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ORIGINAL ARTICLE

# Precursor times of abnormal *b*-values prior to mainshocks

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**Abstract** Seismic observations exhibit the presence of abnormal *b*-values prior to numerous earthquakes. The time interval from the appearance of abnormal *b*-values to the occurrence of mainshock is called the precursor time. There are two kinds of precursor times in use: the first one denoted by $T$ is the time interval from the moment when the *b*-value starts to increase from the normal one to the abnormal one to the occurrence time of the forthcoming mainshock, and the second one denoted by $T_p$ is the time interval from the moment when the abnormal *b*-value reaches the peak one to the occurrence time of the forthcoming mainshock. Let $T^*$ be the waiting time from the moment when the abnormal *b*-value returned to the normal one to the occurrence time of the forthcoming mainshock. The precursor time, $T$ (usually in days), has been found to be related to the magnitude, $M$, of the mainshock expected in a linear form as $\log(T)=q+rM$ where $q$ and $r$ are the coefficient and slope, respectively. In this study, the values of $T$, $T_p$, and $T^*$ of 45 earthquakes with $3\leq M\leq 9$ occurred in various tectonic regions are compiled from or measured from the temporal variations in *b*-values given in numerous source materials. The relationships of $T$ and $T_p$, respectively, versus $M$ are inferred from compiled data. The difference between the values of $T$ and $T_p$ decreases with increasing $M$. In addition, the plots of $T^*/T$ versus $M$, $T^*$ versus $T$, and $T^*$ versus $T$-$T^*$ will be made and related equations between two quantities will be inferred from given data.

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**Keywords** Gutenberg–Richter frequency-magnitude law · *b*-value · Precursor time · Waiting time

## 1 Introduction

Gutenberg and Richter (1944) proposed an important frequency-magnitude (FM) law for earthquakes in the form $\log N=a-bM$, where $M$ is the earthquake magnitude and $N$ is the discrete frequency of events with magnitudes in a small unit from $M$ to $M+\delta M$ or the cumulative frequency of the events with magnitudes $\geq M$. This law is denoted by the GR law hereafter and the *b*-value is an important parameter representing seismicity. An example of the FM distribution is displayed in Fig. 1 for earthquakes, specified with local magnitudes, occurred in northern Taiwan. The GR law is valid in the magnitude range between $M_1$ and $M_2$ which are the lower and upper bounds of the linear portion. The *b*-value varies from area to area and also depends upon the time interval of earthquake data in use. Its values generally vary from 0.8 to 1.2.

Mitigation of seismic risk is not only the seismologists' major research topics but also societal needs, especially for seismically active regions such as Taiwan. One of the most significant and direct ways to mitigate seismic risk is the prediction of a forthcoming

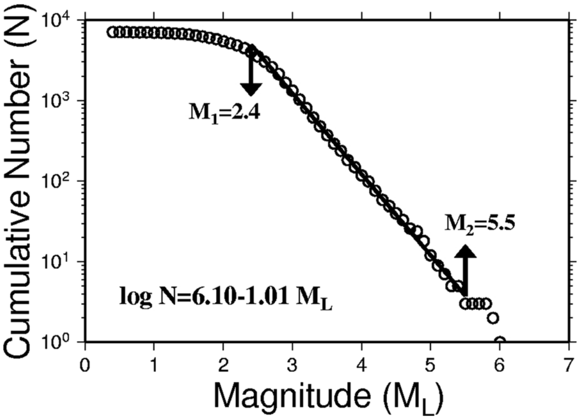


**Fig. 1** An example of the plot of log($N$) versus $M_L$, where $M_L$ is the local magnitude

large earthquake through the observations and physical modeling of precursors. Earthquake prediction research has long been an important issue in the countries that are frequently attacked by large earthquakes, for example, China and Japan (Mogi 1985; Knopoff 1990; Chen et al. 1992; Aki 1995). Of course, some seismologists (e.g., Geller 1997; Geller et al. 1997) argued the low possibility of predicting an earthquake. Actually, up to date, the success rate to predict an earthquake is still very low. Numerous seismic precursors have been investigated for earthquake prediction. The temporal variation in $b$-value is one of significant precursors for volcano activities and earthquake occurrences. Gorshkov first observed a decrease in $b$-value before the Russian Bezymianny eruption in 1956 (see Aki 1985). Suyehiro (1966) first found a change of the $b$-value before and after an earthquake and considered the $b$-value anomaly to be a precursor. Some researchers (e.g., Knopoff et al. 1992a; and Cao and Gao 2002) also observed such a phenomenon. Fiedler (1974) found an increase in $b$-value before the 1967 Caracas earthquake. Numerous studies also exhibit an increase in $b$-value prior to a mainshock (Rikitake 1975a,b, 1979, 1984, 1987; Li et al. 1978; Smith 1981, 1986; Wyss et al. 1981; Kiek 1984; Aki 1985; Chen et al. 1984a,b; Chen et al. 1990; Sahu and Saikia 1994; Chen 2003; Tsai et al. 2006; DasGupta et al. 2007, 2012; Wu et al. 2008; Lin 2010; Moatti et al. 2013). The studies of abnormal $b$-values about earthquakes in Taiwan can be found in Wang et al. (2015). Chen et al. (1984b) also found that the occurrence times and magnitudes of high $b$-values are different when the study areas of events in use are distinct. This is due to area-dependence of evaluating $b$-value.

Figure 2 schematically displays the temporal variation in $b$-value based on observations. The $b$-value starts to increase from the normal value at time $t_1$, reaches the peak one at time $t_2$, then begins to decrease from the peak one at $t_2$, and returns to the normal one at time $t_3$. As $t > t_3$, the $b$-value varies around the normal one or rightly decreases with time until the occurrence of the forthcoming mainshock at time $t_4$. The time interval from the appearance of abnormal $b$-values to the occurrence of mainshock is called the precursor time. Commonly, there are two kinds of precursor times in use: the first one denoted by $T$ is counted from the moment when the $b$-value starts to increase from the normal one to the abnormal one to the occurrence time of the forthcoming mainshock, that is, $T = t_4 - t_1$. The second one denoted by $T_p$ is counted from the moment when the abnormal $b$-value reaches the peak one to the occurrence time of the forthcoming mainshock, that is, $T_p = t_4 - t_2$. Obviously, the time difference $T\text{-}T_p$ is the time interval within which the $b$-value increases from the normal value to the peak one. In addition, $T^* = t_4 - t_3$ is defined to be the waiting time of the forthcoming mainshock and measured from the moment when the $b$-value returns to normal one and decreases again to the occurrence time of the forthcoming mainshock.

Of course, there are some opposite observations about the variations in $b$-values. In rock mechanics experiment using dry rock, Weeks et al. (1978) found that the $b$-value directly decreases from the normal one to a smaller one prior to the occurrence of a large slip. Wyss and Lee (1973) observed a decrease in $b$-value

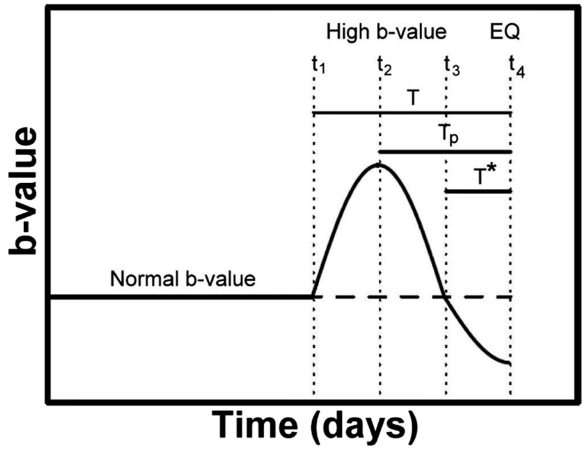


**Fig. 2** A general pattern of time variation in $b$-value: $t_1$ = the starting time of abnormal $b$-value; $t_2$ = the time of the highest $b$-value; $t_3$ = the ending time of abnormal $b$-value; $t_4$ = the occurrence time of mainshock; $T = t_4 - t_1$, $T_p = t_4 - t_2$, and $T^* = t_4 - t_3$

from the normal one to a smaller one before several earthquakes in California. Chen et al. (1984b) observed the same phenomenon before the $M$7.1 Zhaotong earthquake of May 11, 1974. Imoto (1991) studied the temporal variations of $b$-values for microearthquake activity prior to seven $M>6$ earthquakes occurred in the Kanto, Tokai, and Tottori areas. The temporal variation of $b$-values for the Sep. 1984, $M$6.8 western Nagano earthquake in the Tokai area shows a decrease in $b$-values about 2 years before the occurrence of the mainshock. Similar phenomena were also observed before the Feb. 1983, $M$6.0 southwestern Ibaraki earthquake, the Oct. 4, 1985 $M$6.1 southern Ibaraki earthquake, the Dec. 17, 1987 $M$6.7 Chiba earthquakes, and the Oct. 31, 1983 $M$6.2 Tottori earthquake. From the temporal variation in $b$-value within the subducting slab prior to the 2003 $M$8.0 Tokachi-oki earthquake, Japan, Nakaya (2006) stressed a decrease of $b$-values before the mainshock. In the Longmenshan fault zone along which accommodated the May 12, 2008 Wenchuan, China, $M$8.0 earthquake took place, Zhao and Wu (2008) stressed that the temporal variation of $b$-value before the mainshock shows a weak trend of decreasing, thus being hard to be used as an indicator of the approaching of the mainshock. It is noted that their temporal variation in $b$-values indeed shows the $b$-value first increased from a low value to the peak one and then decreased until the occurrence of the mainshock. Nanjo et al. (2012) observed a decrease in $b$-value in the source regions prior to the 2004 $M$9.2 Sumatra, Southern Asia, earthquake and the 2011 $M$9.0 Tohoku, Japan, earthquake. Since their time period considered for measuring the $b$-values for each great earthquake is very long, several $M\geq 7$ events happened during the time period. It is difficult to identify the respective effect on the variations in $b$-values caused by each event. Their observations are different from that by DasGupta et al. (2007). Schurr et al. (2014) observed a decrease in $b$-value in the source regions prior to the $M$8.1 Iquique, Northern Chile, earthquake of April 1, 2014.

It is significant to relate the precursor time, $T$ or $T_p$, to the magnitude, $M$, of a mainshock expected. Generally, the larger the earthquake magnitude, the longer is the precursor time (cf. Scholz et al. 1973; Rikitake 1975a). Tsubokawa (1969, 1973) first obtained a linear relation between the precursor time of crustal movement and magnitude of mainshock expected in the form of $\log(T)=q+rM$ ($T$ usually in days) where $q$ and $r$ are, respectively, the coefficient and slope of the linear equation. His regression equation for Japanese earthquakes is $\log(T)=-1.88+0.79M$. This indicates that $T$ exponentially increases with $M$. After analyzing the data of various earthquake precursors (including land deformation, tilt and strain, foreshocks, $b$-value, microseismicity, source mechanism, fault creep anomaly, $v_p/v_s$, $v_p$ and $v_s$, geomagnetism, earth current, resistivity, radon, underground water, and oil flow) amounting to 418 in number, Rikitake (1975b) related the precursor time (indeed $T_p$) of a precursor to the magnitude, $M$, of the forthcoming mainshock in the following equation: $\log(T_p)=-1.83+0.76M$. He also stressed that the relationships of $T_p$ versus $M$ are different for different groups of precursors. After examining 192 from 391 cases of 19 types of earthquake precursors, Rikitake (1979, 1984) first defined three classes of precursors. Rikitake (1987) made revisions on the classification of precursors. The $b$-value anomaly belongs to the first kind of precursor, which can be described by a linear equation: $\log(T_p)=q+rM$. It is noted that his classifications are also valid when $T_p$ is replaced by $T$. Main and Meredith (1989) suggested that the different classes of earthquake precursors, as described by Rikitake (1987), can be placed into a fracture mechanics cycle based on the stress intensity.

Smith (1981) obtained the following relationship, i.e., $\log(T)=1.42+0.30M$, from his data of precursor times of abnormal $b$-values. His relationship is different from that inferred by Rikitake (1975b). It is noted that the precursor times measured by Smith (1981) are $T$ and those compiled by Rikitake (1975b) are $T_p$. Hence, the precursor time from the former is in general longer than that from the latter. Smith (1986) also correlated $T^*$ to $T$-$T^*$ in the following relationship: $T^*=0.24(T\text{-}T^*)$, that is, $T^*$ is about one fifth of $T$-$T^*$ for New Zealand's earthquakes.

On the other hand, from the data for abnormal $b$-values compiled from Rikitake (1975b) and Smith (1981) and some new ones, Imoto and Ishiguro (1986) made a conclusion with a different relationship from the abovementioned linear equations. They assumed that $\log(T)$ almost linearly increases with $M$ when $M\leq 6.5$ and decreases with increasing $M$ when $M>6.5$. This problem will be discussed below.

Numerous factors can influence the $b$-value and its changes. A review about the factors can be seen in Frohlich and Davis (1993). Observations and experiments exhibit that the factors are (1) the tectonic conditions (e.g., Miyamura 1962; Wang 1988; Tsapanos

1990; Khan et al. 2011); (2) the rock type, stress, and confining pressure (Scholz 1968; Rikitake 1984; Main et al. 1992); (3) self-similarity of geological structure (King 1983); (4) structural heterogeneity (Mogi 1967, Aki 1985; Main et al. 1992; Patane et al. 1992; Wang et al. 1989); (5) hydrothermal activity (Wang 1988); and (6) types of faulting (Schorlemmer et al. 2005). From acoustic emission experiments, Meredith and Atkinson (1983) assumed an empirical relationship between $b$ and $K$, where $K$ is the stress intensity factor (with a dimension of force/area) of fault-zone materials, in the following form: $b = p - qK$, where $p$ and $q$ are two empirical constants. Main and Meredith (1989) first proposed a model to qualitatively explain the major temporal fluctuations in $b$-value in terms of the underlying physical processes of time-varying applied stress and crack growth under conditions of constant strain rate.

From the simulation results, Main (1992) and Wang (1994, 1995) showed that the $b$-value of the cumulative FM relation is less than that of the discrete FM one. Wang (1995) also found that there is a power-law correlation between $b$ and $s$, which is the stiffness ratio of the 1-D dynamical spring-slider model proposed by Burridge and Knopoff (1967): $b \sim s^{-2/3}$ for the cumulative frequency and $b \sim s^{-1/2}$ for the discrete frequency. Clearly, smaller $s$ results in higher $b$-value.

When the controlling factors change with time prior to a mainshock, the $b$-value would be time-dependent. From laboratory experiments of precursory microcracking in stressed rock samples, Main et al. (1989) considered two types of models from stress-time behavior: the first model for elastic failure (time-predictable model) and the second one for anelastic failure (involving precursory strain energy release). Their models can explain the major temporal fluctuations in $b$-value in terms of the underlying physical processes of time-varying applied stress and crack growth under a constant strain rate. The model predicts two minima in the $b$-value, separated by a temporary maximum of inflexion point; a corollary being that a single broad maximum would be expected in scattering attenuation. These fluctuations in $b$-values are consistent with reported "intermediate-term" and "short-term" earthquake precursors separated by a period of seismic quiescence. From the analyses of small-scale fracture processes expressed by acoustic emissions (AEs) through X-ray computer tomography (CT) scans of faulted rock samples with spatial maps of $b$-values, Goebel et al. (2012) found that geometric asperities identified in CT scan images were connected to regions of low $b$-values, increased event densities and moment release over multiple stick–slip cycles. Their experiments underline several parallels between laboratory findings and studies of crustal seismicity, for example, that asperity regions in lab and field are connected to spatial $b$-value anomalies. These regions appear to play an important role in controlling the nucleation spots of dynamic slip events and crustal earthquakes. Goebel et al. (2012, 2013) investigated variations in seismic $b$-value of acoustic emission events during the stress buildup and release on laboratory-created fault zones. They showed that $b$-values mirror periodic stress changes that occur during series of stick–slip events and are correlated with stress over many seismic cycles, and the amount of $b$-value increase related to slip events indicates the extent of the corresponding stress drop. Hence, they concluded that $b$-value variations can be used to approximate the stress state on a fault.

In this work, first we will compile and measure the precursor times, including $T$, $T_p$, and $T^*$ as displayed in Fig. 3, of abnormal $b$-values prior to forthcoming mainshocks from several source materials. Then, we will examine the correlations between $T$ and $M$, between $T_p$ and $M$, between $T^*/T$ and $M$, between $T^*$ and $T$, and between $T^*$ and $T$-$T^*$ and compare the results of this study with those that exist in previous studies. Although it is significant to explore region dependence of the correlations, only the average correlations are made for 45 mainshocks occurred in different tectonic provinces due to limited data.

## 2 Data

In this study, we compile the precursor times of abnormal $b$-values reported by Rikitake (1975b, 1978) and Smith (1981), who took the data from several source materials, and those measured by Hasegawa et al. (1975), Smith (1981), Wyss et al. (1981), Ma (1978, 1982), Wyss et al. (1981), Chen et al. (1984b), Srivastava et al. (1984), Chen et al. (1990), Imoto (1991), Sahu and Saikia (1994), Tsai et al. (2006), DasGupta et al. (2007), Zhao and Wu (2008), Lin (2010), and Chan et al. (2012). It is noted that the values of $T$ and $T_p$ have been considered by different researchers to be the precursor times. This produces the difference in the precursor times among different articles. In this study, we clearly separate the two precursor

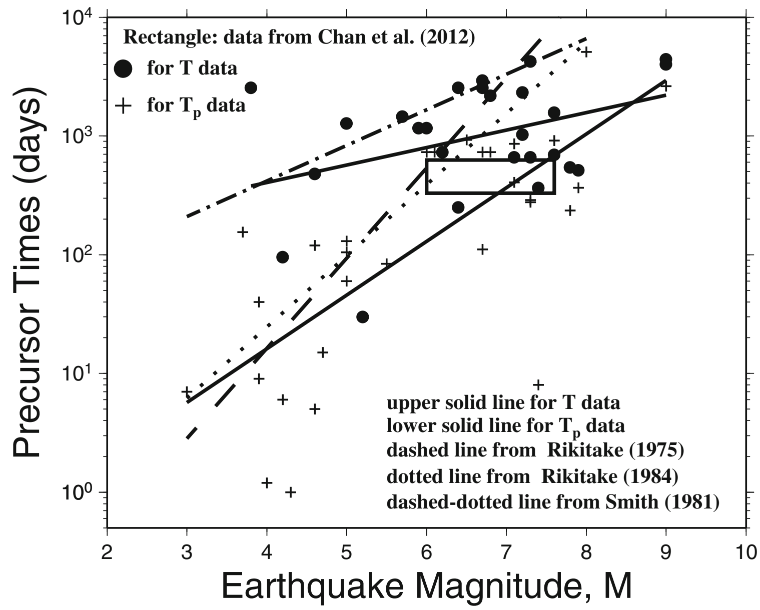


**Fig. 3** The plot of log($T$) versus earthquake magnitude, $M$, for abnormal $b$-values observed by numerous authors. The inferred linear relations are displayed by *solid lines*: the upper one for the $T$ data and the lower one for the $T_p$ data. The *dashed line* denotes the linear relationship inferred by Rikitake (1975b), the *dotted line* by Rikitake (1984), and the *dashed-dotted* by Smith (1981)

times. Some authors also measured the values of $T^*$. We also measure the values of $T$, $T_p$, and $T^*$ for some events from the temporal variations of $b$-values displayed in related articles. Such values are marked by the superscript letter in Table 1. We can measure the values of $T$, $T_p$, and $T^*$ only from the diagrams of temporal variations in $b$-values displayed in related articles. Since the diagrams are usually plotted in a unit of year or month, there are measurement errors which can be up to 30 days.

Rikitake (1975b) compiled a large data set of various earthquake precursors of 11 disciplines (including the $b$-value) as mentioned previously amounting to 418 in number accumulated from different source materials. For the $b$-value anomalies, he took the data of events with $3.0 \leq M \leq 6.5$ from Bufe (1970), Scholz et al. (1973), Wyss and Lee (1973), and Fiedler (1974). The precursor time shown in Rikitake (1975b) is $T'$ rather than $T$. In this study, eleven data of the $T_p$ values of precursory $b$-value are taken from Rikitake (1975b) and listed in Table 1.

From the temporal variation in $b$-values for the 1970 $M$6.2 southeast Akita, Japan, earthquake measured by Hasegawa et al. (1975), we evaluate the value of $T$. The result is listed in Table 1.

Smith (1981) measured the values of $T$ for six earthquakes with $3.8 \leq M \leq 6.8$. The six data are used in this study and listed in Table 1. We cannot measure the values of $T_p$ and $T^*$ for the six events, because he did not provide the temporal variations of $b$-values.

From the temporal variation in $b$-values for the November 29, 1975 $M$7.2 Kalapana, Hawaii, USA, earthquake made by Wyss et al. (1981), we measured the values of $T$ and $T^*$. The measured results are listed in Table 1.

The values of $T$ and $T^*$ for six Chinese earthquakes with $3.8 \leq M \leq 7.8$ were measured by Ma (1978) and those for two Chinese earthquakes with $M = 7.1$ and 7.9, respectively, done by Ma (1982). We evaluated the values of $T_p$ from the temporal variations in $b$-values shown in Ma (1978, 1982). The values of $T$, $T_p$, and $T^*$ are listed in Table 1.

From the temporal variations in $b$-values of four Chinese earthquakes made by Chen et al. (1984b), we measured the values of $T$ and $T^*$ for two of four events. For the $M$7.1 Zhaotong earthquake of May 11, 1974, the $b$-value directly decreased from the first measured one. We measured the values of $T$, $T_p$, and $T^*$ of the two events from the temporal variations displayed in Chen et al. (1984b), and the results are listed in Table 1.

Srivastava et al. (1984) observed a decrease in the $b$-value about 3.5 months and 15 days, respectively, prior to two small earthquakes (with $M = 5.0$ in 1968 and $M = 4.9$ in 1970, respectively) in the Himalayan Foothill region. Nevertheless, their temporal variations in $b$-values show the presence of higher $b$-values before it started to decrease. Their measured values of $T_p$ of the two events are listed in Table 1.

Imoto (1991) measured the temporal variations of $b$-values for microearthquake activity prior to seven $M > 6$

**Table 1** Data of earthquakes with magnitudes used in this study: location, occurrence year, magnitude, epicenter, three precursor times $T$, $T_p$, and $T^*$ (all in days) as displayed in Fig. 2

| Earthquake | Year | $M$ | Epicenter | $T$, $T_p$, $T^*$ (days) | Remarks |
|---|---|---|---|---|---|
| Fairbank, Alaska, USA | 1970 | 3.0 | – | (– ,7, –) | Rikitake (1975b) from Scholz et al. (1973) |
| Fairbank, Alaska, USA | 1970 | 5.0 | – | (–, 60, –) | Rikitake (1975b) from Scholz et al. (1973) |
| Danville, California, USA | 1970 | 4.3 | 37.8° N 121.9° W | (–, 1, –) | Rikitake (1975b) from Bufe (1970) |
| Danville, California, USA | 1970 | 4.0 | 37.8° N 121.9° W | (–, 1.2, –) | Rikitake (1975b) from Bufe (1970) |
| Danville, California, USA | 1970 | 3.7 | 37.8° N 121.9° W | (–,155,–) | Rikitake (1975b) from Wyss and Lee (1973) |
| Hollister, California, USA | 1971 | 3.9 | 36.7° N 121.3° W | (–, 40, –) | Rikitake (1975b) from Wyss and Lee (1973) |
| Hollister, California, USA | 1971 | 3.9 | 36.7° N 121.3° W | (–, 9, –) | Rikitake (1975b) from Wyss and Lee (1973) |
| Bear Valley North, California, USA | 1972 | 5.0 | 36.6° N 121.2° W | (–, 130, –) | Rikitake (1975b) from Wyss and Lee (1973) |
| Bear Valley South, California, USA | 1972 | 4.6 | 36.5° N 121.1° W | (–, 120, –) | Rikitake (1975b) from Wyss and Lee (1973) |
| Bear Valley South, California, USA | 1972 | 4.6 | 36.5° N 121.1° W | (–, 5, –) | Rikitake (1975b) from Wyss and Lee (1973) |
| Venezuela, Caracas | 1967 | 6.5 | 10.6° N67.3° W | (–, 930, –) | Rikitake (1975b) from Fiedler (1974) |
| Tejon Pass, New Zealand | 1961 | 5.0 | – | (1277.5, –, –) | Smith (1981) |
| Caracas, New Zealand | 1967 | 6.7 | – | (2556, –, –) | Smith (1981) |
| Inangahua, , New Zealand | 1968 | 6.8 | – | (2191, –, –) | Smith (1981) |
| San Fernando, , New Zealand | 1971 | 6.4 | – | (2556,–, –) | Smith (1981) |
| Region E, , New Zealand | 1971 | 5.9 | – | (1168, –, –) | Smith (1981) |
| Region C, , New Zealand | 1973 | 5.7 | – | (1460, –, –) | Smith (1981) |
| Region E, , New Zealand | 1971 | 6.0 | – | (1168, –, –) | Smith (1981) |
| Pukaki, New Zealand | 1978 | 3.8 | – | (2555, –, –) | Smith (1981) |
| Pukaki, New Zealand | 1978 | 4.6 | – | (480, –, –) | Smith (1981) |
| Tangshan, China | 1976 | 7.8 | 39.4 °N 118.2° E | (545, 235, 90) | Ma (1978) |
| Haicheng, China | 1975 | 7.3 | 40.7° N 122.8° E | (665, 276, 270) | Ma (1978) |
| Shintai, China | 1966 | 7.2 | 37.8° N 115° E | (1030, –, 425) | Ma (1978) |
| Bohai, China | 1969 | 7.4 | 38.2° N 119.4° E | (365, 8, 0) | Ma (1978) |
| Yanqing, China | 1967 | 5.5 | 40.7° N 115.8° E | (–, 84, 4) | Ma (1978) |
| Fongnan, China | 1975 | 4.2 | 39.5° N 118.0° E | (96, 6, 0) | Ma (1978) |
| Yongshan, China | 1974 | 7.1 | 28.2° N 104.1° E | (665, 406, 300) | Ma (1982) |
| Luhuo, China | 1973 | 7.9 | 31.5° N 100.4° E | (515, 366, 120) | Ma (1982) |
| Zhaotong, China | 1974 | 7.1 | – | (–, 861[a], –) | Chen et al. (1984b) |
| Longling, China | 1976 | 7.6 | – | (695[a], –, 333[a]) | Chen et al. (1984b) |
| Wenchuan, China | 2008 | 8.0 | | (–, 5110[a], –) | Zhao and Wu (2008) |
| Southeast Akita, Japan | 1970 | 6.2 | – | (730, –, –) | Hasegawa et al. (1975) |
| Southwestern Ibraki, Japan | 1983 | 6.0 | 35.9° N 140.1° E | (–, 730, –) | Imoto (1991) |
| Tottori, Japan | 1983 | 6.2 | 35.4° N 133.9° E | (–, 730, –) | Imoto (1991) |
| Nagano, Japan | 1984 | 6.8 | 35.8° N 137.6° E | (–, 730, –) | Imoto (1991) |
| Southern Ibraki, Japan | 1985 | 6.1 | 35.9° N 140.1° E | (–, 730, –) | Imoto (1991) |
| Hyogo-Ken Nanbu (Kobe), Japan, | 1995 | 6.9 | 34.58° N 135.01° E | (2637[a], 1111[a], –) | Enescu and Ito (2001) |
| Tohoku, Japan | 2011 | 9.0 | 38.32° N 142.37° E | (4450[a], 4085[a], –) | Tormann et al. (2015) |
| Kanto, Japan | 1987 | 6.7 | 35.4° N 133.9° E | (–, 730[a], –) | Imoto (1991) |
| Western Himalayan Foothills | 1968 | 5.0 | – | (–, 105[a], –) | Srivastava et al. (1984) |
| Western Himalayan Foothills | 1970 | 4.7 | – | (–, 15[a], –) | Srivastava et al. (1984) |
| India-Myanmar Border Region | 1988 | 7.3 | 25.1° N 95.1° E | (4259[a], 288[a], –) | Sahu and Saikia (1994) |
| Taipingshan, Taiwan | 1983 | 6.4 | – | (251, –, 5.7[a]) | Chen et al. (1990) |
| Chi-Chi, Taiwan | 1999 | 7.6 | 23.9° N 120.8° E | (1577[a], 913, 555[a]) | Tsai et al. (2006) |

**Table 1** (continued)

| Earthquake | Year | $M$ | Epicenter | $T$, $T_p$, $T^*$ (days) | Remarks |
|---|---|---|---|---|---|
| Taoyuan, Taiwan | 2008 | 5.2 | 23.2° N 120.7° E | (30, –, 0) | Lin (2010) |
| Kalapana, Hawaii, USA | 1975 | 7.2 | – | (2340, –, 697[a]) | Wyss et al. (1981) |
| Sumatra, Southern Asia | 2004 | 9.0 | 3.2° N, 95.8° E | (4015[a], 2628[a], 1819[a]) | DasGupta et al. (2007) |

[a] The values of $T$, $T_p$, and $T^*$are measured by the authors of this study

earthquakes that occurred in the Kanto, Tokai, and Tottori areas, Japan. The temporal variation of *b*-values for the Sep. 1984, *M*6.8 western Nagano earthquake in the Tokai area shows a decrease in *b*-values about 2 years before the occurrence of the main shock. Similar phenomena were also observed before the Feb. 1983, *M*6.0 southwestern Ibaraki earthquake, the Oct. 4, 1985 *M*6.1 southern Ibaraki earthquake, the Dec. 17, 1987 *M*6.7 Chiba earthquakes, and the Oct. 31, 1983 *M*6.2 Tottori earthquake. Their precursor times are indeed $T_p$ and the measured values are all ~730 days and listed in Table 1.

Sahu and Saikia (1994) evaluated the temporal–spatial distribution of *b*-values in the preparation zone (21°–25.5° N, 93°–96° E) before the August 6, 1988 *M*7.3 India–Myanmar Border Region earthquake. Results show that the *b*-value increased gradually from 1976 to a maximum value of 1.33 during July, 1987, followed by a short-term drop before the occurrence of the earthquake. From Fig. 2 of their paper, we can only evaluate the values of $T \approx 4259$ days and $T_p \approx 288$ days. The results are listed in Table 1.

Enescu and Ito (2001) studied temporal variation in *b*-values for the events with $M \geq 2$ prior to the *M*6.9 Hyogo-Ken Nanbu (Kobe), Japan, earthquake of January 17, 1995. The mean *b*-value is about 0.8. There are two significant changes in *b*-value: a decrease from 1984 to 1986 and an increase followed by a decrease from 1991 to 1994. The former change might be associated with the occurrences of many moderate-sized earthquakes, including events with $M \geq 5$ in the study area. From 1991 to 1994, there is an increase in *b*-value, followed by a decrease just before the Kobe earthquake. This anomaly in *b*-value corresponds closely with the seismic quiescence pattern. The large decrease in *b*-value that occurred during 1994 has a precursory character and correlates well with the increased seismic activity detected just before the Kobe earthquake. From Fig. 12b of Enescu and Ito (2001), we can see that the *b*-value increased from the beginning of 1988 and reached the peak in the beginning of 1992, then decreased until the occurrence of the mainshock. Hence, we can measure the values of $T$ and $T_p$ as $T \approx 2936$ days and $T_p \approx 1111$ days. The two values are listed in Table 1

DasGupta et al. (2007) analyzed the seismicity pattern and evaluated the temporal variation of *b*-values of earthquakes in north Sumatra–Great Nicobar region, India to search the precursor for the December 26, 2004 *M*9.0 Sumatra, Southern Asia, earthquake. We evaluated the values of $T$, $T_p$, and $T^*$ from their temporal variation of *b*-values. The results are listed in Table 1. Nanjo et al. (2012) also measured the *b*-values prior to the *M*9 Tohoku, Japan, earthquake of March 11, 2011 from the earthquake data recorded by the Japan Metrological Bureau and those prior to the *M*9.2 Sumatra earthquake of December 26, 2004 from the global data. Since they considered a very long time period for each great earthquake, several $M \geq 7$ events happened during the time period of measures of the *b*-values. It is difficult to distinguish the temporal variations in *b*-values for two consecutive events. Hence, their measured values are not included in this study.

Tormann et al. (2015) studied the spatial–temporal distributions of *b*-values for the *M*9 Tohoku-oki, Japan, earthquake of March 11, 2011. They considered four time periods that separate data before, in between, and after several very large events: T1—before the rupture of the Tokachi-oki *M*8 event, T2—3 months after the Tokachi-oki event up to the Tohokuoki *M*9 event; T3—3 months of the Tohoku-oki aftershocks; and T4—2013 onward. From the events within the 10-m-slip contour, the average *b*-value increased between T1 and T2 with an average *b*-value of $0.81 \pm 0.02$ and T3 with an average *b*-value of $1.08 \pm 0.04$ recovered by ~77 % in T4 with an average *b*-value of $0.87 \pm 0.03$. This might seem surprising on such a short timescale, as it indicates that stress conditions are noticeably rebuilding in the greater asperity area within 3 years from the *M*9 earthquake. However, the inference of a rapid stress recovery is consistent with a similarly interpreted observation from postmainshock stress rotations, which suggest a reloading of the asperity on the order of 6 % of the stress drop within the first 8 months. To better understand the medium-term impact of the Tohoku-oki event on *b*-values, they assessed the observed changes between pre-Tohoku-oki and 2013 onward (between T1 + T2 and T4), and compared those changes with the *b*-value differences observed between pairs of 2-year-long subsets of the catalog—that is, periods that are not dominated by large earthquake sequences. They added to this comparison the strong changes observed between pre-Tohoku-oki and the first 3 months of aftershocks (T1 + T2 versus T3), and also the impact of the Tokachi-oki event. From Fig. 3a of Tormann et al. (2015), we can see that the *b*-value increased up to the peak. This phenomenon lasted for about 1 year, and then the *b*-value decreased until the occurrence of the 2011 *M*9 Tohoku-oki earthquake. Hence, we measured the values of $T$ (≈4450 days) and $T_p$ (≈4085 days), which are listed in Table 1.

For the May 12, 2008 *M*8.0 Wenchuan, China, earthquake, which ruptured along the Longmenshan fault zone, Zhao and Wu (2008) evaluated the temporal variation of *b*-value from 1981 to the occurrence time of the mainshock. As shown in Figure 5b of their paper, the *b*-values varied very much and the temporal variation exhibits several local peaks during the time interval in study. It is difficult to measure the values of $T$ and $T^*$, because the authors did not plot the average *b*-value in that figure. Nevertheless, the value of $T_p$ can be estimated from the time of local peak *b*-value nearest the occurrence time of the mainshock. The value is ~14 years or ~5110 days, which is listed in Table 1.

For Taiwan's earthquakes, abnormally high *b*-values appeared in general about 1 year or one more years prior to the mainshock (Chen et al. 1990; Chen 2003; Tsai et al. 2006; Lin 2010). We measured the values of $T$, $T_p$, and $T^*$ from the temporal variations in *b*-values of three earthquakes: the May 10, 1983 *M*6.4 Taipingshan earthquake by Chen et al. (1990), the September 20, 1999 *M*7.6 Chi-Chi earthquake by Tsai et al. (2006), and the March 4, 2008 *M*5.2 Taoyuan earthquake by Lin (2010). The results are listed in Table 1. Chan et al. (2012) investigated the spatial and temporal variations of *b*-values before 23 earthquakes with $M \geq 6$ in Taiwan from 1999 to 2009. They estimated the spatial distribution of *b*-values in 5 years with a unit of 1 year before respective mainshocks. Results exhibit that the epicenter of each earthquake in study was located predominately in the area with low *b*-values. The *b*-values were slightly lower during the year prior to the target earthquakes than those in the periods 2 to 5 years earlier. Nevertheless, from the 5-year time variations (with a unit in 1 year) in average *b*-values of 23 earthquakes (see Fig. 5 of their paper), we can see that the highest average *b*-value appeared in the fourth time period prior to the forthcoming earthquakes. Although the average *b*-value in the fourth time period is only slightly higher than those in the first three time periods, results still suggest the presence of slightly higher *b*-values about 1 year prior to the forthcoming respective mainshocks. Their data are used in this study and displayed in Fig. 4, yet not listed in Table 1.

The values of $T$, $T_p$, and $T^*$ measured from temporal variations of *b*-values for 47 earthquakes with $3 \leq M \leq 9$ occurred in various tectonic regions are taken into account. In order to obtain a stable *b*-value, the area and time window of earthquakes in use must be large enough. Because the individual research groups as mentioned above used different time windows and different statistical methods to evaluate the *b*-values, there are time errors of several 10 days or several months in the temporal variations of *b*-values. This error makes us unable to accurately predict a forthcoming mainshock.

## 3 Results

All data in use are listed in Table 1. The data points of precursor time versus earthquake magnitude obtained

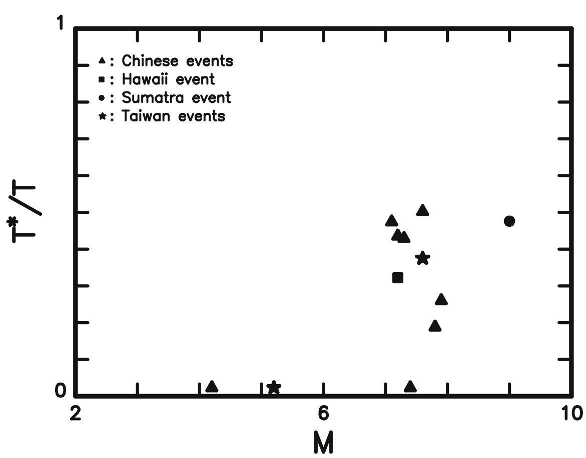


**Fig. 4** The *solid circles* denote the data points of $T^*/T$ versus $M$ (symbol: "*triangle*" for Chinese events; "*square*" for the Hawaii earthquake; "*diamond*" for the Sumatra earthquake; and "*star*" for Taiwan events)

from the table are plotted in Fig. 3 with two different symbols for two kinds of precursor times: solid circles for the $T$ data and crosses for the $T_p$ data. The data obtained from Chan et al. (2012) for 23 Taiwan earthquakes are also displayed in Fig. 3 with a rectangle. Obviously, the data points are quite dispersed. This might be due to two reasons: the first one is the differences in evaluations of $b$-values using different time and space windows. The second one is due to regional dependence of $b$-values. Nevertheless, the data points can be essentially separated into two groups even though some of them are mixed up together. The upper and lower groups belong to the $T$ and $T_p$ data sets, respectively. It can be seen that the data point associated with the 1969 $M$7.4 Bohai, China, earthquake (Ma 1978) (denoted by a cross) departs quite away from the main trend of data points for $T_p$.

In spite of the dispersion of data points and regional dependence of $b$-values as mentioned above, the linear relationships of precursor time versus earthquake magnitude are still inferred from the two data sets for the purpose of obtaining global averages. The resultant regression equations are as follows: $\log(T) = (2.02 \pm 0.49) + (0.15 \pm 0.07)M$ for the $T$ data set and $\log(T_p) = (-0.60 \pm 0.45) + (0.45 \pm 0.73)M$ for the $T_p$ data set. Obviously, standard deviations of the constant and the slope of linear portion for the two cases are acceptable. The two regression equations are displayed by the upper and lower solid lines, respectively, in Fig. 3. Since the data for $M > 9$ is lacking, the constrain of the regression equations for large $M$ is low. Hence, the two solid lines cross each other at $M = 9$. In addition, the data of 23 Taiwan events with $M \geq 6$ given by Chan et al. (2012) shown in the small rectangle are not taken to infer the two equations.

Figure 4 shows the plot of $T^*/T$ versus $M$. The ratio is lower for $M < 7$ earthquakes than for $M > 7$ ones. Although the data points are quite dispersed, we can still delineate an increasing trend of $T^*/T$ with $M$.

Figure 5 exhibits the data points of $T^*$ versus $T$. Clearly, most of data points are distributed along a line. The linear equation inferred from all data points are as follows: $T^* = (-127.17 \pm 414.00) + (0.53 \pm 0.03)T$. The standard deviation for the coefficient is quite high due to the large uncertainty of $T$ and $T^*$. This regression

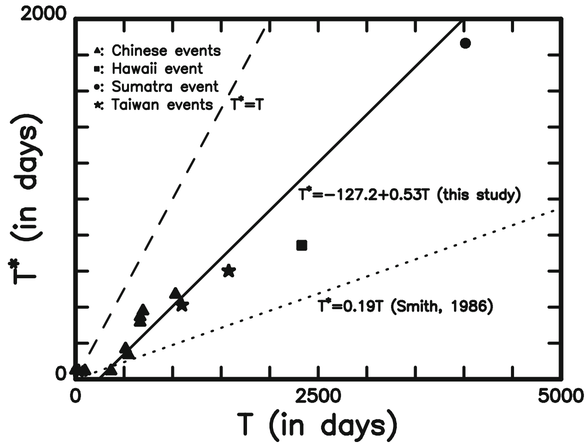


**Fig. 5** The *solid circles* denote the data points of $T^*$ versus $T$. The *solid line* represents the $T^*$-$T$ relationship inferred from the data. The *dotted line* denotes the $T^*$-$T$ relationship inferred by Smith (1986). The *dashed line* is the bisection one (symbol: "*triangle*" for Chinese events; "*square*" for the Hawaii earthquake; "*diamond*" for the Sumatra earthquake; and "*star*" for Taiwan events)

equation suggests that on average, the waiting time plus 127.2 days is about half of the precursor time.

Figure 6 exhibits the data points of $T^*$ versus $T$-$T^*$. Clearly, most data points are distributed along a line. The inferred linear equation from all data points are $T^* = (-99.58 \pm 187.71) + (0.73 \pm 0.05)(T\text{-}T^*)$. The standard deviation for the coefficient is quite high due to the large uncertainty of $T$ and $T^*$. On the average, the waiting time plus 100 days is about three fourth of the occurrence time of $b$-value anomalies.

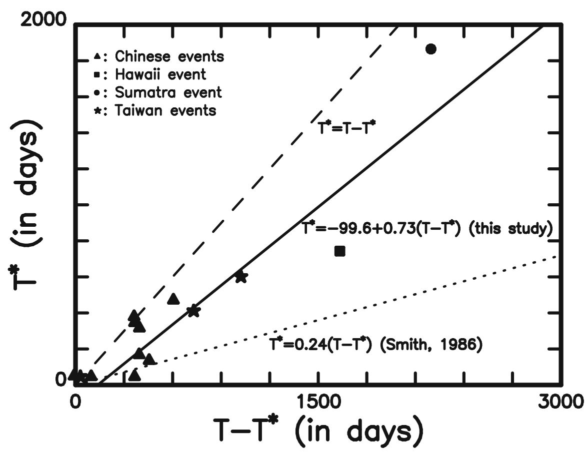


**Fig. 6** The *solid circles* denote the data points of $T^*$ versus $T$-$T^*$. The *solid line* represents the relationship of $T^*$ versus $T^*$-$T$ inferred from the data. The *dotted line* denotes the relationship inferred by Smith (1986). The *dashed line* is the bisection one (symbol: "*triangle*" for Chinese events; "*square*" for the Hawaii earthquake; "*diamond*" for the Sumatra earthquake; and "*star*" for Taiwan events)

## 4 Discussion

Obviously, all the data are collected from different publications made by previous researchers. Hence, the methods for calculating the *b*-values are not uniformed, together with different error bands. There are two common ways to calculate the *b*-value. In the first method, the *b*-value is the slope value of the linear data point portion of log*N* versus *M* in the range between $M_1$ and $M_2$ as shown in Fig. 1 by using the common least-squared method. The second is based on the maximum likelihood method. Aki (1965) proposed the formulae $b=\log(e)/(M_a-M_1)$, where $M_a$ is the average magnitude, to evaluate the *b*-value. In general, the *b*-value evaluation stability is higher from the second method than from the first. From practical calculations, Knopoff (2000) found that when $M_1$ is fixed, the maximum likelihood *b*-value estimate is a function of the variable upon $M_2$ and the *b*-value calculation stability increases with $M_2$. Of course, the choice of $M_1$ also influences the *b*-value estimate. It is usually necessary to consider the completeness of data for the estimate of *b*-value. The completeness of selected data will depend upon the threshold of magnitudes, i.e., $M_1$, the study area, choice of the sizes of bins for calculation, etc. In practice, it is usually necessary to take a time window for selecting complete data. The time window could be months to years and depend upon the study areas: for example, 3 to 6 months for Taiwan's earthquakes (Wang 1988; Chen et al. 1990) and 1 year for California's events (Wyss and Lee 1973). In order to improve the calculation stability, the moving time window technique, for which the *b*-value is measured using a *n*-event window sliding by *n*' events, has been used by some researchers. The *b*-values listed in Table 1 were estimated from different data sets through different time windows. In order to explore the calculation error of the *b*-value, the technique proposed by Shi and Bolt (1982) has been used by some researchers. From simulation results, Main (1992) and Wang (1994, 1995) showed that the *b*-value calculated from the cumulative FM relation is less than that from the discrete FM one. For a certain mainshock, the effect is minor when one of the two ways is used for estimating the *b*-values for the entire study time period.

Although there are numerous factors in influencing the estimates of *b*-values, we cannot test the *b*-value precursors as listed in Table 1 because the values were obtained from different sources. It is noted that the magnitude scales are not unified in the data set. Various magnitude scales can result in bias on the results. However, the magnitude scales for most of data listed in Table 1 are mainly the surface wave magnitude, $M_s$, or the moment magnitude, $M_w$. The two magnitude scales are almost the same, because the latter is defined based on the former by Hanks and Kanamori (1979). Hence, the influence is limited. As mentioned above, for some earthquakes, we measure the values of *T*, $T_p$, and *T** from the diagrams of temporal variations in *b*-values. Since the diagrams are usually plotted in a unit of year or month, there are measurement errors. The previously-mentioned problems can also influence the temporal variation in *b*-values and make it be somewhat different from Fig. 2 as displayed in some publications. This will affect the evaluations of *T*, $T_p$, and *T**. Hence, the temporal variations in *b*-values which are clearly different from Fig. 2 are not included in this study, because we cannot evaluate their precursor times.

Although the data points for both *T* and $T_p$ data sets as shown in Fig. 3 are dispersed due to the previously mentioned problems, a linear trend can still be delineated for the data points of respective sets. As mentioned above, the linear regression equations of precursor time versus earthquake magnitude inferred from the two data sets are as follows: $\log(T)=(2.02\pm0.49)+(0.15\pm0.07)M$ for the *T* data set and $\log(T_p)=(-0.60\pm0.45)+(0.45\pm0.73)M$ for the $T_p$ data set. Since the data points are clearly dispersed, the two can mainly exhibit the correlations between *T* and *M* and between $T_p$ and *M*. Hence, it is necessary to be careful in using the two equations for the purpose of prediction. The two equations are displayed, respectively, by the upper and lower solid lines in Fig. 3. The precursor times for Taiwan's earthquakes measured by Chan et al. (2012) are just in between the two lines. For the two equations, the slope value is smaller for the *T* data than for the $T_p$ data. This means that the increasing rate of precursor time with earthquake magnitude is lower for *T* than for $T_p$. The data points and the two equations exhibit that the difference of *T* and $T_p$, i.e., $T$-$T_p$, decreases with increasing *M*. This means that the time needed for an increase of *b*-value from the initial abnormal one to the peak one is shorter for larger-sized earthquake than for smaller-sized ones.

As mentioned above, Rikitake (1975b) proposed a linear equation to relate the precursor time, $T_p$ (in days), of a precursor to the magnitude, *M*, of the forthcoming mainshock, that is, $\log(T_p)=-1.83+0.76M$. After reanalyzing the data, Rikitake (1984) suggested a new linear equation: $\log(T_p) = -1.01 + 0.60M$. On the other hand, Smith (1981) observed an increase in $b$-value from the normal value to the peak one and then a decrease from the peak one to the normal or even a lower one before the forthcoming mainshock within the vicinity of its source area. Hence, his precursor time is the $T$. He related $T$ (in days) to $M$ in the following form: $\log(T) = 1.42 + 0.30M$. His results show that the abnormal $b$-values appear, at least, 1 year before the respective forthcoming mainshocks when $M \geq 4$.

Obviously, there are differences between Smith's and Rikitake's equations. Such differences might be due to three reasons: the first reason is that the two equations were inferred from different precursor times, $T_p$ in Rikitake (1975b) and $T$ in Smith (1981). The second reason is that the two equations were obtained from different earthquakes that occurred in different regions. The precursor time could vary area to area. The third reason is that Rikitake's equation was inferred from all kinds of precursors he collected and Smith's was only from the $b$-value anomaly.

The linear relationships inferred by Rikitake (1975b), Rikitake (1984), and Smith (1981) from their individual data sets are displayed, respectively, a dashed line for $\log(T) = -1.83 + 0.76M$, a dotted line for $\log(T) = -1.01 + 0.60M$, and a dashed-dotted line for $\log(T) = 1.42 + 0.30M$ in Fig. 3. The two solid lines of this study are obviously deviated from the dashed and dotted lines inferred by Rikitake (1975b, 1984). This means that the two regression equations inferred by Rikitake (1975b, 1984) cannot be applied to describe the data points of $T$ or $T_p$ versus $M$ of this study. Although the upper solid line for the $T$ data of this study is close to the dash-dotted line inferred by Smith (1981), the slope value of this study is lower than Smith's. This means that the precursor time for larger-sized earthquakes is shorter for this study than for Smith's. This can be seen from Fig. 3 in which numerous larger-sized events are located below the dash-dotted line.

As mentioned above, Imoto and Ishiguro (1986) assumed that $\log(T)$ almost linearly increases with $M$ when $M \leq 6.5$ and decreases with increasing $M$ when $M > 6.5$. Their result is obviously different from the two equations of this study. From their Fig. 9, we can see that the data points with the precursor time $T$ for the events with $M \leq 6.5$ were taken from Smith (1981) and those with the precursor time $T_p$ for the events with $M > 6.5$ from Rikitake (1975b). Clearly, they put two different precursor times together for one purpose. This can lead to an incorrect conclusion, because $T_p$ is usually smaller than $T$ as mentioned above. In addition, in their work, lack of data for very large-sized events, such as the December 28, 2004 $M$9 Sumatra, Southern Asia, earthquake and the May 12, 2008 $M$8 Wenchuan, China, earthquake as used in this study, could also result in a bias.

It is significant to explore the implications of the relationships of precursor time versus earthquake magnitude. From the $E_s$–$M$ law proposed by Gutenberg and Richter (1944): $\log(E_s) = 11.8 + 1.5M$, where $E_s$ is the seismic wave energy, we have $M \sim (2/3)\log(E_s)$. This leads to $\log(T) \sim bM \sim (2b/3)\log(E_s)$. $E_s$ is related to the strain energy, $\Delta E$, in the following form: $E_s = \eta\Delta E$ (see Kanamori and Heaton 2000), where $\eta$ is the seismic efficiency. Then, $T$ is correlated to $\Delta E$ in the following power-law form: $T \sim \Delta E^{2b\eta/3}$. This means that the precursor time increases, in a power-law form, with strain energy in a fault zone.

The seismic moment is defined to be $M_o = \mu\bar{u}A$ where $\mu$, $\bar{u}$, and $A$ are, respectively, the rigidity, average displacement, and area of a fault zone. $M_o$ scales with $L$ (fault length) in the following ways (see Kanamori and Anderson 1975): (1): $M_o \sim L^2$ for small events; (2) $M_o \sim L^2$ for medium-sized events: and (3) $M_o \sim L$ for larger-sized events. In general, $M_o \sim L^{\mathrm{n}}$ ($n = 1$ or 2). From the law $\log(M_o) = 16.1 + 1.5M$ proposed by (Purcaru and Berckhemer 1978), we have $M \sim (2\mathrm{n}/3)\log(L)$. This gives $T \sim L^{2b\mathrm{n}/3}$. This means that the precursor time increases, in a power-law form, with length of a fault zone.

The plot of $T^*/T$ versus $M$ is displayed in Fig. 4. Although the data points are quite dispersed, $T^*/T$ still increases somewhat with $M$. For two Taiwan earthquakes with $M = 5.2$ and 6.4 and two Chinese events with $M = 4.2$ and 7.4, the value of $T^*/T$ is particularly low and approaches zero. For those events, the mainshock occurred almost immediately after the $b$-value returned to the normal one. Figure 4 suggests that the waiting time is in general shorter for the earthquakes with $M < 7$ than for those with $M > 7$.

Figure 5 exhibits that the data points are almost distributed along a line. The inferred linear equation for all data points are: $T^* = (-127.17 \pm 414.00) + (0.53 \pm 0.03)T$, which is shown by a solid line in Fig. 5. This means that $T^* + 127.2$ is about half of $T$. From the equation between $T^*$ and $T$-$T^*$ for seven New Zealand earthquakes inferred by Smith (1986) as mentioned above, we can deduce the correlation between $T^*$ and $T$ in the following form: $T^* = 0.19T$, which is displayed by a dotted line in Fig. 5. Obviously, the two equations are quite different from each other, and the dotted line departs very much from and is below the solid line in Fig. 5. The $T^*$-$T$ relationship inferred from the earthquakes of New Zealand cannot be applied to those of other tectonic regions.

Since, $T$-$T^*$ and $T^*$ are, respectively, the time interval of presence of abnormal $b$-values and the waiting time of the occurrence of mainshock after the abnormal $b$-value returned to the normal one. We can predict $T^*$ if there is a good and reliable relationship between $T^*$ and $T$-$T^*$. Figure 6 exhibits the data points of $T^*$ versus $T$-$T^*$ from Table 1. Regardless of few data points, the data points follow a linear trend and the linear equation inferred from the data points are $T^*=(-99.58 \pm 187.71)+(0.73\pm 0.05)(T\text{-}T^*)$. This means that $T^*+100$ is about three fourth of $T$-$T^*$. This equation is quite significant for the purpose of predicting a forthcoming earthquake. For seven earthquakes of New Zealand with $5.40\leq M\leq 6.33$, Smith (1986) suggested a relationship between the two quantities in the form $T^*=0.24(T\text{-}T^*)$. This means that the waiting time of an earthquake is about one quarter of the time interval of abnormal $b$-values. Figure 6 shows that the dotted line for Smith's equation is below the solid line for the equation of this study. Figures 5 and 6 suggest that for a certain magnitude, the waiting time is shorter for an earthquake in New Zealand than for that in other tectonic regions.

The previous results suggest that the difference between $T$ and $T_p$, i.e., $T$-$T_p$, decreases with increasing $M$. In other words, the time needed for an increase of $b$-value from the initial abnormal one to the peak one is shorter for larger-sized earthquake than for smaller-sized ones. In addition, the observation that $T^*/T$ is smaller for $M<7$ earthquakes than for $M>7$ ones indicates that the waiting time is shorter for smaller-sized events than for larger-sized ones. The two results are significant for earthquake prediction.

Understanding the mechanism causing the $b$-value anomalies prior to forthcoming mainshocks and the abovementioned relationships is an important issue for earthquake prediction research. However, the mechanism is not yet clear and must be studied on the basis of basic studies as mentioned in Sect. 1 in advance.

## 5 Summary

Seismic observations exhibit the presence of abnormal $b$-values prior to forthcoming mainshocks. The precursor (or occurrence) time, $T$ (usually in days), of the moment of initial appearance of a precursor prior to the occurrence time of a forthcoming mainshock has been found to be related to the magnitude, $M$, of the event in a linear form as $\log(T)=q+rM$ where $q$ and $r$ are, respectively, the coefficient and slope of the linear equation. The values of $q$ and $r$ depend upon the type of precursor on the study and the study area. Essentially, there are two kinds of precursor times even used: the first one (denoted by $T$) is the time interval from the moment when the $b$-value starts to increase from the normal one to the occurrence time of the forthcoming mainshock, and the other (denoted by $T_p$) is the time interval from the moment when the $b$-value reaches the peak one to the occurrence time of the forthcoming mainshock. Of course, the former is longer than the latter. In addition, the waiting time from the moment when the abnormal $b$-value returned to the normal one to the occurrence time of the mainshock is also important and denoted by $T^*$. In this study, we compile and measure the values of $T$, $T_p$, and $T^*$ from the temporal variations of $b$-values measured for 45 earthquakes with $3\leq M\leq 9$ that occurred in various tectonic regions from several source materials. Given data are used to study the relationships between $T$ or $T_p$ and $M$, between $T^*/T$ and $M$, between $T^*$ and $T$, and between $T^*$ and $T$-$T^*$.

Results show several main concluding points: (1) the regression equations of precursor time versus earthquake magnitude are $\log(T)= (2.02\pm 0.49)+(0.15 \pm 0.07)M$ for the $T$ data set and $\log(T_p)=(-0.60 \pm 0.45)+(0.45\pm 0.73)M$ for the $T_p$ data set, and the two relationships are different from those inferred by Rikitake (1975a,b, 1984) and Smith (1981). (2) The plot of $T^*/T$ show an increase of $T^*/T$ somewhat with $M$, and the ratio is lower for $M<7$ earthquakes than for $M>7$ ones. (3) $T^*$ relates to $T$ in the following equation: $T^*=(-127.17\pm 414.00)+(0.53\pm 0.03)T$. (4) $T^*$ relates to $T$-$T^*$ in the following equation: $T^*=(-99.58 \pm 187.71)+(0.73\pm 0.05)(T\text{-}T^*)$, which is different from that inferred by Smith (1986) from seven New Zealand events, and thus for a certain magnitude, the waiting time is shorter for an earthquake in New Zealand than for that in other tectonic regions.

The inferred relationships indicate the following points: (1) both $T$ and $T_p$ increase with $M$; (2) the time, i.e., $T$-$T_p$, needed for an increase of $b$-value from the initial abnormal one to the peak one decreases with increasing $M$, thus indicating that $T$-$T_p$ is shorter for larger-sized earthquake than for smaller-sized ones; (3)

The precursor time increases, in a power-law form, with strain energy and length of a fault zone; and (4) the waiting time, $T^*$, is longer for larger-sized events than for smaller-sized ones. These results are very significant for earthquake prediction. Of course, it is necessary to further study region dependence of the abovementioned correlations and the mechanism causing the $b$-value anomalies and those relationships of precursor times and magnitudes.

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