overtime hours may increase the required workforce size to cover the staffing requirements and vice versa. The mathematical formulation of the integrated staffing and scheduling problem under study is the following:

# **Notation** *Sets*

- *N* set of employees (index *i*)
- *D* set of days (index *d*)
- *H* set of demand periods per day (index *h*)
- *S* set of shifts (index *j*)
- *S*− set of shifts for which the prior shift extension covers a demand period on the previous day
- *S*+ set of shifts for which the subsequent shift extension covers a demand period on the next day
- $T_{dj}^{\circ|o}$  set of shifts that cannot be assigned the day after day *d* and shift *j* (index *s*)
- $T_{dj}^{\circ|'}$  set of prior shift extensions that cannot be assigned after day *d* and shift *j* (index *s*) with  $T_{dj}^{\circ|o} \cap T_{dj}^{\circ|'} = \emptyset$
- $T_{dj}^{\circ}$  set of subsequent shift extensions that cannot be assigned after day *d* and shift *j* (index *s*) with  $(T_{dj}^{\circ|o} \cup$  $T_{dj}^{\circ |'}$ )  $\cap T_{dj}^{\circ |''} = \varnothing$
- $T_{dj}^{''|o}$  set of shifts that cannot be assigned the day after a subsequent shift extension on day *d* and shift *j* (index *s*)
- *T*  $\frac{\eta}{\eta}$  $\overline{\phantom{a}}$ set of prior shift extensions that cannot be assigned the day after a subsequent shift extension on day *d* and shift *j* (index *s*) with  $T_{dj}^{''|0} \cap T_{dj}^{''|'} = \emptyset$
- $T_{dj}^{''\parallel''}$  set of subsequent shift extensions that cannot be assigned the day after a subsequent shift extension on day *d* and shift *j* (index *s*) with  $(T''_{dj} \cup T''_{dj} \cap T''''_{dj} \cap T''''_{dj} \neq \emptyset$

*Deterministic parameters*

- *lh* duration of a demand period *h*
- $\beta_j$  number of demand periods in shift *j*
- $\beta'_{j}$  number of demand periods in the prior shift extension of shift *j*
- $\beta_j^{\prime\prime}$  number of demand periods in the subsequent shift extension of shift *j*
- $z_{jh}$  1 if shift *j* covers demand period *h*, 0 otherwise
- $z'_{jh}$  1 if the prior shift extension of shift *j* covers demand period *h*, 0 otherwise
- $z_{jh}^{''}$  1 if the subsequent shift extension of shift *j* covers demand period *h*, 0 otherwise
- *c*<sup>w</sup> hourly regular wage cost
- *c*w*<sup>o</sup>* hourly overtime wage cost
- *c*w*<sup>u</sup>* cost for understaffing a demand period
- $R_{dh}^w$  expected staffing requirement for demand period *h* on day *d*
- *R*w,extra *dh* extra staffing requirement for demand period *h* on day *d*
	- $a_{id}$  expected availability of employee *i* on day *d*, 1 if the employee is available and 0 otherwise
	- $l_{id}^{\max}$ maximum number of regular and overtime hours that can be assigned to employee *i* on day *d*
- $l_{id}^{w, \max}$ maximum number of regular hours that can be assigned to employee *i* on day *d*
- $l_{id}^{wo, \text{max}}$ maximum number of overtime hours that can be assigned to employee *i* on day *d*
- $\eta_i^{\min}$ *<sup>i</sup>* minimum number of regular and overtime hours that have to be assigned to employee *i*
- $\eta_i^{w, \text{max}}$ maximum number of regular hours that can be assigned to employee *i*
- $\eta_i^{wo, \max}$ *<sup>i</sup>* maximum number of overtime hours that can be assigned to employee *i*
- $\theta_i^{w, \text{max}}$ maximum number of consecutive assignments for employee *i*

# *Stochastic parameters*

 $\tilde{R}_{dh}^w$  stochastic staffing requirement for demand period *h* on day *d*

# **Mathematical formulation**

 $\tilde{a}_{id}$  stochastic availability of employee *i* on day *d*; 1 if the employee is available, 0 otherwise

#### *Variables*

- $\zeta_i$  1 if employee *i* is hired, 0 otherwise
- $x_{i,j}^w$  1 if employee *i* is assigned to shift *j* on day *d*, 0 otherwise
- $x_{i,j}^{wo}$  1 if employee *i* is assigned to overtime during a complete shift *j* on day *d*, 0 otherwise
- $x_{idj}^{wo'}$ 1 if employee *i* is assigned to overtime during a prior shift extension of shift *j* on day *d*, 0 otherwise
- $x_{i,j}^{wo''}$ 1 if employee  $i$  is assigned to overtime during a subsequent shift extension of shift *j* on day *d*, 0 otherwise
	- $x_{id}^v$  1 if employee *i* receives a day off on day *d*, 0 otherwise
- <span id="page-6-8"></span><span id="page-6-7"></span><span id="page-6-6"></span><span id="page-6-5"></span><span id="page-6-4"></span><span id="page-6-3"></span><span id="page-6-2"></span><span id="page-6-1"></span><span id="page-6-0"></span> $n_{dh}^{wu}$ the number of employees short during demand period *h* on day *d*
- Minimise *i*∈*N*  $\sum$ *d*∈*D*  $\sum$ *j*∈*S*  $l_h\beta_j c^w x_{i d j}^w + \sum$ *d*∈*D*  $\sum$ *h*∈*H*  $c^{wu}n_{dh}^{wu}$ *dh* (1a)

$$
+\sum_{i\in N}\sum_{d\in D}\sum_{j\in S}l_h\beta_j c^{wo}x_{idj}^{wo}+\sum_{i\in N}\sum_{d\in D}\sum_{j\in S}l_h\beta_j' c^{wo}x_{idj}^{wo'}+\sum_{i\in N}\sum_{d\in D}\sum_{j\in S}l_h\beta_j' c^{wo}x_{idj}^{wo''}
$$
(1b)

subject to

$$
\sum_{i \in N} \sum_{j \in S} \tilde{a}_{id} z_{jh} (x_{idj}^w + x_{idj}^{wo}) + \sum_{i \in N} \sum_{j \in S \setminus S^-} \tilde{a}_{id} z'_{jh} x_{idj}^{wo'} + \sum_{i \in N} \sum_{j \in S^-} \tilde{a}_{id} z'_{jh} x_{id+1j}^{wo'} + \sum_{i \in N} \sum_{j \in S^+} \tilde{a}_{id} z''_{jh} x_{id-1j}^{wo''} + n_{dh}^{wa} \ge \tilde{R}_{dh}^w + R_{dh}^{w, extra} \qquad \forall d \in D, \forall h \in H
$$
 (2)

$$
\sum_{j \in S} \left( x_{idj}^w + x_{idj}^{wo} \right) + x_{id}^v = \zeta_i \qquad \qquad \forall i \in N, \forall d \in D \qquad (3)
$$

$$
x_{i,j}^{wo'} \le x_{i,j}^w + x_{i,j}^{wo}
$$
\n
$$
\forall i \in N, \forall d \in D, \forall j \in S \tag{4a}
$$

$$
x_{i,j}^{wo''} \le x_{i,j}^w + x_{i,j}^{wo}
$$
  $\forall i \in N, \forall d \in D, \forall j \in S$  (4b)

$$
x_{i,j}^w + x_{i,j}^{wo} + \sum_{s \in T_{dj}^{\circ l'}} x_{id+1s}^{wo'} + \sum_{s \in T_{dj}^{\circ l}\circ} (x_{id+1s}^w + x_{id+1s}^{wo}) + \sum_{s \in T_{dj}^{\circ l'}} x_{id+1s}^{wo''} \le 1 \qquad \forall i \in N, \forall d \in D, \forall j \in S \qquad (5a)
$$

$$
x_{idj}^{wo''} + \sum_{s \in T_{dj}^{n'j'}} x_{id+1s}^{wo'} + \sum_{s \in T_{dj}^{n'j}} (x_{id+1s}^{w} + x_{id+1s}^{wo}) + \sum_{s \in T_{dj}^{n'j''}} x_{id+1s}^{wo''} \le 1 \qquad \forall i \in N, \forall d \in D, \forall j \in S \quad (5b)
$$

<span id="page-7-5"></span>
$$
\sum_{j \in S} (h_{i}\beta_{j} (x_{i,j}^{w} + x_{i,j}^{wo}) + l_{i}\beta_{j}'x_{i,j}^{wo'} + l_{i}\beta_{j}^{w}x_{i,j}^{wo''}) \leq l_{id}^{\max}
$$
\n
$$
\sum_{j \in S} l_{i}\beta_{j}x_{i,j}^{w} \leq l_{id}^{w,\max}
$$
\n
$$
\sum_{j \in S} (l_{i}\beta_{j}x_{i,j}^{wo} + l_{i}\beta_{j}'x_{i,j}^{wo'} + l_{i}\beta_{j}'x_{i,j}^{wo''}) \leq l_{id}^{w,\max}
$$
\n
$$
\sum_{j \in S} (l_{i}\beta_{j}x_{i,j}^{wo} + l_{i}\beta_{j}'x_{i,j}^{wo'} + l_{i}\beta_{j}'x_{i,j}^{wo''}) \leq l_{id}^{w,\max}
$$
\n
$$
\sum_{d \in D} \sum_{j \in S} (l_{i}\beta_{j}(x_{i,j}^{w} + x_{i,j}^{wo}) + l_{i}\beta_{j}'x_{i,j}^{wo''}) \geq \eta_{i}^{\min}\zeta_{i}
$$
\n
$$
\sum_{d \in D} \sum_{j \in S} (l_{i}\beta_{j}x_{i,j}^{wo} \leq \eta_{i}^{w,\max}\zeta_{i}
$$
\n
$$
\sum_{d \in D} \sum_{j \in S} (l_{i}\beta_{j}x_{i,j}^{wo} \leq \eta_{i}^{w,\max}\zeta_{i}
$$
\n
$$
\sum_{d \in D} \sum_{j \in S} (l_{i}\beta_{j}x_{i,j}^{wo} \leq \eta_{i}^{w,\max}\zeta_{i}
$$
\n
$$
\sum_{d \in D} \sum_{j \in S} (l_{i}\beta_{j}x_{i,j}^{wo} \leq \eta_{i}^{w,\max}\zeta_{i}
$$
\n
$$
\sum_{d \in D} \sum_{j \in S} (l_{i}\beta_{j}x_{i,j}^{wo} \leq \eta_{i}^{w,\max}\zeta_{i}
$$
\n
$$
\sum_{d \in D} \sum_{i \in N, \forall d \in D} (9)
$$
\n
$$
\zeta_{i} \in \{0, 1\}
$$
\n
$$
\forall i \in N, \forall d
$$

We solve a deterministic version of this stochastic model formulation by assuming that the stochastic staffing requirements equal the expected staffing requirements ( $\tilde{R}^w_{dh} = R^w_{dh}$ ) and that the employees are available on each day  $(\tilde{a}_{id}$  =  $a_{id} = 1$ ). We obtain a baseline personnel shift roster by solving this model with the commercial software package Gurobi [\(Gurobi Optimization 2015\)](#page-22-24).

The general objective in model  $(1-10)$  $(1-10)$  is to minimise the employee wage costs and the cost for understaffing particular demand periods. Equation [\(1a\)](#page-6-1) minimises the wage costs for assigning workers to regular duties and the understaffing costs. Equation [\(1b\)](#page-6-2) minimises the cost for assigning overtime during complete shifts and during a prior shift extension or a subsequent shift extension.

In order to construct a workable personnel shift roster, a specified number of employees should be scheduled for every demand period [Eq. [\(2\)](#page-6-3)]. The staffing requirements include the minimum staffing requirements for working duties ( $\tilde{R}_{dh}^w$ ) and the staffing requirements for extra working duties ( $R_{dh}^{\hat{w}, \text{extra}}$ ). Note that the staffing requirements  $R_{dh}^{w, \text{extra}}$ are only employed for certain time-based buffer strategies

<span id="page-7-8"></span><span id="page-7-7"></span><span id="page-7-6"></span><span id="page-7-4"></span><span id="page-7-3"></span><span id="page-7-2"></span><span id="page-7-1"></span><span id="page-7-0"></span>(cf. Sect. [4\)](#page-11-0). The staffing constraints are relaxed since understaffing is allowed.

We impose different time-related constraints on the schedule of a single employee. Equation [\(3\)](#page-6-4) stipulates that each hired employee receives an assignment on each day. A prior and subsequent daily shift time extension of a regular duty can only be assigned to employees who work the corresponding shift duty [Eq.  $(4a)$  and  $(4b)$ ]. Moreover, a minimum rest period between two consecutive duties is imposed by the constraints  $(5a)$  and  $(5b)$ . Equation  $(5a)$  is the common constraint that prohibits certain duty assignments to succeed a particular working duty. This constraint does not completely ensure the satisfaction of the minimum rest period since a shift can be extended by an overtime subsequent shift extension, which is considered by Eq.  $(5b)$ . Notice that the succession constraint that restricts the type of duty that follows a prior shift extension is implicitly modelled by constraint  $(5a)$ . Furthermore, every employee can work a maximum number of hours per day  $[Eq. (6a)]$  $[Eq. (6a)]$  $[Eq. (6a)]$ . This maximum is further refined into two constraints that impose a maximum on the number of regular hours per day [Eq. [\(6b\)](#page-7-2)] and a maximum on

the number of overtime hours per day  $[Eq, (6c)]$  $[Eq, (6c)]$  $[Eq, (6c)]$ . We also impose constraints on the number of hours assigned over the total planning period. Every hired employee has to work a minimum number of hours  $[Eq. (7)]$  $[Eq. (7)]$  $[Eq. (7)]$ . Equation  $(8)$  imposes a maximum on the number of hours an employee can work. Constraints [\(8a\)](#page-7-6) and [\(8b\)](#page-7-7), respectively, impose a maximum on the number of regular hours and overtime hours for a single employee over the total planning period. Irrespective of the duration of the assignments, Eq. [\(9\)](#page-7-8) ensures that the number of consecutive working duties is limited for every employee.

Equation [\(10\)](#page-7-0) defines the integrality conditions for each variable.

#### <span id="page-8-0"></span>**3.2 Operational allocation phase**

The operational allocation phase considers the baseline personnel roster on a day-by-day basis and comprehends a discrete-event simulation component (Sect. [3.2.1\)](#page-8-1) and an adjustment component (Sect. [3.2.2\)](#page-8-2). The baseline personnel shift roster is subject to operational variability, and we simulate the uncertainty of demand and capacity by making use of particular stochastic distributions (cf. Fig. [1\)](#page-3-0). As a result, the supply and demand for staff may be imbalanced and schedule changes may be required to restore the workability and/or feasibility of the personnel shift roster. In order to obtain meaningful and statistically significant results, we repeat this day-by-day process of simulations and adjustments 100 times. As such, we investigate 100 different scenarios that may occur in reality for each baseline personnel shift roster.

#### <span id="page-8-1"></span>*3.2.1 Simulation of operational variability*

The discrete-event simulation component simulates the uncertainty of demand  $(R'_{dh}^{w})$  and capacity  $(a_{id})$  for each demand period on the day under consideration. We imitate this operational variability using the GNU Scientific Library [\(Gough 2009](#page-22-25)) and assume that the simulated parameters  $R'_{dh}$ <sup>'w</sup> and *aid* represent real-time or perfect information that is no longer subject to uncertainty.

## **Uncertainty of demand**

The uncertainty of demand is simulated for every demand period of the day under consideration. This means that the demand for employees is independent between demand periods, which is based on the assumption of an exogenous demand characterised by independent and identically distributed inter-arrival and service times [\(de Bruin et al. 2010](#page-22-26); [Paul and Lin 2012](#page-22-27)). Hence, the demand can differ between demand periods within a single standard shift. In order to avoid too large differences in our simulation experiment and to control the demand variability in general, we impose a lower and upper bound on the possible values for the staffing requirements (Eqs.  $(11)$  and  $(12)$ ). In these equations, the variability is expressed with a parameter  $(\sigma)$  that ranges from one (low variability) to  $+\infty$  (high variability). Given the ass[umption](#page-22-28) [that](#page-22-28) [the](#page-22-28) [demand](#page-22-28) [is](#page-22-28) [Poisson-distributed](#page-22-28) [\(](#page-22-28)Yeh and Lin [2007;](#page-22-28) [Ahmed and Alkhamis 2009](#page-21-4)[;](#page-22-9) Ingels and Maen-hout [2015](#page-22-9)), we simulate the demand uncertainty and accept the simulated staffing requirements if equation  $(13)$  is satisfied.

$$
LB_h = R_{dh}^w - \sigma \qquad \qquad \forall h \in H \tag{11}
$$

$$
UB_h = R_{dh}^w + \sigma \qquad \qquad \forall h \in H \tag{12}
$$

<span id="page-8-5"></span><span id="page-8-4"></span><span id="page-8-3"></span>
$$
R_{dh}^{w} \in [LB_h, UB_h] \qquad \forall h \in H \tag{13}
$$

#### **Uncertainty of capacity**

We simulate the uncertainty of capacity to imitate the employee availability. The parameter *aid* indicates whether an employee *i* is available to work on day  $d (a_{id} = 1)$  or not  $(a_{id} = 0)$ . Based on the probability of absenteeism  $(P_{id}(X =$ 0)), we obtain the value for *aid* by applying a Bernoulli distribution for each employee. The probability  $P_{id}(X = 0)$ depends on the number of days the employee has already bee[n](#page-22-9) [unexpectedly](#page-22-9) [absent](#page-22-9) [before](#page-22-9) [day](#page-22-9) *d* (Ingels and Maen-hout [2015](#page-22-9)). When this number increases for an employee, the probability for absenteeism on day *d* decreases.

#### <span id="page-8-2"></span>*3.2.2 Balancing supply and demand*

As a result of the new information obtained by the simulation of supply and demand, the personnel planner needs to evaluate whether the day roster needs to be adjusted. These adjustments are guided by the reactive allocation strategies, which include the allocation of unscheduled overtime [\(Bard and Purnomo 2005\)](#page-22-16). Moreover, we consider cancellations and r[eassignments](#page-22-16) [\(Abdelghany et al. 2008](#page-21-1)[;](#page-22-16) Bard and Purnomo [2005;](#page-22-16) [Ingels and Maenhout 2015](#page-22-9)), and conversions of a day off to a working duty [\(Bard and Purnomo](#page-22-16) [2005](#page-22-16)). Based upon the reactive allocation strategies, we formulate a deterministic mathematical decision model for the operational allocation phase that considers a planning period of a single day. This means that only the assignments corresponding to the six demand periods that cover the day under consideration are included in the decision model (cf. Fig. [2\)](#page-3-1). The assignments for the other days are considered to be fixed. The mathematical formulation of the operational allocation problem under study is given below. Note that we only define those sets, parameters and variables that are specific to the operational allocation phase to avoid duplication.

# **Notation**

*Sets*

- *Bid* set of shifts that cannot be assigned to employee *i* on day *d* (index *j*)
- $B_{id}^{'}$  set of shifts for which the prior shift extension cannot be assigned to employee  $i$  on day  $d$  (index  $j$ )
- $B_{id}^{''}$  set of shifts for which the subsequent shift extension cannot be assigned to employee *i* on day *d* (index *j*)
- $T_{d+1j}^{'''}$  set of subsequent shift extensions that cannot be assigned the day before a prior shift extension on day  $d + 1$  and shift *j* (index *s*)

#### *General parameters*

- *d* day under consideration in the operational planning horizon
- *M* a large number
- $y_{i d j}^{\alpha'}$  1 if employee *i* is allowed to receive a prior shift extension corresponding to shift  $j \in S^-$  on day  $d + 1$ , 0 otherwise
- $y_{i d j}^{\alpha''}$  1 if employee *i* is allowed to receive a subsequent shift extension corresponding to shift  $j \in S^+$  on day *d* − 1, 0 otherwise
- *y*<sup>*f*</sup><sub>*idj*</sub> 1 if employee *i* is forced to work shift *j* ∈ *S*<sup>−</sup> on day *d*, 0 otherwise
- $y_{i,d}^{\min}$ the total number of hours employee  $i$  is forced to work on day *d*
- $y_{id}^{w, \text{max}}$ the maximum number of regular hours employee *i* can work on day *d*
- $y_{id}^{wo, \text{max}}$ the maximum number of overtime hours employee *i* can work on day *d*

# **Mathematical formulation**

- $a_{id}$  1 if employee *i* is available on day *d*, 0 otherwise
- $R'_{dh}$  simulated staffing requirement for demand period *h* on day *d*

#### *Roster change parameters*

- $\bar{x}_{idj}^w$  1 if employee *i* was assigned to shift *j* on day *d* in the baseline personnel roster, 0 otherwise
- $\bar{x}_{idj}^{wo}/\bar{x}_{idj}^{wo'}/\bar{x}_{idj}^{wo''}$ 1 if employee *i* was assigned to overtime during shift *j*/prior shift extension of shift *j*/subsequent shift extension of shift *j* on day *d* in the baseline personnel roster, 0 otherwise
	- $\bar{x}_{id}^v$  1 if employee *i* received a day off on day *d* in the baseline personnel roster, 0 otherwise
- <span id="page-9-1"></span><span id="page-9-0"></span> $c_{i,j}^{\omega\delta}/c_{i,j}^{\omega\delta'}/c_{i,j}^{\omega\delta''}$  roster change cost for assigning employee *i* to shift *j*/prior shift extension of shift *j*/subsequent shift extension of shift *j* on day *d* with  $c_{i d j}^{w \delta} > 0$  if  $\bar{x}_{i d j}^{w} + \bar{x}_{i d j}^{w o} = 0$ and  $c_{i d j}^{w \delta} = 0$  otherwise, with  $c_{i d j}^{w \delta'} > 0$  if  $\bar{x}_{idj}^{wo'} = 0$  and  $c_{idj}^{w\delta'} = 0$  otherwise, with  $c_{i d j}^{w \delta''} > 0$  if  $\bar{x}_{i d j}^{w o''} = 0$  and  $c_{i d j}^{w \delta''} = 0$  otherwise
	- $c_{id}^v$  cancellation cost for employee *i* on day *d* with  $c_{id}^v > 0$  if  $\bar{x}_{id}^v = 0 \wedge a_{id} = 1$  and  $c_{id}^v = 0$  otherwise

Minimise  $\sum$ i∈N  $\sum$ j∈S  $\left(\left(\mathbf{l}_\mathrm{h}\beta_\mathrm{jc} \mathrm{c}^{\mathrm{w}} + \mathrm{c}_{\mathrm{id} \mathrm{j}}^{\mathrm{w}\delta}\right) \mathrm{x}_{\mathrm{id} \mathrm{j}}^{\mathrm{w}} + \left(\mathbf{l}_\mathrm{h}\beta_\mathrm{jc}^{\mathrm{w}\mathrm{o}} + \mathrm{c}_{\mathrm{id} \mathrm{j}}^{\mathrm{w}\delta}\right) \mathrm{x}_{\mathrm{id} \mathrm{j}}^{\mathrm{w}\mathrm{o}}\right) + \sum$ i∈N  $c_{\rm id}^{\rm v}x_{\rm id}^{\rm v}+\sum$ h∈H cwunwu dh  $+\sum$ *i*∈*N*  $\sqrt{5}$ *j*∈*S*\*S*<sup>−</sup>  $\left(l_h \beta'_j c^{wo} + c_{i d j}^{w \delta'}\right) x_{i d j}^{w o'} + \sum$ *j*∈*S*<sup>−</sup>  $\left(l_h \beta'_j c^{wo} + c_{id+1j}^{w\delta'}\right) x_{id+1j}^{wo'}$  $\setminus$  $+\sum$ *i*∈*N*  $\sqrt{2}$ *j*∈*S*\*S*<sup>+</sup>  $\left(l_h \beta_j^{\prime\prime} c^{wo} + c_{i d j}^{w \delta^{\prime\prime}}\right) x_{i d j}^{w o^{\prime\prime}} + \sum$ *j*∈*S*<sup>+</sup>  $\left(l_h \beta''_j c^{wo} + c^{w\delta''}_{id-1j}\right) x^{wo''}_{id-1j}$  $\setminus$ (14)

subject to

$$
\sum_{i \in N} \sum_{j \in S} z_{jh} \left( x_{idj}^w + x_{idj}^{wo} \right) + \sum_{i \in N} \sum_{j \in S \setminus S^-} z'_{jh} x_{idj}^{wo'} + \sum_{i \in N} \sum_{j \in S^-} z'_{jh} x_{id+1j}^{wo'} + \sum_{i \in N} \sum_{j \in S \setminus S^+} z''_{jh} x_{idj}^{wo''} + \sum_{i \in N} \sum_{j \in S^+} z''_{jh} x_{id-1j}^{wo''} + n_{dh}^{wu} \ge R_{dh}^{iw} \qquad \forall h \in H
$$
\n(15)

<span id="page-10-1"></span>
$$
\sum_{j \in S} \left( x_{idj}^w + x_{idj}^w \right) + x_{id}^v = 1 \tag{16}
$$

$$
x_{i,j}^{wo'} \le x_{i,j}^w + x_{i,j}^{wo}
$$
  $\forall i \in N, \forall j \in S \setminus S^-$  (17a)

<span id="page-10-3"></span><span id="page-10-2"></span>
$$
x_{id+1j}^{wo'} \le y_{idj}^{\alpha'} \left(1 - \sum_{s \in \underline{T}_{d+1j}^{(v')}} x_{ids}^{wo''}\right)
$$
  $\forall i \in N, \forall j \in S^-$  (17b)

<span id="page-10-4"></span>
$$
x_{i,j}^{wo''} \le x_{i,j}^w + x_{i,j}^{wo}
$$
 (17c)

<span id="page-10-5"></span>
$$
x_{id-1j}^{wo''} \le y_{idj}^{\alpha''} \left( 1 - \sum_{s \in T_{d-1j}^{"'} } x_{ids}^{wo'} \right) \qquad \forall i \in N, \forall j \in S^+ \tag{17d}
$$

$$
\sum_{j \in B_{id}} \left( x_{idj}^w + x_{idj}^{wo} \right) + \sum_{j \in B'_{id}} x_{idj}^{wo'} + \sum_{j \in B''_{id}} x_{idj}^{wo''} = 0
$$
\n
$$
\forall i \in N \tag{18}
$$

$$
\sum_{j \in S} (x_{idj}^w + x_{idj}^{wo}) + \sum_{j \in S \setminus S^-} x_{idj}^{wo'} + \sum_{j \in S^-} x_{id+1j}^{wo'} + \sum_{j \in S \setminus S^+} x_{idj}^{wo''} + \sum_{j \in S^+} x_{id-1j}^{wo''} \leq Ma_{id} \qquad \forall i \in N
$$
 (19)

$$
x_{i,j}^w + x_{i,j}^w \ge y_{i,j}^f a_{i,d} \qquad \qquad \forall i \in N, \forall j \in S^-
$$
 (20)

$$
\sum_{j \in S} \left( l_h \beta_j \left( x_{idj}^w + x_{idj}^{wo} \right) + l_h \beta_j' x_{idj}^{wo'} + l_h \beta_j'' x_{idj}^{wo''} \right) \ge y_{id}^{\min} a_{id} \qquad \forall i \in N \tag{21}
$$

$$
\sum_{j \in S} \left( l_h \beta_j \left( x_{i,j}^w + x_{i,j}^{wo} \right) + l_h \beta_j' x_{i,j}^{wo'} + l_h \beta_j'' x_{i,j}^{wo''} \right) \le l_{id}^{\max} \qquad \forall i \in N \tag{22a}
$$

$$
\sum_{j \in S} l_h \beta_j x_{i,j}^w \le y_{i,j}^{w, \max} \qquad \qquad \forall i \in N \tag{22b}
$$

$$
\sum_{j \in S} \left( l_h \beta_j x_{idj}^{wo} + l_h \beta_j' x_{idj}^{wo'} + l_h \beta_j'' x_{idj}^{wo''} \right) \le y_{id}^{wo, \max} \qquad \qquad \forall i \in N \qquad (22c)
$$

$x_{idj}^{w}$ $\in$ {0, 1}	$\forall i \in N, \forall j \in S$
$x_{idj}^{wo'} \in \{0, 1\}$	$\forall i \in N, \forall j \in S$
$x_{idj}^{wo'} \in \{0, 1\}$	$\forall i \in N, \forall j \in S$
$x_{idj}^{wo''} \in \{0, 1\}$	$\forall i \in N, \forall j \in S \setminus S^{-}$
$x_{idj}^{wo''} \in \{0, 1\}$	$\forall i \in N, \forall j \in S^{-}$
$x_{idj}^{wo''} \in \{0, 1\}$	$\forall i \in N, \forall j \in S \setminus S^{+}$
$x_{id}^{wo''} \in \{0, 1\}$	$\forall i \in N, \forall j \in S \setminus S^{+}$
$x_{id}^{wo''} \in \{0, 1\}$	$\forall i \in N, \forall j \in S^{+}$
$x_{id}^{w}$ $\in \{0, 1\}$	$\forall i \in N$
$x_{id}^{w}$ $\in \{0, 1\}$	$\forall i \in N$
$x_{id}^{w}$ $\in \{0, 1\}$	$\forall i \in N$
$x_{id}^{w}$ $\in \{0, 1\}$	$\forall i \in N$

We assign regular shift duties, overtime duties and days off by solving this operational allocation model [Eqs. [\(14–](#page-9-0) [23\)](#page-10-0)] with the commercial optimisation package Gurobi [\(Gurobi Optimization 2015\)](#page-22-24). The objective function [Eq.

<span id="page-10-12"></span><span id="page-10-11"></span><span id="page-10-10"></span><span id="page-10-9"></span><span id="page-10-8"></span><span id="page-10-7"></span><span id="page-10-6"></span><span id="page-10-0"></span>[\(14\)](#page-9-0)] minimises the actual total cost that arises on day *d* in the operational allocation phase. This cost is comprised of the wage cost for regular and overtime duties, the change and cancellation cost for duties assigned in the baseline per-

sonnel roster and the cost for understaffing particular demand periods.

The staffing requirements  $[Eq. (15)]$  $[Eq. (15)]$  $[Eq. (15)]$  postulate that a sufficient number of employees is present during each demand period, which ensures that the workers are assigned to the right demand periods to cover the demand for staff. This constraint is relaxed as understaffing is allowed, which is penalised in the objective function. Note that the staffing requirements  $R_{dh}^{w}$  in this operational decision model are the result of the simulated operational variability and can differ from the expected demand for staff  $(R_{dh}^w)$ used to construct the baseline personnel roster ( $\tilde{R}_{dh}^w$  = *R*<sup>w</sup> *dh*).

The other constraints embody time-related rules imposed on the duty roster of a single employee. Equation  $(16)$  stipulates that each employee needs to receive either a shift assignment or a day off on day *d*. A standard shift duty may be extended with a prior shift extension [Eq.  $(17a)$ ,  $(17b)$ ] or a subsequent shift extension [Eq.  $(17c, (17d))$  $(17c, (17d))$  $(17c, (17d))$  $(17c, (17d))$ ] only if the worker is assigned to the corresponding standard shift. Equation  $(17a)$  and  $(17c)$  regulate the daily shift extensions corresponding to standard shifts on day *d*. Equation [\(17b\)](#page-10-3) and [\(17d\)](#page-10-5), respectively, determine whether a prior shift extension corresponding to a shift on day  $d + 1$  and a subsequent shift extension corresponding to a shift on day *d* − 1 can be assigned on day *d*. Moreover, these equations ensure that we do not violate the minimum rest period between assignments that cover demand periods on day *d*. However, we also need to impose the minimum rest period between assignments that cover demand periods on different days. Constraint [\(18\)](#page-10-6) ensures that the sequence constraints imposed on the baseline personnel roster (cf. Eqs.  $(5a)$ ,  $(5b)$  and  $(9)$ ) are satisfied given the assignments on the other days of the planning horizon by the definition of the sets  $B_{id}$ ,  $B'_{id}$  and  $B''_{id}$ . Equation [\(19\)](#page-10-7) imposes that an employee can only receive work duties when he is available. The parameter *aid* indicates for each employee whether this employee is available to perform a working duty on day *d* and is the result of the simulated operational variability. Given that the prior shift extension corresponding to a standard shift on day *d* can cover a demand period on day  $d - 1$ , an employee should be assigned to that standard shift if this extension was assigned to that employee on day  $d - 1$  [Eq. [\(20\)](#page-10-8)]. Based on the assignments on the other days, each employee needs to work a minimum number of hours on day *d* if available [Eq. [\(21\)](#page-10-9)] in order to respect the minimum number of working hours over the complete planning horizon. Equation [\(22a\)](#page-10-10) limits the number of hours an employee can work on one particular day. Equation [\(22b\)](#page-10-11), [\(22c\)](#page-10-12) impose a restriction upon the allowed number of regular and overtime hours on day *d* in order to satisfy the maximum number of regular and overtime duty hours over the complete planning horizon. Constraint [\(23\)](#page-10-0) defines the integrality conditions of the variables that



<span id="page-11-2"></span>**Fig. 3** Building blocks of the planned and actual performance

correspond to demand periods that lie within the planning period.

## <span id="page-11-1"></span>**3.3 Robustness evaluation**

We evaluate the personnel shift rosters based on their planned and actual performance. Figure [3](#page-11-2) gives a brief overview of the different components of both performance measures.

The planned performance evaluates the outcome after the integrated strategic staffing and tactical scheduling phase and reflects the planned cost, the number of hired employees and the existent overstaffing of the baseline personnel roster. The planned cost comprises the understaffing cost and the total assignment cost, i.e., the wage cost and the overtime wage cost.

The actual performance assesses the outcome of the operational allocation phase and evaluates the actual cost, the overstaffing and the overtime utilisation of the eventual personnel roster. The actual cost includes the cost for understaffing, the change cost and the total assignment cost, i.e., the wage cost, the overtime wage cost and the cancellation cost. The overtime utilisation comprises the scheduled overtime utilisation and the unscheduled overtime duties. The scheduled overtime utilisation reports the percentage of scheduled overtime periods that are actually utilised in the operational allocation phase. Furthermore, we identify the number of demand periods during which unscheduled overtime is reactively allocated.

#### <span id="page-11-0"></span>**4 Computational experiments**

In this section, we provide insight into our computational experiments and the robustness of the time-based proactive and reactive strategies. We describe our test design and parameter settings in Sect. [4.1](#page-12-0) and outline the different experiments and their results in Sect. [4.2.](#page-12-1) All tests were carried out on an Intel Core processor 2.5 GHz and 4 GB RAM.

# <span id="page-12-0"></span>**4.1 Test design**

In this section, we describe the parameter settings of the test instances for our computational experiments. Note that all test instances have a planning period of 7 d.

#### *Shift characteristics*

Each day consists of six demand periods with a duration of 4 h  $(l_h)$ . Hence, each day contains three non-overlapping shifts with a duration of 8 hours ( $l_h \times \beta_i$ ) and three prior shift extensions ( $l_h \times \beta'_j$ ) and subsequent shift extensions ( $l_h \times \beta''_j$ ) with a duration of 4 hours (cf. Fig. [2\)](#page-3-1).

#### *Staffing requirements*

We generate staffing requirements based on three indicators defined in literature assuming a fixed hiring level of 10 employees. [Vanhoucke and Maenhout](#page-22-29) [\(2009](#page-22-29)) define the *total coverage constrainedness (TCC)*, the *day coverage distribution (DCD)* and the *shift coverage distribution (SCD)* to characterise the staffing requirements. Each of these indicators has a value between zero and one.

- The *TCC-value* gives an expression of how the total staffing requirements compare to the maximum staffing requirements employees can theoretically cover. As the *TCC-value* increases, the total staffing requirements, that need to be covered with a given number of employees, rise. In this study, we consider *TCC-values* of 0.30, 0.40 and 0.50.
- The *DCD-value* reflects how the staffing requirements are distributed over the days in the planning period. A value of one indicates that the staffing requirements are focused on one or a few days. A value of zero means that the staffing requirements are equally distributed over all days in the planning period. We investigate *DCD-values* of 0.00, 0.25 and 0.50.
- The distribution of the daily staffing requirements over the shifts is indicated by the *SCD-value*. A value of one results in staffing requirements that are focused during one shift, while a value of zero equally distributes the staffing requirements over all shifts. We consider *SCD-values* of 0.00, 0.25 and 0.50.

Since the basic assignments in the integrated strategic staffing and tactical scheduling phase (Sect. [3.1\)](#page-5-0) occur on a shift level, we generate the staffing requirements per shift. Next, we transfer these requirements to the demand periods that correspond to that shift  $(R_{dh}^w)$ .

#### *Time-related constraints*

We define the parameter values for the time-related constraints below, i.e.,

- The maximum number of regular and overtime hours that can be assigned to employee *i* on day *d* ( $l_{id}^{\max}$ ) is 12.
- The maximum number of regular hours that can be assigned to employee *i* on day  $d$  ( $l_{id}^{w, \text{max}}$ ) is 8.
- The maximum number of overtime hours that can be assigned to employee *i* on day *d* ( $l_{id}^{wo, \text{max}}$ ) is 12.
- The minimum number of regular and overtime hours that have to be assigned to employee *i*  $(\eta_i^{\min})$  is 32.
- The maximum number of regular hours that can be assigned to employee *i*  $(\eta_i^{w,\max})$  is 40.
- The maximum number of overtime hours that can be assigned to employee *i*  $(\eta_i^{wo, \text{max}})$  is 12.
- The maximum number of consecutive assignments for employee *i*  $(\theta_i^{w,\max})$  is 5.

#### *Objective function*

The objective function coefficients used during the integrated strategic staffing and tactical scheduling phase (eqs.  $(1)-(10)$  $(1)-(10)$  $(1)-(10)$ ) and the operational allocation phase (eqs.  $(14)-(23)$  $(14)-(23)$  $(14)-(23)$ ) are defined as follows:

- General objective function coefficients
	- The hourly regular wage cost  $(c^w)$  is 1.25.
	- The hourly overtime wage cost  $(c^{wo})$  is 1.875.
	- The cost for understaffing a demand period  $(c^{wu})$  is 10.
- Objective function coefficients specific to the operational allocation phase
	- The roster change cost for assigning employee *i* to shift *j* on day *d* ( $c_{idj}^{w\delta}$ ) is 5.
	- The roster change cost for assigning employee *i* to a prior shift extension/a subsequent shift extension of shift *j* on day *d*  $(c_{\textit{idj}}^{\textit{w\delta'}}/c_{\textit{idj}}^{\textit{w\delta''}})$  is 2.5.
	- The cancellation cost for employee *i* on day  $d(c_{id}^v)$ is 5.

# <span id="page-12-1"></span>**4.2 Computational results**

In this section, we describe the computational experiments and results. Unless otherwise stated, the computational results are averaged over all settings in the test design (Sect. [4.1\)](#page-12-0). We compare the results for three types of baseline personnel rosters that differ based on the applied time-based proactive scheduling and reactive allocation strategy, i.e.,

– The *basic baseline roster* does not include any (un)scheduled overtime. This means that we do not allow employees to be assigned to overtime duties in the integrated strategic staffing and tactical scheduling phase or in the operational allocation phase.

- The *minimum cost baseline roster* does include scheduled and unscheduled overtime. Hence, employees can extend their daily working time and total working time by overtime duties in the integrated strategic staffing and tactical scheduling phase and in the operational allocation phase.
- The *time buffer baseline roster* is very similar to the *minimum cost baseline roster*. The sole difference is that we introduce specific staffing requirements in the integrated strategic staffing and tactical scheduling phase on top of the minimum staffing requirements  $(R_{dh}^w)$ . These requirements entail staffing requirements for extra working duties  $(R_{dh}^{w, \text{extra}})$ . Hence, a capacity buffer is installed based upon the number of available employees and the intrinsic time buffer, which is determined by the overtime policy and the time-related restrictions imposed on the schedule of a single worker. The capacity buffer is defined based upon the research of [Ingels and Maenhout](#page-22-9) [\(2015\)](#page-22-9) that identifies a fixed ratio of the minimum staffing requirements as a good strategy to install a buffer capacity. We round the staffing requirements for the extra working duties to the nearest integer [Eq. [\(24\)](#page-13-0)].

$$
R_{dh}^{w, \text{extra}} = \text{round}[0.25 \times R_{dh}^{w}] \quad \forall d \in D, \forall h \in H \quad (24)
$$

These extra staffing requirements  $(R_{dh}^{w,\text{extra}})$  and the expected staffing requirements  $(R_{dh}^w)$  are available online.<sup>[1](#page-13-1)</sup>

Note that each of these rosters is obtained by solving model [\(1](#page-6-0)[–10\)](#page-7-0) with Gurobi [\(Gurobi Optimization 2015](#page-22-24)). Given that certain instances could not be solved within a reasonable time however, we impose an MIP-gap of 5%. This MIP-gap enables us to construct baseline personnel shift rosters, which require an average CPU time of 0.658 seconds. In order to solve the operational allocation model [Eqs. [\(14–](#page-9-0) [\(23\)](#page-10-0)] with Gurobi [\(Gurobi Optimization 2015](#page-22-24)), we do not impose an MIP-gap because the solution times are negligible.

In the following section, we discuss the impact of overtime, as a time buffer, on the planned performance of a personnel roster. Section [4.2.1](#page-13-2) reveals the benefits on the planned performance of introducing scheduled overtime in the baseline personnel roster. In Sect. [4.2.2,](#page-14-0) we assess the impact of overtime on the actual performance in the operational allocation phase and investigate the trade-off between the hiring policy and the overtime policy from different perspectives. In Sect. [4.2.3,](#page-17-0) we determine the extra number of employees required to improve the effectivity of the time buffer baseline roster. The impact of the variability of demand on the actual performance is studied in Sect. [4.2.4.](#page-19-0)



 $\blacksquare$ Basic baseline roster  $\blacksquare$ Minimum cost baseline roster  $\blacksquare$ Time buffer baseline roster

<span id="page-13-4"></span>**Fig. 4** Impact of overtime on the planned cost

#### <span id="page-13-2"></span>*4.2.1 The impact of overtime on the planned performance*

<span id="page-13-0"></span>The total available overtime budget is calculated based on the number of hired employees( $\sum_{i \in N} \zeta_i$ ), the number of overtime hours employees are allowed to work  $(\eta_i^{wo, \max})$  and the hourly overtime wage cost  $(c^{wo})$ , i.e., the overtime budget =  $c^{wo} \times \sum_{i \in N} \zeta_i \times \eta_i^{wo, \max}$ . This budget can be distributed over scheduled and unscheduled overtime in the integrated strategic staffing and tactical scheduling phase and the operational allocation phase, respectively. In order to control this distribution, an additional constraint [Eq. [\(25\)](#page-13-3)] is imposed on the integrated strategic staffing and tactical scheduling decision model  $[(Eqs. (1–10))]$  $[(Eqs. (1–10))]$  $[(Eqs. (1–10))]$  $[(Eqs. (1–10))]$ . This constraint ensures the construction of a baseline personnel roster where only a fraction *f* of the overtime budget may be used.

<span id="page-13-3"></span>
$$
\sum_{i \in N} \sum_{d \in D} \sum_{j \in S} \left( l_h \beta_j c^{wo} x_{idj}^{wo} + l_h \beta_j' c^{wo} x_{idj}^{wo'} + l_h \beta_j'' c^{wo} x_{idj}^{wo''} \right)
$$
  
 
$$
\leq f \times \text{overtime budget}
$$
 (25)

In Fig. [4,](#page-13-4) we show the impact on the planned cost for different values for the parameter  $f$ , i.e., 0.00, 0.20, 0.40, 0.60, 0.80 and 1.00. This parameter determines the percentage of the overtime budget that is available in the integrated strategic staffing and tactical scheduling phase for scheduled overtime. Figure [4](#page-13-4) shows that this parameter *f* has no impact on the basic baseline roster as overtime is not incorporated. The planned cost for the minimum cost baseline roster and the time buffer baseline roster shows a steady decrease as the budget for scheduled overtime increases. Allotting the overtime completely in the integrated staffing and scheduling phase, i.e., (Scheduled OT)% = 100%, results in an average cost decrease of 5.2% compared to the scenario without scheduled overtime, i.e., (Scheduled OT)% =  $0\%$ . This improvement is due to the decrease in the understaffing, which compensates the rise in the wage cost for overtime, which is shown in Table [1.](#page-14-1) Note that the results in Fig. [4](#page-13-4) and Table [1](#page-14-1) are obtained with a fixed hiring level of 4 employees. We observe the same trend for a higher number of hired

<span id="page-13-1"></span><sup>1</sup> [http://www.projectmanagement.ugent.be/?q=research/personnel\\_](http://www.projectmanagement.ugent.be/?q=research/personnel_scheduling) [scheduling](http://www.projectmanagement.ugent.be/?q=research/personnel_scheduling)

<span id="page-14-1"></span>**Table 1** Evolution of the average planned performance metrics over the minimum cost and time buffer baseline roster

	(Scheduled OT)%							
	$0\%$	20%	40%	60%	80%	100%		
Hired employees	4.00	4.00	4.00	4.00	4.00	4.00		
Understaffing	18.47	17.06	15.84	13.98	12.89	11.56		
Overstaffing	0.02	0.04	0.01	0.00	0.01	0.01		
Overtime	0.00	2.00	3.80	6.27	7.71	9.43		



 $\blacksquare$  Basic baseline roster  $\blacksquare$  Minimum cost baseline roster  $\blacksquare$  Time buffer baseline roster

<span id="page-14-2"></span>**Fig. 5** Impact of overtime on the optimal hiring level in terms of planned cost

employees but as the number of hired employees increases, the impact of scheduled overtime decreases.

The incorporation of scheduled overtime in the minimum cost and time buffer baseline roster leads to an increased assignment flexibility, which is not existent in the basic baseline roster. This flexibility results in a planned cost that is smaller for the minimum cost baseline roster than for the basic baseline roster. The planned cost for the time buffer baseline roster is significantly higher compared to the basic and minimum cost baseline roster. This is the result from the definition of specific staffing requirements  $[Eq. (24)]$  $[Eq. (24)]$  $[Eq. (24)]$  on top of the minimum staffing requirements, which causes a higher number of regular and overtime assignments and a higher understaffing of demand periods.

The additional assignment flexibility introduced by allowing overtime also influences the optimal hiring level when the planned cost is minimised (Fig. [5\)](#page-14-2). The optimal hiring level for a basic baseline roster is 7 employees, whereas the optimal hiring level for the minimum cost baseline roster is 6 employees. Moreover, the planned cost for the minimum cost baseline roster with 6 hired employees is better than the planned cost for the basic baseline roster with 7 hired employees. Hence, the flexibility offered by scheduling overtime allows the organisation to hire a lower number of employees without repercussions in terms of the planned cost. This observation is not valid for the time buffer baseline roster because of the definition of the extra staffing requirements [Eq. [\(24\)](#page-13-0)]. For this roster type, the optimal hiring level is 8 employees.

<span id="page-14-3"></span>**Table 2** Optimal percentage of scheduled overtime for the different hiring levels

	Number of hired employees							
	4	5	6		8		10	
Basic baseline roster 0%		$0\%$	$0\%$	$0\%$	$0\%$	$0\%$	$0\%$	
Minimum cost baseline roster				$100\%$ $100\%$ 60\% 40\% 20\% 20\% 0\%				
Time buffer baseline $100\%$ $100\%$ $100\%$ $100\%$ 80\% 60\% $60\%$ 40\% roster								

Figure [5](#page-14-2) indicates the minimal planned cost for each hiring level and type of baseline roster. These minima are obtained for varying percentages of the overtime budget available in the integrated strategic staffing and tactical scheduling phase. Table [2](#page-14-3) indicates, for each hiring level, the percentage of the overtime budget that should be available in the integrated strategic staffing and tactical scheduling phase to obtain the best results that are displayed in Fig. [5.](#page-14-2) Since the basic baseline roster does not allow overtime, the optimal percentage of scheduled overtime is always 0%. For the minimum cost and time buffer baseline roster, we observe a decreasing trend. As more employees are hired, the percentage of scheduled overtime required to obtain a minimal planned cost decreases. This decrease is more pronounced for the minimum cost baseline roster than for the time buffer baseline roster. As mentioned before, this is due to the definition of the extra staffing requirements [Eq. [\(24\)](#page-13-0)], which increases the number of required working duties.

#### <span id="page-14-0"></span>*4.2.2 The trade-off between hiring and overtime*

In this section, we investigate the trade-off between the hiring budget and the overtime budget based on the performance of the baseline personnel rosters in the operational allocation phase. In order to obtain insight in this trade-off, we performed two experiments that differ in the determination of the total personnel budget.

In the first experiment, we assess the impact of the distribution of the overtime budget between scheduled and unscheduled overtime for several fixed workforce sizes. This experiment approaches the trade-off from the perspective of a *varying personnel budget*, i.e., the hiring budget and the overtime budget are determined by the fixed workforce sizes. In the second experiment, we investigate the distribution of a *fixed personnel budget* over the hiring budget and the overtime budget.

#### **Scheduled versus unscheduled overtime**

We set the number of hired employees fixed and investigate the impact of different distributions between the allowed scheduled and unscheduled overtime. In order to fix the



 $0\% -100\% = 20\% -80\% = 40\% -60\% = 60\% -40\% = 80\% -20\% = 100\% -0\%$ 

<span id="page-15-2"></span>**Fig. 6** Impact of scheduled versus unscheduled overtime on the actual cost

workforce size, we additionally impose Eq. [\(26\)](#page-15-0) on the integrated strategic staffing and tactical scheduling decision model [Eqs.  $(1-10)$  $(1-10)$ ] to construct a baseline personnel roster. The fixed workforce sizes are varied in the experiment and range between 4 and 10 workers. The minimum hiring level is determined based on the minimum value of the TCCindicator in the test design. A minimum TCC-value of 0.30 corresponds to a total staff demand of 21 shift duties and 168 hours (=  $21 \times l_h \beta_j$ ). A workforce size of 4 employees may cover a total of 160 hours (=  $4 \times \eta_i^{w, max}$ ) in regular time. A lower hiring level would create too much understaffing. Since we generate the staffing requirements assuming that 10 employees are available, a maximum hiring level of 10 workers is considered.

$$
\sum_{i \in N} \zeta_i = \text{fixed workforce size}
$$
 (26)

In order to distribute the overtime budget into scheduled and unscheduled overtime in a controlled manner, we also add Eq. [\(25\)](#page-13-3) to the integrated staffing and scheduling decision model [Eqs.  $(1-10)$  $(1-10)$ ] to construct a baseline roster. As a result of this constraint, we may have some flexibility to allocate unscheduled overtime in the operational allocation phase. In order to ensure that we do not exceed the total overtime budget, we impose Eq.  $(27)$  on the operational allocation decision model [Eqs. [\(14](#page-9-0)[–23\)](#page-10-0)] that considers day *d* in the planning horizon. Note that this equation takes the unscheduled overtime that was already allotted before day *d* into account.

$$
\sum_{d=1}^{d-1}
$$
utilised unscheduled budget<sub>d</sub>  
+ 
$$
\sum_{i \in N} \sum_{j \in S} l_h \beta_j c^{wo} x_{idj}^{wo} (1 - \bar{x}_{idj}^{wo})
$$
  
+ 
$$
\sum_{i \in N} \sum_{j \in S \setminus S^-} l_h \beta_j' c^{wo} x_{idj}^{wo'} (1 - \bar{x}_{idj}^{wo'})
$$

+ 
$$
\sum_{i \in N} \sum_{j \in S^{-}} l_h \beta'_j c^{wo} x_{id+1j}^{wo'} (1 - \bar{x}_{id+1j}^{wo'})
$$
  
+ 
$$
\sum_{i \in N} \sum_{j \in S \setminus S^{+}} l_h \beta''_j c^{wo} x_{idj}^{wo''} (1 - \bar{x}_{idj}^{wo''})
$$
  
+ 
$$
\sum_{i \in N} \sum_{j \in S^{+}} l_h \beta''_j c^{wo} x_{id-1j}^{wo''} (1 - \bar{x}_{id-1j}^{wo''})
$$
  

$$
\leq (1 - f) \times \text{overtime budget}
$$
 (27)

<span id="page-15-1"></span>Figure [6](#page-15-2) displays the impact of the distribution of scheduled and unscheduled overtime ((Scheduled OT)%- (Unscheduled OT)%) on the actual cost for several workforce sizes. Since the basic baseline roster does not consider overtime, the presented results are the average actual cost over the minimum cost and time buffer baseline roster. The chart shows that for small workforce sizes the actual cost decreases as more overtime is scheduled in the baseline personnel roster. As the number of hired employees increases, the impact of including scheduled overtime reduces because the need for overtime diminishes. At some point, starting from a moderate workforce size, the actual cost is minimal if a combination of scheduled and unscheduled overtime is utilised. Thus, for a low number of employees, it is important to include as much overtime as possible in the integrated strategic staffing and tactical scheduling phase to reduce the planned understaffing. A higher number of hired employees, however, automatically results in less scheduled overtime and it is therefore beneficial to reserve a fraction of the overtime budget for allocating unscheduled overtime in the operational allocation phase.

<span id="page-15-0"></span>We provide the results of the individual components of the actual performance in Table [3](#page-16-0) averaged over all hiring levels. The table indicates the evolution of the understaffing, overstaffing, changes, cancellations and overtime for the different overtime distributions. The table reveals that the understaffing and overstaffing are minimal for a combination of both scheduled and unscheduled overtime. The number of cancellations is minimal if the complete overtime budget is reserved for the operational allocation phase. As more overtime budget is reserved for the operational allocation phase, the reactive flexibility increases, which results in a higher number of changes. Even though the scheduled overtime utilisation increases if more overtime is scheduled in the integrated strategic staffing and tactical scheduling phase, the number of overtime periods is maximal if a combination of scheduled and unscheduled overtime is utilised.

We show the impact of the workforce size on the actual cost in Fig. [7.](#page-16-1) The figure shows the average results over all possible distributions for the scheduled and unscheduled overtime. It is clear that, for each baseline roster type (basic, minimum cost and time buffer), a workforce size that is either too low or too high leads to a deterioration in the actual cost. Based on the obtained results, we can conclude that the optimal hiring level is 6 employees. Note that this level represents an

<span id="page-16-0"></span>



 $\blacksquare$  Basic baseline roster  $\blacksquare$  Minimum cost baseline roster  $\blacksquare$  Time buffer baseline roster

<span id="page-16-1"></span>**Fig. 7** Impact of the workforce size on the actual cost

average result. This optimum certainly depends on the total staffing requirements, i.e., the TCC-value. A TCC-value of 0.30, 0.40 and 0.50, respectively, leads to an optimal hiring level of 5, 6 and 8 employees.

Moreover, Fig. [7](#page-16-1) reveals that it is most beneficial to employ the minimum cost baseline roster for a low workforce size and the time buffer baseline roster for a high workforce size. As more employees are hired, the flexibility in the personnel roster construction increases and the personnel planner is better able to satisfy the extra staffing requirements [Eq. [\(24\)](#page-13-0)]. This reduces the planned understaffing and facilitates the performance of the baseline personnel roster in the operational allocation phase. This improved actual performance is expressed by a smaller understaffing with a lower number of changes and unscheduled overtime periods.

## **The hiring and overtime budget**

In the second experiment, we assume a fixed number of budgeted working hours and discuss the impact of the distribution of this budget over regular and overtime hours, which correspond to the hiring and overtime budget. The fixed number of budgeted working hours is determined based upon the minimum required number of working hours and the additional workforce buffer to cope with the operational variability, i.e., (1+buffer ratio) ×  $\sum_{d \in D} \sum_{h \in H} l_h R_{dh}^w$ .

The incorporation of the buffer ratio to determine the budgeted working hours facilitates the inclusion of an implicit buffer and an explicit buffer to hedge the personnel roster against operational variability. The buffer ratio enables an implicit buffer for the basic and minimum cost baseline rosters because it allows that a higher number of working hours may be assigned than required. It creates an explicit buffer for the time buffer baseline roster because it allows this roster to better satisfy the extra staffing requirements  $(R_{dh}^{w, \text{extra}})$ .

The number of required working hours may be distributed over the hiring budget, and the overtime budget and in our computational experiments we explore three different scenarios, i.e.,

- *Scenario 1*: We assign the complete budget to the hiring budget, i.e., we hire a maximum number of employees that only work during regular time. Given the allowed number of working hours  $\eta_i^{w,max}$  for a single worker, we can calculate the required number of full-time equivalents  $\zeta^{\text{max}}$  to execute all duties in regular time without overtime.
- *Scenario 2*: We distribute the budget between the hiring budget and overtime budget by reducing the hiring budget determined in scenario 1 by a full-time equivalent, i.e., the hiring budget drops to  $\zeta^{\text{max}} - 1$  workers, and by increasing the allowed number of overtime hours with  $\eta_i^{w, \text{max}}$  hours.
- *Scenario 3*: We distribute the budget between the hiring budget and overtime budget by reducing the hiring budget determined in scenario 1 by two full-time equivalents, i.e., the hiring budget drops to  $\zeta^{\text{max}} - 2$  workers, and by increasing the allowed number of overtime hours with  $2 \times \eta_i^{w, \text{max}}$  hours.

Hence, we add Eqs.  $(25)$  and  $(26)$  to the integrated strategic staffing and tactical scheduling decision model [Eqs.  $(1-10)$  $(1-10)$ ] for each of these scenarios. The overtime budget is determined as the allowed number of overtime hours times the overtime cost per hour.

<span id="page-17-1"></span>**Table 4** Evolution of planned and actual performance metrics for the minimum cost baseline roster



Table [4](#page-17-1) compares the average planned and actual performance for the three scenarios starting from the minimum cost roster as baseline roster. The table reveals that scenario 2 outperforms the other scenarios for both the planned and actual performance. Hence, it is useful to include a limited budget for overtime at the expense of the hiring budget. Hence, situations in which more employees are hired without overtime (scenario 1) and situations in which a low number of employees can work a high number of overtime hours (scenario 3) should be avoided and lead to inferior results.

When the hiring budget is reduced in the integrated strategic staffing and tactical scheduling phase, the table reveals that the number of hired employees and the overstaffing decrease. The impact on the planned understaffing is rather ambiguous and is lowest for scenario 2. The actual performance in the operational allocation phase shows that the understaffing is again the lowest for scenario 2, whereas the number of performed changes is the highest. The number of overstaffed demand periods decreases while the overtime budget increases as less employees are hired. Remarkably, this lower number of employees does not result in a lower number of cancellations. On the contrary, the number of cancellations increases when the workforce size decreases.

# <span id="page-17-0"></span>*4.2.3 The optimal workforce buffer for the time buffer baseline roster*

In the two experiments in Sect. [4.2.2,](#page-14-0) we imposed the same personnel budget restrictions for the basic, minimum cost and time buffer baseline roster. However, the time buffer baseline roster installs a capacity buffer on top of the minimum number of staffing requirements. In order to meet the larger demand for staff, this time buffer baseline roster requires extra personnel budget, i.e., hiring budget and overtime budget. This extra budget enables the time buffer baseline roster to become an effective strategy and outperform the basic and minimum cost baseline roster in terms of actual cost. In this experiment, the objective is to determine how large the available personnel budget for the time buffer baseline roster should be relative to the personnel budget for the basic and minimum cost baseline roster. Hence, we aim to determine the optimal level of additional employees, i.e., hiring and overtime budget, needed for the time buffer baseline roster. In order to avoid that an implicit buffer could be created for the basic and minimum cost baseline roster, the starting point in this analysis is the minimum required number of employees to cover the minimum staffing requirements  $(R_{dh}^w)$ . This can be calculated based on the minimum required number of working hours and the allowed number of working hours



<span id="page-18-0"></span>**Fig. 8** Impact of an employee buffer for the time buffer baseline roster. (a) TCC=0.30. (b) TCC=0.40. (c) TCC=0.50

for a single employee in regular time and in overtime, i.e.,  $\left[ \sum_{d \in D} \sum_{n \in H} \hat{l}_h R_{dh}^w / (\eta_i^{w, max} + \eta_i^{wo, max}) \right]$ . Hence, a TCC value of 0.30, 0.40 or 0.50 leads to a minimum of 4, 5 or 6 employees required, respectively. In our experiments, we vary the additional number of employees that are hired on top of this minimum number and we determine the impact of the number of additional employees on the actual cost. Note that we do not impose restrictions on the distribution of overtime over the integrated strategic staffing and tactical scheduling phase  $[Eq. (25)]$  $[Eq. (25)]$  $[Eq. (25)]$  and the operational allocation phase [Eq. [\(27\)](#page-15-1)]. The results are displayed in Fig. [8,](#page-18-0) which shows the results for the basic baseline roster, the minimum cost baseline roster and the time buffer baseline roster with a different number of extra employees for different values of the TCC-indicator.

Figure [8](#page-18-0) expresses the trade-off between hiring extra employees and the actual cost for the time buffer baseline roster. Figure [8a](#page-18-0) shows the evolution for a varying employee buffer size and a TCC-value of 0.30 and indicates that the best results are obtained without a buffer. The time buffer baseline roster is outperformed by the minimum cost baseline roster in this case. The optimal buffer for a TCC-value of 0.40 and 0.50 is, respectively, 1 and 2 employees and the actual cost for the time buffer baseline roster is smaller than that for the minimum cost baseline roster. Thus, it is useful to hire additional employees if the total staffing requirements are sufficiently large.

Table [5](#page-19-1) reveals the results for the individual components of the planned and actual performance. As the number of employees increases, the understaffing of staffing requirements decreases in the integrated strategic staffing and tactical scheduling phase. Naturally, we also observe a reduction in the scheduled overtime and an augmentation in the overstaffing. This results in a reduction of the actual understaffing in the operational allocation phase. Similarly, the availability of extra employees decreases the need for unscheduled overtime and reduces the number of cancellations in the operational allocation phase. However, these advantages of additional employees are negated through an increase in the overstaffing and the number of changes.

<span id="page-19-1"></span>

	Planned performance							
	Employee buffer				3 4 8.00 9.00 0.24 0.09 1.83 5.04 2.87 4.84			
	0		2					
Hired employees	5.00	6.00	7.00			10.00		
Understaffing (in periods)	6.12	2.27	0.66			0.01		
Overstaffing (in periods)	0.02	0.07	0.48			9.95		
Overtime (in periods)	12.38	10.96	8.02			1.63		
		Actual performance						

**Table 5** Evolution of planned and actual performance metrics for the time buffer baseline roster



**Strategy 1** Demand variability per shift



**Strategy 2** Demand variability per period





<span id="page-19-2"></span>**Fig. 9** Impact of the variability of demand

## <span id="page-19-0"></span>*4.2.4 The impact of the variability of demand*

The experiments in Sects. [4.2.2](#page-14-0) and [4.2.3](#page-17-0) are based on the assumption that the demand variability per demand period is not restricted and can range up to a value of  $+\infty$  [Eqs.  $(11-13)$  $(11-13)$ ]. In this section, we investigate the impact of different degrees of variability of demand by varying the value of parameter  $\sigma$  in Eqs. [\(11\)](#page-8-3) and [\(12\)](#page-8-4). We distinguish 6 uncertainty scenarios that differ in the degree of variability, i.e.,  $\sigma \in \{1, 2, 3, 4, 5, +\infty\}$ . Moreover, the demand variability is simulated according to two simulation strategies, i.e.,

– In the first strategy, the demand variability is simulated for the defined *standard shift duties*, which consist of two consecutive demand periods (cf. Fig. [2\)](#page-3-1). Hence, the simulated staffing requirements remain the same for both demand periods within a standard shift.

– The second strategy simulates the staffing requirements per *demand period* and includes a higher intrinsic variability because the staffing requirements can differ from demand period to demand period.

Figure [9](#page-19-2) displays the actual cost for the different uncertainty scenarios and the two simulation strategies. When the uncertainty rises, the actual cost displays a convex behaviour, i.e., the actual cost increases at a decreasing rate. The main difference between the two simulation strategies is the height

<span id="page-20-0"></span>



of the actual cost. Since the first strategy comprises a lower overall variability, the actual cost is lower than for the second strategy.

In order to obtain further insight in the impact of the uncertainty scenarios for the two simulation strategies, we repeat the experiments of Sects. [4.2.2](#page-14-0) and [4.2.3.](#page-17-0) In general, the findings of these experiments are confirmed for different degrees of variability. The detailed results of the actual cost for different variability degrees and hiring levels are displayed in Table [6](#page-20-0) for both the minimum cost baseline roster and the time buffer baseline roster. The italics indicate whether the minimum cost baseline roster or the time buffer baseline roster performs better for a given combination of demand variability and workforce size. In general, the construction of a time buffer baseline roster is beneficial when the workforce size is relatively high and the demand variability increases. This is in particular the case when the demand strategy is simulated per demand period (cf. Fig. [9\)](#page-19-2). When the number of employees is relatively low, the personnel planner should not introduce additional staffing requirements on top of the minimum staffing requirements as the minimum cost baseline roster performs best.

The inclusion of a small workforce buffer on top of the minimally required number of employees leads to an actual cost improvement for the time buffer baseline roster. However, the construction of a time buffer baseline roster is only beneficial when the total staffing requirements and the degree of variability are sufficiently high in comparison to the minimum cost baseline roster.

# <span id="page-21-3"></span>**5 Conclusions**

In this paper, we discuss the impact of overtime as a time buffer strategy on the robustness of a personnel roster. In the personnel planning process, a decision on the overtime budget is typically taken in the staffing phase as this decision is interconnected with the hiring policy in an organisation and overtime may reduce the required number of employees. Overtime is defined as the extension of the daily working time and/or of the total working time over the planning period. In the personnel planning process, decisions taken in the higher-level hierarchical phases impact lower-level phases. The overtime budget has undoubtedly an impact on the operational allocation phase where operational variability arises and overtime may offer some flexibility to solve schedule disruptions in a reactive way. In this perspective, we explore the trade-off between the number of hired employees and the overtime budget. Additionally, we investigate the impact of the overtime policy, which determines how overtime is used in the personnel planning process. Overtime may be introduced in the integrated staffing and scheduling phase to construct the baseline personnel roster as a proactive scheduling strategy and in the operational allocation phase as a reactive allocation strategy to balance supply and demand. The research methodology embodies a three-step methodology that consists of the construction of a baseline personnel roster, the daily simulation and adjustment decision model and an evaluation step. We assess three types of personnel rosters, which differ in the availability of overtime and the definition of specific staffing requirements for overtime duties as a capacity buffer to hedge against uncertainty.

The results of the computational experiments show that the introduction of overtime reduces both the planned and actual cost. The degree in which overtime should be included in the baseline personnel roster depends on the number of hired employees. A low number of hired employees requires that more overtime is proactively scheduled to reduce the planned understaffing while a larger number of hired employees benefits from a combination of scheduled and unscheduled overtime. The definition of additional staffing requirements as a capacity buffer is most useful when the workforce size is higher than the minimum required workforce size. We investigated the size of this workforce buffer on top of the minimum number of required employees. Moreover, the additional staffing requirements are only relevant when the demand variability is sufficiently high.

Future research should focus on the expansion and adaptation of the three-step methodology. First, the development of dedicated algorithms based on the techniques of robust optimisation should be studied in order to construct baseline personnel shift rosters under uncertainty. Second, the simulation may also consider dependencies between the demand of subsequent demand periods, i.e., an endogenous demand, by means of stochastic processes.

**Acknowledgements** We acknowledge the support for the doctoral research project fundings by the Bijzonder Onderzoekfonds (BOF, Ghent University) under contract number 01N00712 and the National Bank of Belgium.

# **References**

- <span id="page-21-2"></span>Abdelghany, A., Ekollu, G., Narasimhan, R., & Abdelghany, K. (2004). A proactive crew recovery decision support tool for commercial airlines during irregular operations. *Annals of Operations Research*, *127*, 309–331.
- <span id="page-21-1"></span>Abdelghany, K., Abdelghany, A., & Ekollu, G. (2008). An integrated decision support tool for airlines schedule recovery during irregular operations. *European Journal of Operational Research*, *185*(2), 825–848.
- <span id="page-21-0"></span>Abernathy, W., Baloff, N., & Hershey, J. (1973). A three-stage manpower planning and scheduling model a service sector example. *Operations Research*, *21*, 693–711.
- <span id="page-21-4"></span>Ahmed, M., & Alkhamis, T. (2009). Simulation optimization for an emergency department healthcare unit in Kuwait. *European Journal of Operational Research*, *198*(3), 936–942.
- <span id="page-22-16"></span>Bard, J., & Purnomo, H. (2005). Hospital-wide reactive scheduling of nurses with preference considerations. *IIE Transactions*, *37*, 589– 608.
- <span id="page-22-3"></span>Burke, E., De Causmaecker, P., Vanden Berghe, G., & Van Landeghem, H. (2004). The state of the art of nurse rostering. *Journal of Scheduling*, *7*, 441–499.
- <span id="page-22-18"></span>Camden, M. C., Price, V. A., & Ludwig, T. D. (2011). Reducing absenteeism and rescheduling among grocery store employees with point-contingent rewards. *Journal of Organizational Behavior Management*, *31*(2), 140–149.
- <span id="page-22-13"></span>Campbell, G. M. (2012). On-call overtime for service workforce scheduling when demand is uncertain. *Decision Sciences*, *43*(5), 817–850.
- <span id="page-22-26"></span>de Bruin, A. M., Bekker, R., van Zanten, L., & Koole, G. M. (2010). Dimensioning hospital wards using the Erlang loss model. *Annals of Operations Research*, *178*(1), 23–43.
- <span id="page-22-7"></span>De Causmaecker, P., & Vanden Berghe, G. (2003). Relaxation of coverage constraints in hospital personnel rostering. *Lecture Notes in Computer Science*, *2740*, 129–147.
- <span id="page-22-22"></span>De Causmaecker, P., & Vanden Berghe, G. (2011). A categorisation of nurse rostering problems. *Journal of Scheduling*, *14*, 3–16.
- <span id="page-22-6"></span>Dowsland, K., & Thompson, J. (2000). Solving a nurse scheduling problem with knapsacks, networks and tabu search. *Journal of the Operational Research Society*, *51*, 825–833.
- <span id="page-22-19"></span>Easton, F., & Rossin, D. (1997). Overtime schedules for full-time service workers. *Omega*, *25*(3), 285–299.
- <span id="page-22-12"></span>Ehrgott, M., & Ryan, D. M. (2002). Constructing robust crew schedules with bicriteria optimization. *Journal of Multi-Criteria Decision Analysis*, *11*(3), 139–150.
- <span id="page-22-0"></span>Ernst, A., Jiang, H., Krishnamoorthy, M., & Sier, D. (2004). Staff scheduling and rostering: A review of applications, methods and models. *European Journal of Operational Research*, *153*, 3–27.
- <span id="page-22-25"></span>Gough, B. (2009). *GNU scientific library reference manual—third edition* (3rd ed.). Godalming: Network Theory Ltd.
- <span id="page-22-24"></span>Gurobi Optimization, Inc. (2015). Gurobi optimizer reference manual. [http://www.gurobi.com/documentation/.](http://www.gurobi.com/documentation/)
- <span id="page-22-9"></span>Ingels, J., & Maenhout, B. (2015). The impact of reserve duties on the robustness of a personnel shift roster: An empirical investigation. *Computers and Operations Research*, *61*, 153–169.
- <span id="page-22-20"></span>Ionescu, L., & Kliewer, N. (2011). Increasing flexibility of airline crew schedules. *Procedia Social and Behavioral Sciences*, *20*, 1019– 1028.
- <span id="page-22-4"></span>Koutsopoulos, H. N., & Wilson, N. H. (1987). Operator workforce planning in the transit industry. *Transportation Research Part A General*, *21*(2), 127–138.
- <span id="page-22-2"></span>Li, N., & Li, L. X. (2000). Modeling staffing flexibility: A case of China. *European Journal of Operational Research*, *124*(2), 255–266.
- <span id="page-22-23"></span>Lobo, V. M., Fisher, A., Ploeg, J., Peachey, G., & Akhtar-Danesh, N. (2013). A concept analysis of nursing overtime. *Journal of Advanced Nursing*, *69*(11), 2401–2412.
- <span id="page-22-21"></span>Maenhout, B., & Vanhoucke, M. (2013). An artificial immune system based approach for solving the nurse re-rostering system. *Lecture Notes in Computer Science*, *7832*, 97–108.
- <span id="page-22-27"></span>Paul, J. A., & Lin, L. (2012). Models for improving patient throughput and waiting at hospital emergency departments. *The Journal of Emergency Medicine*, *43*(6), 1119–1126.
- <span id="page-22-17"></span>Schalk, R., & Van Rijckevorsel, A. (2007). Factors influencing absenteeism and intention to leave in a call centre. *New Technology Work and Employment*, *22*(3), 260–274.
- <span id="page-22-10"></span>Shebalov, J., & Klabjan, D. (2006). Robust airline crew pairing: Moveup crews. *Transportation Science*, *40*, 300–312.
- <span id="page-22-11"></span>Tam, B., Ehrgott, M., Ryan, D. M., & Zakeri, G. (2011). A comparison of stochastic programming and bi-objective optimisation approaches to robust airline crew scheduling. *OR Spectrum*, *33*(1), 49–75.
- <span id="page-22-14"></span>Tam, B., Ryan, D., & Ehrgott, M. (2014). Multi-objective approaches to the unit crewing problem in airline crew scheduling. *Journal of Multi-Criteria Decision Analysis*, *21*(5–6), 257–277.
- <span id="page-22-5"></span>Tan, J. (2003). Curvilinear relationship between organizational slack and firm performance: Evidence from Chinese state enterprises. *European Management Journal*, *21*(6), 740–749.
- <span id="page-22-15"></span>Thiel, M. P. (2005). *Team-oriented Airline Crew Scheduling and Rostering: Problem Description, Solution Approaches, and Decision Support*. PhD thesis, University of Paderborn, Germany.
- <span id="page-22-8"></span>Topaloglu, S., & Selim, H. (2010). Nurse scheduling using fuzzy modelling approach. *Fuzzy Sets and Systems*, *161*, 1543–1563.
- <span id="page-22-1"></span>Van den Bergh, J., Beliën, J., De Bruecker, P., Demeulemeester, E., & De Boeck, L. (2013). Personnel scheduling: A literature review. *European Journal of Operational Research*, *226*, 367–385.
- <span id="page-22-29"></span>Vanhoucke, M., & Maenhout, B. (2009). On the characterization and generation of nurse scheduling problem instances. *European Journal of Operational Research*, *196*, 457–467.
- <span id="page-22-28"></span>Yeh, J.-Y., & Lin, W.-S. (2007). Using simulation technique and genetic algorithm to improve the quality care of a hospital emergency department. *Expert Systems with Applications*, *32*(4), 1073–1083.