

Are new data suggesting a revision of the current M_w and M_e scaling formulas?

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Abstract The paper looks first into the history of the derivation of the currently common formulas for calculating seismic moment magnitude M_w and energy magnitude M_e into the type of data and relationships available in these years and the parameter assumptions made. The general relationship between M_w and M_e is analysed and formulated in physical terms. The original M_w - and M_e -defining relationships are then confronted with equivalent relationships derived on the basis of rich modern magnitude data measured according to recently accepted International Association of Seismology and Physics of the Earth's Interior (IASPEI) standards for (a) 20-s surface-wave data and (b) broadband body P wave data as well as M_0 and E_S data based on digital broadband waveform inversion or integration. The agreement between old and new data and derived relationships is of different quality. The Richter $\log E_S$ - M_S relationship, which has been instrumental for deriving the current standard M_w formula, could be very well reproduced with orthogonally regressed $M_S(20)$ and $\log E_S$ data,

provided that the latter were not corrected for source mechanism-dependent radiation. In contrast, the relationships between old and modern m_B - $\log E_S$ as well as m_B - $M_S(20)$ data pairs deviate significantly from the respective Gutenberg and Richter relationships. Also the average E_S/M_0 ratio assumed by Kanamori when deriving his M_w formula differs from those of respective recent data sets. But the various differences between old and new data and data relationships compensate each other partially when deriving related M_w and M_e formulas. Therefore, they do not justify the modification of the existing scaling formulas, also for very pragmatic reasons. On the other hand, most striking is the so far not yet considered and by far best correlation that exists between the IASPEI body-wave magnitude standard $m_B(\text{BB})$ and seismic energy E_S , both estimated via P wave broadband records. The scatter of the $\log E_S$ - $m_B(\text{BB})$ data pair plots is only half of that of $\log E_S$ - $M_S(20)$. This questions the appropriateness of the current exclusive scaling of teleseismic M_e to the practically monochromatic long-period 20-s surface-wave magnitude M_S . The potential advantage of a complementary M_e formula, which scales the currently common teleseismic broadband P wave E_S data to P wave broadband $m_B(\text{BB})$, as well as the benefit of fast joint determination and interpretation of M_w and M_e in general, is discussed.

Prof. Peter Bormann passed away on 11 February 2015 when this manuscript had been reviewed with recommendation of minor revisions. The requested revisions were kindly made by Dr. Domenico Di Giacomo, who volunteered for preparing the final version of the paper. The Editor would like to express his sincere thanks to Dr. Domenico Di Giacomo. This is a small tribute to Prof. Bormann's memory.

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1 Introduction

For pure scientific purposes, it is usually considered best and sufficient to classify earthquake size solely according to the physically defined seismic moment M_0 , or to the product between rupture area and average slip, termed seismic potency P . However, both M_0 and P are pure static measures of earthquake size, i.e. of the earthquake's cumulative tectonic effect. For seismic hazard and risk assessment, more relevant would be the knowledge of the amount of radiated seismic wave energy E_S , of its high-frequency content in particular. E_S is closely related to stress drop and rupture velocity, i.e. to the rupture dynamics and kinematics and the related earthquake strength in terms of its potential of cause shaking damage. Therefore, the complementary determination and joint assessment of both M_0 and E_S is particularly useful for a more realistic quantification and assessment of the actual type of seismic or tsunami hazard (e.g., Newman and Okal 1998; Choy and Kirby 2004; Bormann and Di Giacomo 2011). Yet, there is a great disadvantage for practical application, early warning of people at risk included, of these linear earthquake size and strength classifiers. For relevant damaging earthquakes, when expressed in common physical international standard (SI) units for energy (Joule) and moment (Newton-metre), the latter reach numbers in the range from terra (10^{12}) to yotta (10^{24}) and even 10^7 times more in centimetre-gram-second (cgs) units, till now still widely used in the USA. Such astronomical numbers are hardly comprehensible and tractable for both ordinary people and decision-makers. This makes them not suitable for public information, guiding rapid and widely accepted disaster assessment and response actions or for calculating and communicating generally understandable seismic activity and probability of reoccurrence rates.

Therefore, the physical measures M_0 and E_S have been scaled to empirically derived logarithmic magnitude classifiers. Values of the latter vary in about the same range as the realistically discernible and thus relevant degrees of macroseismic intensities and related perceptibility and damage patterns or as the fractions of strong-motion ground accelerations in terms of earth gravity g . No wonder that the Global Earthquake Model (GEM) initiative insisted to get from the International Seismological Center (ISC) as a deliverable of particular importance for their efforts to improve probabilistic seismic hazard assessment on a global scale a

homogenized Global Instrumental Earthquake Catalogue (1900–2009). It classifies earthquake size magnitude-wise solely according to moment magnitude M_w , also for events for which no direct M_0 measurements were available (Storchak et al. 2015). Then proxy estimates of M_w via classical magnitudes had to be made (Di Giacomo et al. 2015). But the problem is that there exist several empirical magnitude scales. They are based on different types of seismic waves and ranges of periods measured. Therefore, it is not a trivial question to decide to which type of magnitude M_0 and E_S should be scaled best in order to derive meaningful and reasonably reliable conversion relationships and with which purpose in view. This will be illustrated in this paper by way of scaling $\log E_S$ data and thus M_e either to teleseismic surface or body-wave magnitudes.

First, the paper looks critically into the history of tackling these problems. It also investigates whether the currently common formulas to scale M_0 and E_S to magnitude are still appropriate and backed by standardized modern data. In fact, the original intermagnitude and magnitude-energy relationships used as well as some of the parameter assumptions made when deriving the formulas for calculating M_w and M_e are based on not well-constrained data sets, or questionable for some other reasons. The new data presented here have been measured according to recently adopted and in future to be applied International Association of Seismology and Physics of the Earth's Interior (IASPEI) magnitude measurement standards. Moreover, these data have been regressed with more appropriate statistical procedures than most of the classical relationships. The agreement between the magnitude relationships derived on the basis of old and new data is of different quality. It is better for M_w than for M_e . By far the best correlation exists, however, as reasoned already by Gutenberg (1956), when scaling $\log E_S$ to the new IASPEI (2005, 2013) broadband body-wave magnitude m_B (BB). The latter is based, with slight modifications, on Gutenberg (1945a, b) original medium- to long-period body-wave magnitude concept and thus an important complement to the dominating narrow-band short-period m_b since the mid 1970s. Reasons for discrepancies between old and new relationships for scaling M_0 and E_S to magnitude and recommendable actions will finally be discussed.

2 History

In the 1970s, Kanamori was interested in estimating the seismic energy E_S released in great earthquakes with rupture lengths of 100 km or more. At that time, neither E_S nor seismic moment data M_0 based on direct high-quality instrumental recordings were available en masse. The M_0 values available to Kanamori had mostly been estimated using long-period body waves, surface waves, very long-period free oscillations and geodetic data. In the absence of such data, M_0 of great earthquakes was derived via estimates of the area and slip of surface rupturing events and/or inferred from the 100-s magnitudes determined by Brune and Engen (1969).

Kanamori and Anderson (1975) had shown via dimension analysis that 20-s surface wave magnitudes M_S scaled for values between about 6.5 and 8 rather well with M_0 but that M_S tends to saturate for much larger M_0 values. Therefore, Kanamori (1977) proposed a “work magnitude” M_w scaled to and conceived as a linear extension of the not yet saturated M_S . Pursuing this, he assumed an average shear modulus μ in the crust and upper mantle of about $3\text{--}6 \times 10^4$ MPa and, according to Kanamori and Anderson (1975) and Abe (1975), a more or less constant stress drop $\Delta\sigma$ of large earthquakes ranging between about 2 and 6 MPa. With such values, Kanamori (1977) and Hanks and Kanamori (1979) arrived—via the Wyss and Brune (1968) relationship $E_S/M_0 = \Delta\sigma/2\mu = \text{apparent stress } \tau_a/\mu$ —at $E_S \approx M_0/(2 \times 10^4)$. This agreed on average reasonably well with the few scattering E_S - M_0 data available at that time (Fig. 1).

Inserting this E_S - M_0 relation into the relationship proposed by Richter (1958) between E_S and surface-wave magnitude M_s measured at periods around 20 s, which reads, when corrected for a typo,

$$\log E_S = 1.5M_S + 4.8, \tag{1}$$

it follows

$$\log M_0 = 1.5M_S + 9.1, \tag{2}$$

when E_S and M_0 are given here and in the following in SI units Joule J and Newton-metre Nm, respectively.

When substituting in (2) M_S by M_w and resolving it for M_w yields, then the definition formula for M_w calculations is

$$M_w = (\log M_0 - 9.1)/1.5 = (2/3)(\log M_0 - 9.1) \tag{3}$$

This way of writing has been accepted by IASPEI (2005, 2013) as the standard form. I has been derived by strictly following Kanamori’s way of reasoning but without prior resolving (3) and then rounding the constant with a different degree of precision. In contrast to various slightly different published M_w formulas, e.g. formulas (4) to (6) in Hanks and Kanamori (1979), there all given in units of dyne centimetre, (3) yields unambiguous results. Reasons of the IASPEI Working Group on magnitude measurements in favour of (3) have been outlined by Bormann and Dewey (2014):

The order of operations in the right-hand sides of equations for M_w , subtraction prior to division by 1.5, respectively multiplication by 2/3, avoids an ambiguity that arises if this operation is performed prior to subtraction, which in a certain percentage of cases leads to M_w being different according to whether the moment is expressed in CGS or SI units (Utsu 2002).

The effect of rounding the constant in the resolved formula (3) to one or two decimal places may lead in some cases to a difference of 0.1 magnitude units (m.u.) in the derived magnitude values. Therefore, Bormann and Choy (personal correspondence 2007) also agreed on the same way of standardized writing of the energy magnitude relationship [see formula (8)] instead of the earlier used expanded version with the constant rounded-off to one decimal place, i.e. $M_e = (2/3) \log E_S - 2.9$.

Both the US National Earthquake Information Center (NEIC) and the Global Centroid Moment Tensor (GCMT) Project (<http://www.globalcmt.org>) adopted (3) but with units in dyn cm. On the GCMT search front page, this change is commented as follows:

The moment magnitude is calculated by this software using the formula of Kanamori (1977), $M_w = (2/3) \cdot (\log M_0 - 16.1)$, where M_0 is given in units of dyne-cm. Prior to February 1, 2006, the quantity $(2/3) \cdot 16.1$ was rounded to the value 10.73. For a small number of earthquakes,

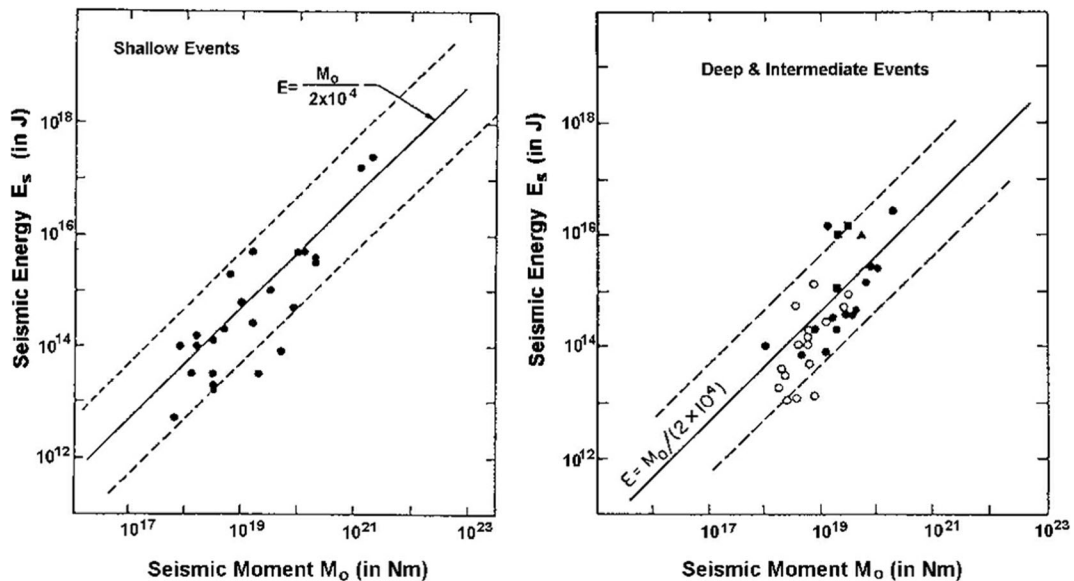


Fig. 1 Relations between seismic moment M_0 and energy E_S for shallow and deep earthquakes (according to Vassiliou and Kanamori 1982). The *solid line* indicates the relation $E_S = M_0 / (2 \times 10^4)$, suggested by Kanamori (1977) on the basis of

elastostatic considerations. Note that the real data may deviate from it by ± 1 order or even more. Copy from Kanamori 1983, *Tectonophysics*, Vol. 93, p. 191, with permission from Elsevier Science

searches conducted after 2006/02/01 will give values for M_w that differ by 0.1 magnitude unit from values given by searches prior to 2006/02/01.

In the following, we will relate only to the standard M_w relationship (3) and the respective M_e relationship (8) in SI units. M_w , although developed as a very adequate non-saturating magnitude scale for great earthquakes, is—because of its linear relationship with $\log M_0$ —also suitable for M_w determinations down to very small earthquakes, consistent and compatible M_0 determination provided. The same holds for the M_e formula (8). Regrettably, however, so far neither the procedures for M_0 nor those for E_S determination are standardized. They may differ significantly depending on observations made in the teleseismic, regional or local range (Lolli et al. 2014) and on earthquake size, duration and complexity (e.g. Tsai et al. 2005), whether the records are analysed in the frequency or time domain (Edwards and Fäh 2013) and in which magnitude-dependent period range (Stein and Okal 2005), and corrected, or not, for empirically confirmed or theoretically expected biases (Boatwright and Choy 1986 and discussion below). This may lead to incompatible results for equal events, with differences up to several tenths of magnitude units but all being labelled as

“ M_w ”. To avoid confusion and misinterpretation of such data, a unique specification of magnitude nomenclature is urgently required. Therefore, we flag in the following magnitude values of equal type differently when they have been produced by different institutions with different procedures. But with respect to M_w , only GCMT data are considered. They are based on a consistent centroid moment tensor inversion procedure according to Dziewonski et al. (1981) with some recent modifications (Ekström et al. 2012).

Because of

$$M_0 = \mu \bar{D} S \quad (4)$$

(with μ —rigidity or shear modulus of the source medium, \bar{D} —average final displacement after the rupture, S —the surface area of the rupture), M_w is principally a pure static measure of earthquake size in terms of the irreversible inelastic deformation in the rupture area. With (1) and (2), the M_w formula (3) can be decomposed according to Bormann and Di Giacomo (2011) into the following general formula:

$$M_w = (\log M_0 - \mathbf{a} + \mathbf{b}) / \mathbf{c}, \quad (5)$$

where \mathbf{a} is the constant and \mathbf{c} the slope in the $\log E_S - M_S$ relation (1) and \mathbf{b} the average ratio $\Theta = \log(E_S / M_0)$.

According to Kanamori’s (1977) condition $E_S \approx M_0 / (2 \times 10^4)$, one gets $\Theta = -4.3$.

In contrast to (1), the only E_S - M relationship proposed and authorized by Gutenberg (1956) (see Fig. 2) and jointly also by Gutenberg and Richter (1956) had been

$$\log E_S = 2.4m_B - 1.2. \tag{6}$$

Gutenberg (1945a, b) m_B is a medium- to long-period body-wave magnitude, measuring the largest ratio $(A/T)_{\max}$ between the displacement amplitude A and period T in the range $T=2$ to 20 s (see Abe 1981, 1984). Thus, m_B is more closely related to seismic energy release, also of smaller earthquakes, than 20-s M_S .

However, with the deployment in the 1960s and 1970s of the World-Wide Seismic Standard Network (WWSSN), which lacked medium-period broadband instrumentation, m_B determination was disbanded in western seismological practice. But the obvious benefits of m_B , being an important complement to the much earlier saturating short-period m_b , let the IASPEI (2005, 2013) Working Group on Magnitude Measurement recommend the reintroduction into global seismological practice of m_B as a broadband P wave body-wave magnitude complement, termed $m_B(\text{BB})$ (Bormann and Saul 2008; Bormann and Dewey 2014). It is based on direct measurement of the largest velocity amplitude $V_{\max} = 2\pi(A/T)_{\max}$ in the period range $0.2 \text{ s} < T < 30 \text{ s}$ and correlates excellent with $\log E_S$ (Fig. 4).

Note that Eq. (1) has been derived by inserting into (6) the Gutenberg and Richter (1956) relationship between body- and surface-wave magnitudes

$$m_B = 0.63M_S + 2.5. \tag{7a}$$

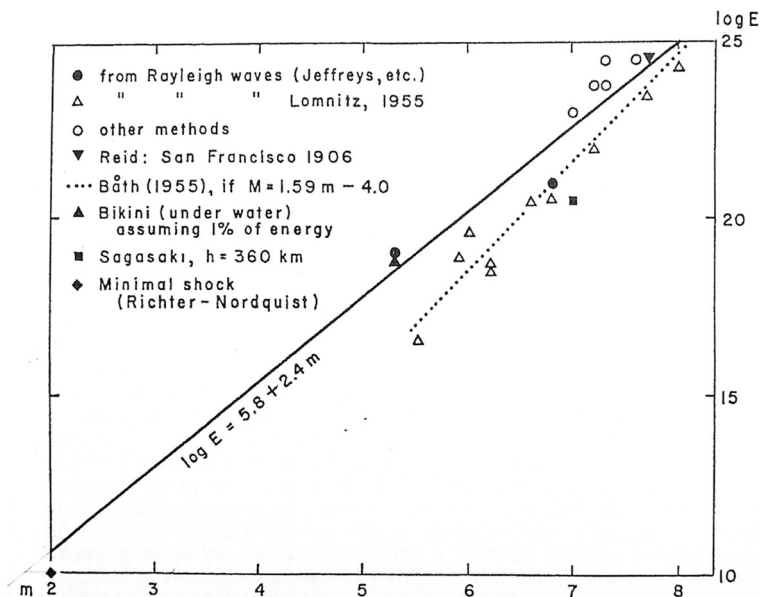
Thus, (7a), although not yet well constrained by data, has become the fundamental classical intermagnitude relationships that provided the very basis for the derivation of (3). Therefore, it is important to know that (7a) could be reproduced rather well by orthogonal regression of hundreds of IASPEI standard magnitudes $m_B(\text{BB})$ over $M_S(20)$ (Bormann et al. 2009):

$$m_B(\text{BB}) = 0.61M_S(20) + 2.63 \pm 0.24. \tag{7b}$$

According to (7a) and (7b), $m_B = M_S$ for $6^{3/4}$, $m_B > M_S$ for smaller and $m_B < M_S$ for larger values.

Two decades later, Choy and Boatwright (1995) were in a better position when proposing the energy magnitude scale M_e . They regressed hundreds of E_S values, calculated via the integration of squared P wave velocity amplitudes over the whole P wave train in the wide period range between about 1 and 100 s (Boatwright and Choy 1986), over 20-s M_S values of the US National Earthquake Information Center (NEIC). $M_S(20)$ has been confirmed by IASPEI (2005, 2013) as one of the two standard surface-wave magnitudes, besides the newly proposed velocity broadband version $M_S(\text{BB})$. The latter, however, is better tuned than $M_S(20)$ to the $\log(A/T)$ -based M_S calibration function according to

Fig. 2 Primary relation between seismic energy E_S and the “unified” magnitude $m = m_B$, with E_S in units of dyne centimetre, as published first by Gutenberg (1956). Note that this fundamental relationship is just an eye fit through only 21 data points. Copy of Fig. 1 in Gutenberg (1956)



Vaněk et al. (1962), IASPEI standard since 1967, applicable in a much wider range of periods and epicentral distance down to 2° , and differs less than $M_S(20)$ from M_w for values <6.5 (Bormann et al. 2013; Bormann and Dewey 2014).

Choy and Boatwright (1995) least square fitted the $\log E_S - M_S(20)$ data available to them with a prescribed slope of 1.5, as in (1). Their intention was to assure as far as possible continuity with this widely accepted and applied Richter formula. But for the constant, they then got 4.4, instead of 4.8, as in (1). Substituting in the so revised formula M_S by M_e and resolving it for M_e , they arrived at (when written in the now recommended standard form for better recognition of its origin)

$$M_e = (\log E_S - a)/c = (\log E_S - 4.4)/1.5. \quad (8)$$

Because of the similar approaches of deriving and scaling M_w and M_e via $M_S(20)$ and due to the ratio $\Theta = \log(E_S/M_0)$ with $E_S/M_0 = \Delta\sigma/2\mu$, Bormann and Di Giacomo (2011) derived the following general relationship between M_e and M_w :

$$\begin{aligned} M_e &= M_w + [\log(\Delta\sigma/2\mu) + 4.7]/1.5 \\ &= M_w + [\log(E_S/M_0) + 4.7]/1.5. \end{aligned} \quad (9)$$

According to (9), the Choy and Boatwright M_e and the Hanks and Kanamori (1979) M_w are equal only for $\log(E_S/M_0) = -4.7$, i.e. for a lower global average stress drop than the one assumed by Kanamori (1977) and later by Hanks and Kanamori when deriving (3). However, for earthquakes with significantly different stress drop, and thus related kinematic-dynamic source parameters such as rupture velocity and radiation efficiency, M_e and M_w may differ by about one magnitude unit (m.u.) or even more (Choy and Kirby 2004; Kanamori and Brodsky 2004; Bormann and Di Giacomo 2011; Choy 2011). This illustrates the importance of measuring, as fast as possible, both M_w and M_e . While the former is just a measure of the cumulative average static tectonic effect of an earthquake, M_e is largely controlled by the variable dynamics and kinematics of individual earthquake ruptures. Accordingly, earthquakes of equal M_w may have a very different potential for causing either shaking damage or generating a tsunami (e.g., Choy and Kirby 2004; Di Giacomo et al. 2008).

3 Modern data

The above basic relationships, on which the current M_w and M_e formulas are based, could not or only marginally be reproduced by orthogonally regressing nowadays available directly measured $\log E_S$ data over standard $m_B(\text{BB})$ and $M_S(20)$ (see Fig. 3 and for regression procedure Castellaro et al. 2006 and Castellaro and Bormann 2007). $\log E_S(\text{GFZ})$ has been calculated with an automatic procedure developed at the GFZ German Research Centre for Geosciences, described by Di Giacomo et al. (2008, 2010a, b). It can run in real time and thus be of great benefit in conjunction with near real-time M_w data or good proxy estimates of M_w in tsunami early warning and related emergency response activities. No corrections for different source radiation patterns are applied to $\log E_S(\text{GFZ})$. The same applies to $m_B(\text{GFZ}) = m_B(\text{BB})$ according to Bormann and Saul (2008), $M_S(\text{USGS}) = M_S(20)$ or other types of classical catalogued magnitudes such as the Gutenberg m_B and M_S and the Richter M_L or to m_b . The plot shows that $\log E_S(\text{GFZ}) - m_B(\text{BB})$ data fit rather well whereas the scatter of $\log E_S(\text{GFZ}) - M_S(\text{USGS})$ is about twice as large. Also the slopes of the two regressions differ strongly.

None of the GFZ $\log E_S - m_B(\text{BB})$ regression relationships agrees with the respective Gutenberg-Richter formula (6), but the orthogonal $\log E_S(\text{GFZ}) - M_S(\text{NEIC})$ relationship agrees almost perfectly with the semi-heuristic Choy and Boatwright (1995) formula. This is not the case, however, for the regression relationships between $m_B(\text{BB})$ and $M_S(20)$ when compared with the Gutenberg-Richter $m_B - M_S$ relationship (7) (Bormann et al. 2013).

The methods for M_0 and E_S determination are not yet standardized. There exist procedural differences that may lead to different results. $\log E_S$ and M_e data published by the NEIC are the result of interactive off-line data analysis. They aim at best precision and the use of these data also for geo-diagnostic research problems (e.g. Choy 2011). This goes well beyond the common operational use of magnitude data. Since theory predicts a dependence of energy radiation on the source mechanism (Boatwright and Choy 1986), corrections, for strike-slip earthquakes in particular, are applied to USGS $\log E_S$ and thus M_e values, as soon as final CMT fault plane values are available. But E_S data corrected for idealized fault radiation pattern may differ significantly from respective uncorrected GFZ data

(Fig. 3), also when calculating $\Theta = \log(E_S/M_0)$ (Fig. 5). Reasons are given below.

Striking are the systematically larger $\log E_S$ values for strike-slip events and the generally larger (approximately doubled) data scatter of the USGS relations. The reason may be just the source mechanism corrections which are supposed to improve the accuracy of the data. They are, however, based on idealized theoretical calculations of source radiation and wave propagation. Generally, plane earthquake rupture surfaces and a homogeneous 1-D Earth velocity model with minimum path P wave propagation are assumed. But CMT fault plane solutions are smoothed average representations of the real rupture surface in space. They are derived from wavelengths larger than the rupture itself, thus considering the source as a point and not as a complex finite rupture. Accordingly, they cannot account for fault bending, branching and/or step overs. The orientation and radiation pattern of such non-planar fault segments and other rough rupture irregularities may significantly differ (up to more than 10°) from the calculated average planar fault plane. This may have a significant influence on the actual angular radiation pattern, especially of high-frequency energy s , which is not accounted for by simplified rupture and wave propagation models. Also the Earth medium along the propagation paths is more or less heterogeneous, resulting in scattering and

multi-pathing, especially of short-period waves which contribute much to the E_S estimates (see the pronounced signal codas in SP records). Therefore, Newman and Okal (1998), Pérez-Campos and Beroza (2001) and Bormann and Di Giacomo (2011) pointed out that the short-period energy will find its way to the stations irrespective of idealized theoretical assumptions. Okal and Talandier (1989) expressed that a “...magnitude concept, which ignores the exact focal geometry, could be a more robust measure of the true size of the event than one correcting for an expected small source excitation, but failing to account for non-geometrical effects”. The rather stable estimation of wave attenuation via coda-Q measurements, independent of the unknown actual source mechanism, is explained by multi-path wave scattering as well. Also Schweitzer and Kväerna (1999) could not confirm a significant source mechanism effect for short-period m_b .

4 Consequences for the M_w and M_e formulas

The new data raise the question whether there is a need to revise the current commonly used formulas (3) and (8) for M_w and M_e estimation. According to Figs. 3 and 4, the following orthogonal regression relationships hold, with RMSO being the

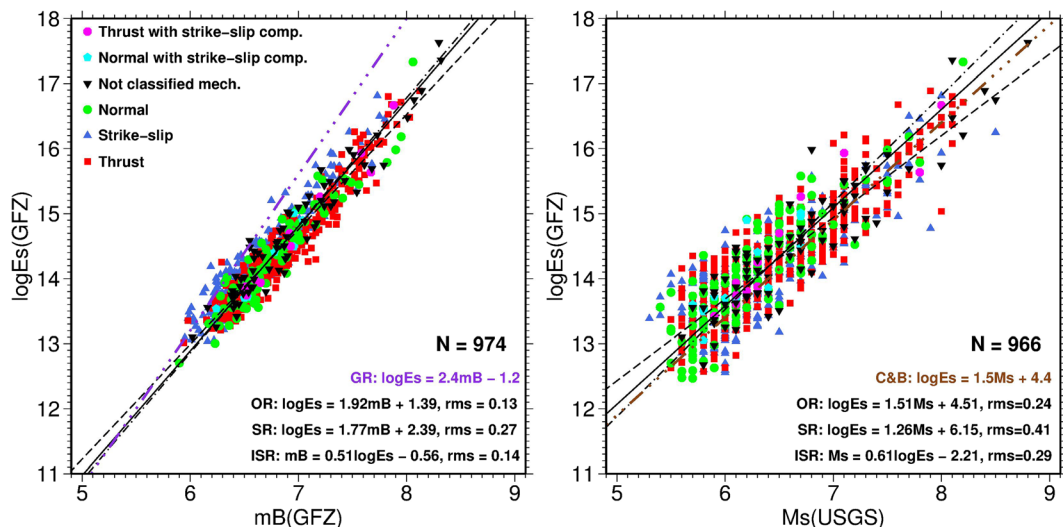


Fig. 3 Orthogonal standard (OR—solid line), standard (SR—broken line) and inverse standard regression (ISR—dot-hatched line) relationships between $\log E_S$ (GFZ) over m_B (GFZ) (left-hand panel) and 20-s M_S (USGS) (right-hand panel); rms root-mean-square data scatter. The E_S values are based on broadband records of earthquakes with different source mechanism. The

new $\log E_S$ -magnitude relationships are compared with the original Gutenberg and Richter (1956) $\log E_S$ - m_B relationship (GR, multi-dot hatched line) and the Choy and Boatwright (1995) $\log E_S$ - M_S relationships (C&B, multi-dot hatched line). Compiled from Figs. 3.85 and 3.86 in Bormann et al. (2013), © IASPEI

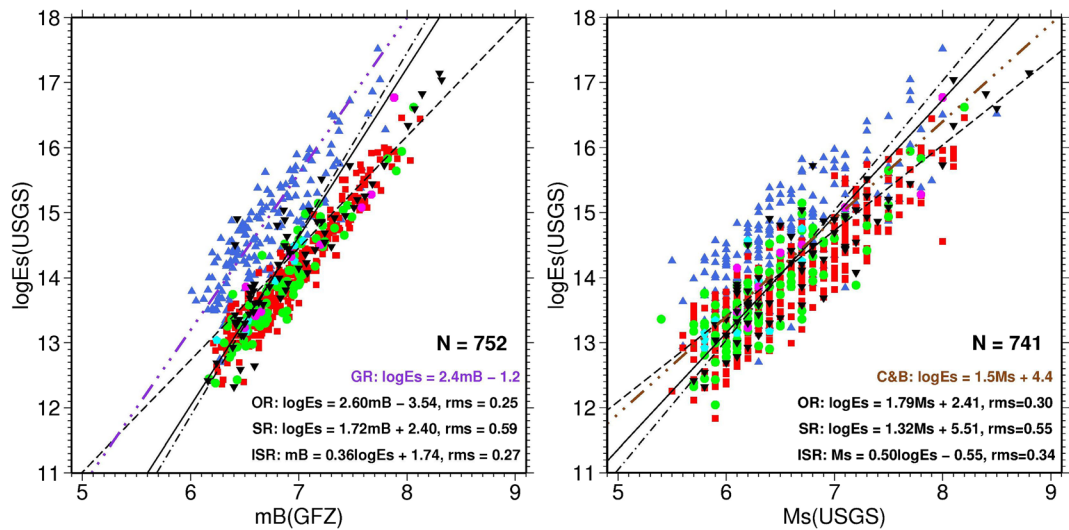


Fig. 4 Linear regression relationships between $\log E_S$ (USGS) over m_B (GFZ) (*left-hand panel*) and $\log E_S$ (USGS) over 20-s M_S (USGS) (*right-hand panel*), based on broadband records of earthquakes with different source mechanism, and their comparison with the original Gutenberg and Richter (1956) $\log E_S$ - m_B and

Choy and Boatwright (1995) $\log E_S$ - M_S relationships (*multi-dot hatched lines*). For legend of earthquake mechanism symbols, see Fig. 3. Compiled from Figs. 3.85 and 3.86 in Bormann et al. (2013), © IASPEI

root-mean-square data scatter orthogonal to the linear regression lines:

$$\log E_S(\text{GFZ}) = 1.51M_S(\text{USGS}) + 4.51 \text{ with RMSO} = 0.24 \quad (10)$$

$$\log E_S(\text{USGS}) = 1.79M_S(\text{USGS}) + 2.41 \text{ with RMSO} = 0.30 \quad (11)$$

$$\log E_S(\text{GFZ}) = 1.92m_B(\text{GFZ}) + 1.39 \text{ with RMSO} = 0.13. \quad (12)$$

According to Fig. 5, the global average values for Θ are

$$\Theta(\text{GFZ}) = -4.6 \quad (13)$$

and

$$\Theta(\text{USGS}) = -4.8, \quad (14)$$

respectively. The answer should be based on quantifying the difference between the values derived by the currently used and possible alternative new formulas. Moreover, new formulas should pay credit to the basic intention, methodology and arguments of Kanamori and Choy and recognize, with respect to the derivation of M_w , also the quantitatively and qualitatively inferior data available at that time. With this in view, we have first to acknowledge what remains valid and what has changed

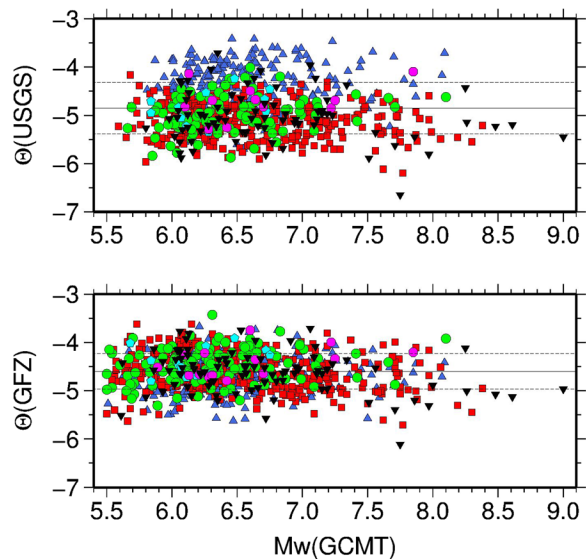


Fig. 5 Comparison between $\Theta = \log(E_S/M_0)$ calculated as the difference between $\log E_S$ of the US Geological Service (USGS), respectively, $\log E_S$ (GFZ) and $\log M_0$ according to the Global Centroid Moment Tensor Catalog (GCMT; <http://www.globalcmt.org/CMTsearch.html>) for earthquakes in the magnitude range $5.5 < M_w \leq 9.0$. The different symbols relate to different types of source mechanism, e.g. *upright triangles* to strike-slip (SS) earthquakes (for detailed legend, see Fig. 3). Note the much larger data scatter of the USGS data set with its distinct separation of SS earthquakes which have been corrected for theoretically expected reduced radiation coefficients according to Boatwright and Choy (1986). Copy of Fig. 3.89 in Bormann et al. (2013), © IASPEI

since the 1970s. According to the ISC-GEM Global Instrumental Earthquake Catalogue (1900–2009), the $M_S(20)$ type of magnitude is the longest available and most complete long-period magnitude (Di Giacomo et al. 2015). Therefore, transforming non-saturating M_0 data of great earthquakes into an equivalent magnitude scale by relating M_w to M_S remains the correct choice. Whether at a later time a scaling to $M_S(BB)$ that is measured in the much wider range of periods between $3 s < T < 60 s$ will be more appropriate may be looked at when masses of standard $M_S(BB)$ values are available. But different to Gutenberg’s and early Kanamori’s time, now hosts of both instrumentally measured M_0 , E_S and thus $\Theta = \log(E_S/M_0)$ values as well as a much better constrained and directly measured $\log E_S$ - M_S relationship are available. When inserting into the general M_w formula (5), which reflects best the essence of Kanamori’s approach, the slope and constant of (10) would yield, together with the Θ of (13), as an alternative M_w formula

$$M_w = (\log M_0 - 9.11) / 1.51. \tag{15}$$

(15) is practically identical with the current standard formula (3) although the constant in (10) and $\Theta = -4.6$ differ more strongly from the respective values assumed by Kanamori. But since the constant in (1), being b in (5), depends on the average value for Θ , being c in (5), their interrelationship balances off this difference. M_w values calculated via (15) for $\log M_0$ between about 17 and 23 (i.e. $M_w \approx 5.3$ to 9.3) differ only within -0.04 and -0.07 m.u. from those calculated via (3). Thus, formula (15) confirms the correctness of the current M_w formula within the IASPEI aimed tolerance limits of < 0.1 m.u. for different procedures of magnitude estimation. Yet, this also hints to the general correctness of regressing $\log E_S(GFZ)$ data—which have not been corrected for source mechanism-dependent radiation coefficients—orthogonally over $M_S(USGS) = M_S(20)$. In contrast, the constants based on orthogonally regressed $\log E_S(USGS)$ data over $M_S(USGS)$ according to Fig. 4 (right) and formula (11) are, together with $\Theta(USGS) = -4.8$, $a = 2.41$, $b = -4.8$ and $c = 1.79$. These values would yield as a new M_w formula

$$M_w = (\log M_0 - 7.21) / 1.79. \tag{16}$$

M_w values calculated via (16) with $\log M_0$ (respectively M_w) values in the same range as above would

differ from those derived with the current M_w standard formula (3) between $+0.20$ und -0.45 m.u. Such large differences, if accepted as correct, might suggest the need to replace formula (3) by (16). Interestingly, however, the more heuristically derived relationship by Choy and Boatwright (1995)

$$\log E_S = 1.5 M_S + 4.4, \tag{17}$$

aimed at preserving the Richter slope of 1.5 for the sake of data continuity, would yield together with the new average $\Theta(USGS) = -4.8$ according to Fig. 5, instead of Kanamori’s (1977) value of $\Theta = -4.3$,

$$M_w = (\log M_0 - 9.2) / 1.5. \tag{18}$$

Again, this is very close to (3). (18) yields, on average, M_w values that are constantly only 0.07 m.u. smaller than those derived with the current standard formula. Thus, the current M_w standard formula is well supported within these error limits by modern instrumentally measured $\log E_S$ data, better than the currently used M_c formula.

With respect to $\log E_S$ and M_c , the following conclusions can be drawn from Figs. 4 and 5:

- Relationship (10), i.e. when scaling $\log E_S(GFZ)$ to $M_S(USGS)$, yields in the whole considered magnitude range between M_S 5.2 and 8.8 values of $\log E_S$ that are only between $+0.16$ and $+0.20$ log units larger than those calculated via (17). This is less than the general uncertainty of energy calculations. According to McGarr and Fletcher (2002), E_S released by earthquakes can probably be estimated only within a factor of 2–3 (i.e. within 0.30 to 0.47 log units), if multi-station high-quality seismic broadband data are available. This corresponds to an uncertainty of about 0.2 to 0.3 m.u. in M_c .
- In contrast, when converting M_S into $\log E_S$ via (11), the differences reach for $M_S = 5.2$ and 8.8 already values of -0.42 and $+0.56$ $\log E_S$ units, respectively. This is unacceptably large.
- Thus, the orthogonal regression between $\log E_S(GFZ)$ and $M_S(USGS)$ constrains the Choy and Boatwright (1995) formula (8) much better than the orthogonal regression between $\log E_S(USGS)$ and $M_S(USGS)$ data.

Substituting in the relationships (10) and (11) M_S by M_e , and resolving them for M_e , yields

$$M_e = [\log E_S(GFZ) - 4.51] / 1.51 \quad (19)$$

and

$$M_e = [\log E_S(USGS) - 2.41] / 1.79 \quad (20)$$

respectively.

Calculating M_e via relationship (19) for $\log E_S$ values of 12 and 18, i.e. M_e between about 5 and 9, yields values that are in this range only between 0.11 and 0.14 m.u. smaller than those calculated with the current M_e formula (8). This is acceptable within the uncertainty range of M_e determinations, not, however, when calculating M_e with (20). Values will then in the considered magnitude range differ from those calculated with (8) between +0.40 and -0.36 m.u. Again it holds that $\log E_S(GFZ)$ data better justify the current semi-heuristic USGS M_e formula than a more correct orthogonal regression of $\log E_S(USGS)$ data over $M_S(USGS)$.

Generally, it holds, however, that E_S and M_e values are more “noisy” than asymptotic long-period M_0 and thus M_w estimates. Reliable energy calculations require inclusion of about one decade of frequencies higher than the corner frequency of the source spectrum (Bormann and Di Giacomo 2011). However, high-frequency waves are more difficult to treat theoretically and much more affected than low-frequency waves by unknown attenuation and velocity heterogeneities along the propagation paths, local site effects in particular.

In conclusion, the current IASPEI standard M_w formula according to Kanamori (1977) and the currently common teleseismic M_e formula according to Choy and Boatwright (1995), both being scaled to $M_S(20)$, yield magnitude values that agree within acceptable limits with alternative orthogonal regression relationships based on almost thousand $\log E_S(GFZ)$ and USGS $M_S(20)$ data as well as on the average of many hundreds of related $\Theta = \log(E_{SGFZ} / \log M_{0GCMT})$ values. This, however, is achieved only by not applying source mechanism corrections to $\log E_S(GFZ)$.

5 Should M_e better be scaled to $m_B(BB)$?

The above notwithstanding, it remains the principal question whether it is reasonable at all to scale E_S , estimated via the analysis of broadband P wave records, to

narrowband long-period 20-s surface wave M_S . Would it not be more appropriate and data compatible to compare such E_S estimates with m_B that is measured too on broadband velocity P wave records in a wide range of periods? The advantage is obvious from Fig. 4. Gutenberg’s (1956) energy-magnitude relationship (Fig. 2) had already been of this type. But such an approach could not be considered at the USGS in the 1980s, because m_B determinations had been disbanded at the NEIC in the 1970s. Yet $m_B(BB)$ is now being accepted and recommended by IASPEI as one of the important complementary broadband standard magnitudes to be determined in future day-to-day practice at regional and global seismological data centres (see Resolution 1 of the 2013 IASPEI Assembly; http://www.iaspei.org/resolutions/resolutions_2013_gutenberg.pdf). This makes an alternative—or at least complementary—scaling of M_e a realistic option again. What are the likely consequences?

Taking the orthogonal $\log E_S(GFZ)$ - $m_B(BB)$ relationship in Fig. 3 (left) as a good starting data set for deriving such a new M_e formula, one would get, by replacing in (12) m_B by M_{emB} and resolving it for the latter,

$$M_{emB} = [\log E_S(GFZ) - 1.39] / 1.92. \quad (21)$$

The factor 1.92 is close to 2 which one would expect if $\log E_S$ is proportional to squared ground velocity amplitudes. It is also very close to the factor of 1.96 in the $\log E_S$ - M_L relationship for California derived by Kanamori et al. (1993). Energy-wise any factor near 2 makes much more sense than the factor 1.5 in the Richter relationship (1). However, such a new type of M_e , tentatively termed M_{emB} in contrast to M_{eMS} scaled M_S , would yield rather different values for the same $\log E_S$ values of 12 and 18 as assumed above:

$$\log E_S = 12 \rightarrow M_{emB} = 5.53; M_{eMS} = 5.06; \Delta M_e = +0.47$$

$$\log E_S = 18 \rightarrow M_{emB} = 8.65; M_{eMS} = 9.07; \Delta M_e = -0.42$$

Because of (7a) and (7b), formula (21) will on average yield larger $\log E_S$ values for high stress-drop earthquakes and those with magnitudes $\ll 6\frac{3}{4}$. The corner period of the source spectrum and thus the maximum of energy radiation occurs then at periods $\ll 20$ s. In contrast, for stronger, low stress-drop and low rupture earthquakes, M_{eMS} , scaled to 20-s surface waves, tends to yield larger $\log E_S$ values. This property would make M_{emB} particularly suitable for applications in earthquake

engineering, engineering seismology and shaking hazard assessment. Moreover, for slow and very slow tsunamigenic and tsunami earthquakes, the difference between M_w and M_{emB} would become even larger than for M_{eMs} and thus even more indicative and alarming. This would make the joint assessment of M_w and M_{emB} even more useful in actual event-related risk assessment, disaster response and management. Ideally, in the years to come, regional and global data centres should aim at testing these conclusions by calculating for several thousands of earthquakes M_e according to these two different but complementary approaches of M_e determination.

6 Summary discussion and conclusions

Classical magnitude and energy relationships, on which currently accepted formulas for calculating M_w and M_e are based, have been derived from either rather insufficient and inhomogeneous data and/or by applying not optimal statistical and correction procedures. Rich homogeneous modern data, amongst them magnitudes measured according to recently accepted and IASPEI (2005, 2013) recommended standards, yield after homogeneous orthogonal regression new relationships. These sometimes differ significantly from the classical M_w - and M_e -defining relationships and related parameter assumptions. However, the mutual relationship of these differences sometimes balances off. Thus, the correctness of Richter's (1958) $\log E_S$ - M_S relationship, originally derived via Gutenberg's $\log E_S$ - m_B and m_B - M_S relationships, could now for the first time be confirmed with well-constrained and orthogonal-regressed, direct instrumentally measured $\log E_S$ (GFZ) over M_S (USGS) = $M_S(20)$ data, not, however, by orthogonally regressing source mechanism-corrected $\log E_S$ (USGS) over M_S (USGS) data.

On the other hand, also the combination of the semi-heuristic Choy and Boatwright $\log E_S$ - M_S relationship (17) with Θ (USGS) = -4.8 instead of Kanamori's (1977) Θ = -4.3 yields an M_w formula that yields values which agree on average within <0.1 m.u. with those derived via the alternative new relationships based on rich modern data. This small average deviation is less than the IASPEI acceptable tolerance for magnitudes of the same type that have been estimated via different procedures.

Also the M_e formula (8) could be well reproduced with formula (19) via formula (10). In the considered M_S

and $\log E_S$ range, the values calculated with both formulas differ less than 0.15 m.u. This is negligible when compared with the uncertainty of about 0.2–0.3 m.u. even of very good M_e determinations. Thus, there is no need for revising the current M_w and M_e formulas. However, the suitability of source mechanism corrections applied to $\log E_S$ data prior to M_e calculation, practice at the NEIC, is questioned.

The best correlation between E_S and magnitudes exists with the newly proposed IASPEI (2005, 2013) broadband body-wave magnitude m_B (BB). It is the modern version of the original medium-period body-wave magnitude proposed by Gutenberg (1945a, b) and Gutenberg and Richter (1956). The easy to measure m_B (BB) allows to estimate M_e with a standard deviation $SD = \pm 0.19$ m.u. only. This is within the inherent error range of direct M_e determinations. When estimating M_e via $M_S(20)$ instead, SD is twice as large. Therefore, scaling body-wave $\log E_S$ estimates derived from velocity broadband P wave records to velocity broadband m_B (BB) body-wave magnitude would be more appropriate and physically more correct than scaling them to long-period and nearly monochromatic 20-s surface-wave M_S . m_B (BB) is measured in a wide range of periods, thus covering a wide range of the magnitude-dependent corner frequencies of seismic source spectra, at which the maximum of seismic energy is radiated (Bormann and Di Giacomo 2011). Using currently available m_B (BB) and $\log E_S$ data, a new M_{emB} formula has been derived. Values calculated with it may differ up to almost 0.5 m.u. from those calculated with the current formula scaled to $M_S(20)$. The new formula more correctly represents, as does m_B (BB), the energy release of medium to smaller size as well as of faster rupturing and high stress-drop earthquakes but underestimates the dominatingly long-period energy release of great, slow and low-stress-drop earthquakes (e.g. Fig. 3 of Bormann and Saul 2008) which is better represented by the current M_e formula scaled to $M_S(20)$. This would make M_{emB} very attractive for engineering seismological and hazard assessment applications and a useful complement to the currently common M_{eMs} .

But generally, one has to accept that both M_w (GCMT) and M_e (USGS), consistently determined and published for 30 to 40 years already in authoritative catalogues, constitute valuable data bases in their own right. Already for this reason alone, their calculation should be continued with the current formulas (3) and

(8). Most important in seismic hazard assessment is not how these M_w or M_c scales were initially calibrated but how we can infer from the available M_w and M_c values' hazard relevant ground motion and source parameters. Numerous subsequent equations are based on the existing formulae, so far mainly for M_w , e.g., estimating model-dependent ground motion parameters such as peak ground acceleration and response spectral accelerations and prediction of next-generation attenuation models (Douglas et al. 2014), determining source parameters from catalogued M_w (Edwards and Fäh 2013), historical earthquake M_w calibration based on M_w -intensity relationships (Stucchi et al. 2013), determining magnitude-frequency relations from catalogued M_w (Wiemer et al. 2009). Changing these formulas now, even if the original calibration data and parameter assumptions were not optimal, would therefore not be beneficial. But important is the understanding of the physical reasons for systematic differences between the various scales. Formula (9) illustrates this for the difference between M_w and M_c . With such an understanding, one may derive from classical empirical magnitudes still reasonably good proxy estimates for the more physically defined modern magnitudes M_w and M_c . Up to now, M_w proxies are still favoured for unifying magnitude data in long-term instrumental earthquake catalogues needed for probabilistic seismic hazard assessment (e.g. Grünthal et al. 2009, Wiemer et al. 2009; Stucchi et al. 2013; Di Giacomo et al. 2015). Hosts of mutual relationships between M_w and other magnitude scales have been published. Summaries with helpful discussions can be found, e.g. in Kanamori (1983), Bormann et al. (2009; 2013), Edwards et al. (2010), Di Giacomo et al. (2015) and Lolli et al. (2014). In future, however, the joint consideration of M_w and M_c is needed for a better understanding of the differences in earthquake rupture kinematics and dynamics and thus for a more realistic fast assessment of the specific nature and related hazard potential of individual earthquakes (e.g. Newman and Okal 1998; Choy and Kirby 2004; Di Giacomo et al. 2010a, b). Good correlation relationships between M_c and classical magnitudes such as m_B , m_b and M_S are still rare although for many significant pre-1990 earthquakes, a comparison between M_w and M_c proxy estimates would be highly desirable too, e.g. via the m_B and M_S values in the Abe (1981, 1984) catalogues. First regression relationships between M_c and the four IASPEI teleseismic standard

magnitudes have been published by Bormann et al. (2009).

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