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ORIGINAL ARTICLE

Determination of a seismometer's generator constant, azimuth, and orthogonality in three-dimensional space using a reference seismometer

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Abstract To accurately predict the performance of a seismometer, knowledge of its key parameters is required. We present a new method that requires a single reference instrument to estimate some of the important parameters of the seismometer, such as the ratio of the generator constants, the orthogonality deviation, and the rotation in space and in the horizontal plane with regards to the reference instrument. The procedure is performed in the three-dimensional spaces where the Euler rotation theorem is applied in order to define a transformation, which is then used to transform the detection of the reference seismometer as well as the detection of the instrument under test. The estimated transformation matrix is defined as an upper triangular matrix, where its elements contain the information regarding the parameters of the tested seismometer, which are then evaluated using the Euler angles. The new method has been verified on a pair consisting of two STS-2 seismometers and on a pair consisting of one CMG-3T and one STS-2 seismometer.

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Abbreviations

ARSO	Slovenian Environment Agency
PSD	Power spectral density
3D	Three dimensional
E–W	East–west
N–S	North–south
Z	Upwards, vertical

1 Introduction

A modern broadband seismometer consists of three built-in sensors, with the orthogonality of its outputs being within a fraction of a degree and where the gain of each component is known to within 1 % (Ekström and Busby 2008). The "gain constant" for a particular sensor is usually given in its calibration sheets. There are two ways to manufacture a three-component seismometer (Wielandt 2002): the sensors can be oriented orthogonal to each other, where the construction of the vertical component usually differs from the construction of the horizontal ones (e.g., Guralp seismometers); or the three sensors can be of identical construction, while their sensitive axes are inclined to the vertical by an angle of 54.7° (e.g., STS-2 seismometers). For this last type of seismometer, the output signals are factory adjusted in order to present the motion in the orthogonal axes. The gain constants for these outputs are not usually known exactly. For example, in the "certificate of calibration" of an STS-2 seismometer, it says only: "generic constants (*x*, *y*, *z* value 1,500±15 V×s/m each)". Otherwise, the values of the generator constants for each component are sometimes given separately in the calibration sheet (e.g. for Guralp seismometers). However, there is no information about the uncertainty related to these data. A generator constant for the particular seismometer can be estimated absolutely by using the three-component calibration table CALTAB_1 installed at the Conrad Observatory (Central Institute for Meteorology and Geodynamics) or a portable calibration table CT-EW1 by Lennartz Electronics. Both calibration tables use a procedure described by Wielandt (2002).

Generally, the manufacturers give just a generic value for the deviation of the axes' orthogonality, while the deviations are not known exactly for a particular seismometer. For an STS-2 seismometer the maximum deviation should be ±0.6°, while for a Guralp seismometer there is no data in the "certificate of calibration"—the relevant information is available on the manufacturer's website. Like with the 3T and the 3ESPC seismometers, one can read: "sensor axes orthogonal to within ±0.05°" (Guralp 2011a; b). Evaluating these two producers, Holcomb writes the following (Holcomb 2002): "The author doubts that any of the manufacturers have actually performed the costly experiments necessary to validate their alignment claims. Instead, the author feels that, in the cases of the Guralp CMG-3T and the Streckeisn STS-2, the manufacturers may have measured the sensitivity axis misalignment in one or two or at most "a few" of their preproduction sensor systems and then assumed that all their production units were assembled to the same accuracy." The same page contains the following statement: "Regardless of the source of the error, it seems to make sense that it is useless to attempt to align the sensor systems with geographic north–south and east–west more accurately than they are constructed internally."

Seismometers are usually oriented towards the east (E–W), north (N–S), and upwards (Z) in order to observe the ground motion in all directions. The sensor orientation can be perform using different instruments and procedures (Davis and Gee 2009), such as gyroscopic theodolites, astronomical methods, differential GPS methods or magnetic field methods. While the orientation of a seismometer on the surface is a relatively simple task, and can be determined even with a compass and a previously known magnetic declination for the site, such is not the case for an installation in deep, underground vaults or for a borehole installation. The orientation in deep, underground vaults can be performing using a Gyroscopic theodolite, which is highly accurate, but this equipment is expensive and is difficult to operate properly (Davis and Gee 2009). To define the orientation of seismometers in deep vaults or boreholes, a reference seismometer that is temporarily installed on the surface with a known orientation can also be used. There are many techniques used to define the horizontal orientation of a "target" seismometer in comparison to a reference instrument (Holcomb 2002; Ringler et al. 2012). An extended study relating to this problem was performed by Holcomb (2002), where he points out two sources of errors that are related to the gain of the sensors and to the imperfections in the sensors' orthogonality. He states that: "if the gains are not equal, errors arise in the calculated azimuths" and "the lack of a perfect alignment of the two horizontal components within a given sensor may contribute to errors in the relative angular measurements". There is another source of error, which is usually neglected: the misalignment of the vertical component. While a modern seismometer is a three-dimensional detector of the moving earth, the problem of a seismometer's orientation was mostly addressed in horizontal space, i.e., in two dimensions only. Under these assumptions, the horizontal planes of both seismometers are supposed to be parallel or aligned. The question then arises, if this can really be true, especially in borehole installations, when the seismometer is fixed in a borehole by sand (Guralp 2006).

It is also desirable for each institution that maintains a local seismic network to have a reference seismometer with well-defined gain constants, transfer functions, deviation of orthogonality, and self-noise in order to control and check the parameters of all the seismometers within an institution (Hutt et al. 2009; Tasič and Runovc 2010). This reference seismometer can then be referred to the "secondary standard" of a particular seismic network and can be used in a procedure for testing seismometers, for example, when they are purchased from the manufacturer, or later, when they are installed at a seismic station. However, nowadays, secondary reference units are very rare and the situations where broadband seismometers are used as control units are more or less coincidental, and the estimated parameters of the tested seismometers are only defined or estimated relative to this particular, noncalibrated, coincidentally chosen "reference" system.

Under field conditions, the procedure to orient a seismometer involves the following three steps (Davis and Gee 2009): determining true north, translating true north to a fiducial line, and finally orienting the sensor to a fiducial line. Each of these steps can add some error in the correct orientation to true North. Because of this, our opinion is that orienting a seismometer in the field with a precision of one tenth of a degree is more than adequate. However, under laboratory conditions, where two collocated seismometers are used in the test, and true north is not important, information about angles with a precision of one hundredth of a degree is adequate. So it would be ideal to have a reference seismometer with orthogonal deviations below 0.01° and with a generator constant having a relative error of less than 0.1 %. But according to the information that we have at present, a more realistic scenario is the following: the orthogonal deviations are below 0.03° and the generator constant has a relative error of less than 0.5 %.

After considering the above-presented problems, we developed a procedure to estimate the gain constants, the relative orientation between the tested and the reference seismometer, and the relative orthogonal deviation using the Euler angles. This can be performed when the matrix that is mapping the seismic data from the reference seismometer to the tested one in three-dimensional (3D) space is known. Such a matrix can be calculated using a procedure described by Tasič and Runovc (2011) or Wielandt (2009).

2 The seismometer's generator constant and azimuth determination

We have two 3-component seismological systems, each consisting of a broadband seismometer and an analog-to-digital converter (or an acquisition unit). Let us mark the first system with the index "q", and the second one with the index "r". We assume that the transfer functions of both systems are constant, within their respective bandwidth, and that the sensors in the seismological system "q" are orthogonal to each other and lie on the axis of a Cartesian coordinate system. The sensors of the system "r" are not completely orthogonal to each other: there are small deviations in the orthogonality. The seismometer "r" is also not aligned in the same direction as the seismometer "q" (Fig. 1). We will also assume that the generator constant is not known for any of the sensors of the systems "r".

A signal detected by the system "r" needs to be transformed in order to detect the same signal as the system "q". The linear transformation can be represented by a matrix. Let us define a matrix **A** that maps the vector $\mathbf{x}^r$ to the vector $\mathbf{x}^q$:

$$\mathbf{x}^q = \mathbf{A}\mathbf{x}^r \rightarrow \begin{bmatrix} x_1^q \\ x_2^q \\ x_3^q \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1^r \\ x_2^r \\ x_3^r \end{bmatrix}. \tag{1}$$

The transformation matrix **A** between the two collocated seismometers can be calculated using the procedure developed by Tasič and Runovc (2011). Under the assumption that the generator constants are not known for the seismometer "r", we can conclude that the matrix **A** does not represent a pure rotation but also contains a scaling factor (the ratio of the generator constants). Accordingly, the following relationship applies:

$$|\det(\mathrm{A})| \neq 1. \tag{2}$$

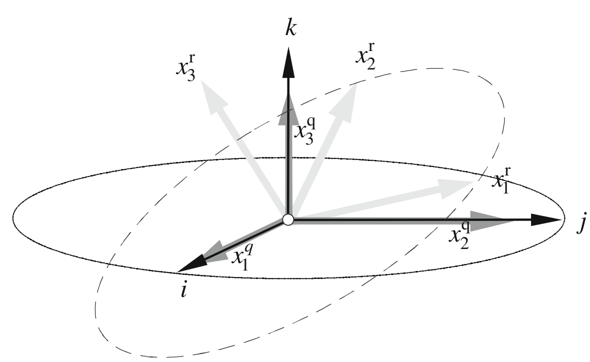

Fig. 1 Sensors of the seismometer "q" lie on the axes of the Cartesian coordinate system. The sensors of the seismometers "r" are not aligned equally in the space as those of the system "q"

The elements of the matrix $\mathbf{A}$ can be:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} a^r_{11}G_1 & a^r_{12}G_2 & a^r_{13}G_3 \\ a^r_{21}G_1 & a^r_{22}G_2 & a^r_{23}G_3 \\ a^r_{31}G_1 & a^r_{32}G_2 & a^r_{33}G_3 \end{bmatrix}, \tag{3}$$

where G_i; i=1, 2, 3, are the ratios for the generator constants between the ith component of a tested and a referenced seismometer g^r_i/g^q_i and a^r_{ij} is the element of the pure rotation matrix $\mathbf{A}^r$ when $|\det(\mathbf{A}^r)|=1$. If a generator constant of the reference system is known, or if the output of the reference seismometer is calibrated, then G_i is just a generator constant of the system under test.

2.1 Simple approach

When the orientation of both seismometers can be set to be equal at a very precise level, we can assume that the off-diagonal elements are small when compared to the diagonal elements. The ratio of the generator constant G_i can then be estimated as:

$$G_i \approx a_{ii};\ \text{for } \mathrm{a_{ij}} \gg \mathrm{a_{ij}}; = 1,2,3 \text{ and } \mathrm{i} \neq \mathrm{j}. \tag{4}$$

Mostly for historical reasons, and in some other cases, we are only interested in the rotation in the horizontal plane. Such is, for example, the case of a borehole installation. Let us set the vertical component in the direction "k". A submatrix $\mathbf{A}_{\mathrm{hor}}$ of the matrix $\mathbf{A}$ can be then applied as follows:

$$\begin{aligned} \mathbf{A}_{\mathrm{hor}} = \mathrm{A}[1,2;1,2] &= \begin{bmatrix} \mathrm{a}_{11} & \mathrm{a}_{12} \\ \mathrm{a}_{21} & \mathrm{a}_{22} \end{bmatrix} \\ &= \begin{bmatrix} \mathrm{G}_1\cos(\alpha) & -\mathrm{G}_2\sin(\alpha) \\ \mathrm{G}_1\sin(\alpha) & \mathrm{G}_2\cos(\alpha) \end{bmatrix}. \end{aligned} \tag{5}$$

When the values of $a_{31}, a_{32}, a_{13}, a_{23}$ are close to zero, we estimate the ratio of the generator constant as:

$$G_i \approx \sqrt{a_{1i} + a_{2i}};\ \text{for } \mathrm{i} = 1,2 \text{ and } \mathrm{G_3} \approx \mathrm{a_{33}}. \tag{6}$$

When the generator constants are known, the rotation in a horizontal plane (given by angle α) can be calculated using the element of $\mathbf{A}_{\mathrm{hor}}$ (Eq. 5) by using elementary inverse trigonometric functions (Holcomb 2002).

2.2 Using Euler angles

In a nonlaboratory environment (e.g., setting a reference system at a remote seismic station or performing the borehole installation of a seismometer), or sometimes even in laboratory conditions, a precise installation is very difficult to achieve. For this reason, the generator constant cannot be estimated in a straightforward way. To calculate the ratio of the generator constants, we will perform a virtual "rotation" of the reference instrument so that the third component of both seismometers (x^q_3 and x^r_3) is aligned on the same axis. This can easily be done by using Euler's rotation theorem (Arfken and Weber 2005). There are several conventions expressing Euler angles, depending on the axes about which the rotations are carried out. In this case, the rotation is presented by the three rotational matrices $\mathbf{R}_\psi$, $\mathbf{R}_\theta$, and $\mathbf{R}_\phi$ (see Fig. 2). The first rotation will be performed by an angle ψ about the axis "k" using the rotation matrix $\mathbf{R}_\psi$:

$$\mathbf{R}_\psi = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{7}$$

The second rotation is carried out by an angle θ about the axis "i" using the rotation matrix $\mathbf{R}_\theta$:

$$\mathbf{R}_\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}. \tag{8}$$

With this rotation, we actually ensure that the axes of the sensors x^q_3 and x^r_3 are aligned. The third rotation is carried out by an angle ϕ about the axis "k", resulting in the rotation matrix $\mathbf{R}_\phi$:

$$\mathbf{R}_\phi = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{9}$$

This last rotation makes the axis of the sensor x^r_2 of the seismometer "r" lie in the plane formed by the axes of just two sensors, x^q_3 and x^q_2, of the rotated seismometer "q". Such a Euler rotation can be marked as [3,1,3]. Equation (1) can now be expressed using the Euler angles as:

$$\mathbf{R}_\phi\mathbf{R}_\theta\mathbf{R}_\psi\mathbf{x}^q = \mathbf{R}_\phi\mathbf{R}_\theta\mathbf{R}_\psi\mathbf{A}\mathbf{x}^r. \tag{10}$$

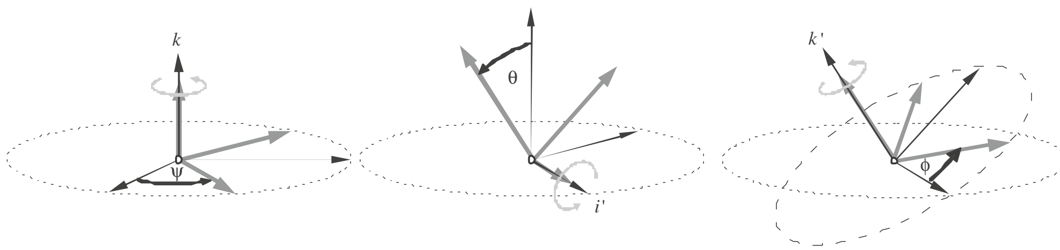

Fig. 2 Rotation using three Euler angles: ψ, θ, and ϕ

The last equation can also be rewritten using the general rotation matrices **R**:

$$\mathbf{R}\mathbf{x}^q = \mathbf{R}\mathbf{A}\mathbf{x}^r. \tag{11}$$

The product of the matrices **R** and **A** gives a new matrix **K** from:

$$\mathbf{R}\mathbf{A} = \mathbf{K}, \tag{12}$$

or when expressed by the elements:

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}. \tag{13}$$

The elements of the general rotation matrix **R**, using the compact notation (c_θ=cos(θ), s_θ=sin(ϕ), …), are then given by:

$$\mathbf{R} = \begin{bmatrix} c_\phi c_\psi - s_\phi c_\theta s_\psi & -c_\phi s_\psi - s_\phi c_\theta c_\psi & s_\phi s_\theta \\ s_\phi c_\psi + c_\phi c_\theta s_\psi & -s_\phi s_\psi + c_\phi c_\theta c_\psi & -c_\phi s_\theta \\ s_\theta s_\psi & s_\theta c_\psi & c_\theta \end{bmatrix}. \tag{14}$$

Using our definition of the rotation of the seismometer "q", the transformation matrix **K** becomes an upper triangular matrix:

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ 0 & k_{22} & k_{23} \\ 0 & 0 & k_{33} \end{bmatrix}. \tag{15}$$

The Euler angles can be calculated by setting k_{21}=k_{31}=k_{32}=0:

$$\left(s_\phi c_\psi + c_\phi c_\theta s_\psi\right)a_{11} + \left(-s_\phi s_\psi + c_\phi c_\theta c_\psi\right)a_{21} + \left(-c_\phi s_\theta\right)a_{31} = 0, \tag{16}$$

$$s_\theta s_\psi a_{11} + s_\theta c_\psi a_{21} + c_\theta a_{31} = 0, \tag{17}$$

$$s_\theta s_\psi a_{12} + s_\theta c_\psi a_{22} + c_\theta a_{32} = 0. \tag{18}$$

The Eqs. (17) and (18) are rewritten as $s_\theta\left(s_\psi a_{11} + c_\psi a_{21}\right) = -c_\theta a_{31}$ and $s_\theta\left(s_\psi a_{12} + c_\psi a_{22}\right) = -c_\theta a_{32}$. After dividing these two expressions, we obtain the relationship for ψ as:

$$\psi = \arctan\left(\frac{a_{22}a_{31} - a_{21}a_{32}}{a_{11}a_{32} - a_{12}a_{31}}\right). \tag{19}$$

The solution for ψ lies in the range [−π/2, π/2]. The notation and the function atan2 are useful, where atan2[y, x] is the arc tangent of the variables x and y. The function atan2 is available in many programming languages (e.g., FORTRAN, JAVA) and computational environments (MATLAB, Mathematica) and it is similar to the arctan of y/x, except that the signs of both arguments are used to determine the quadrant of the result, which lies in the range [−π, π] (Slabaugh 1999). So we obtain:

$$\psi = \mathrm{atan2}\left[(a_{22}a_{31} - a_{21}a_{32}), (a_{11}a_{32} - a_{12}a_{31})\right]. \tag{20}$$

Knowing the angle ψ, the angle θ can be calculated using Eq. (17):

$$\theta = \mathrm{atan2}\left[-a_{31}, \left(s_\psi a_{11} + c_\psi a_{21}\right)\right], \tag{21}$$

or Eq. (18):

$$\theta = \mathrm{atan2}\left[-a_{32}, \left(s_\psi a_{12} + c_\psi a_{22}\right)\right]. \tag{22}$$

In general, Eqs. (21) and (22) give the same result. But under the same circumstances (e.g., $s_\psi a_{11} = -c_\psi a_{21}$ or $s_\psi a_{12} = -c_\psi a_{22}$) two solutions are possible. But only one is correct, where the condition k_{31}=

k_{32}=0 is satisfied. Now the angle ϕ can be calculated using the angle ψ, the angle θ, and Eq. (16):

$$\phi = \mathrm{atan2}\left[\left(\mathrm{s}_\theta \mathrm{a}_{31} - \mathrm{c}_\theta \mathrm{s}_\psi \mathrm{a}_{11} - \mathrm{c}_\theta \mathrm{c}_\psi \mathrm{a}_{21}\right), \left(\mathrm{c}_\psi \mathrm{a}_{11} - \mathrm{s}_\psi \mathrm{a}_{21}\right)\right]. \tag{23}$$

The value of G_3 is simply the k_{33} element of the matrix **K**. But as the orientation of the "rotated" seismometer "*q*" is not predefined, k_{33} can also be negative. After the "rotation", the sensors x_3^q and x_2^q can be set in opposite directions. Because G_3 is always positive, their value is equal to the absolute value of k_{33}:

$$G_3 = \left|s_\theta s_\psi a_{13} + s_\theta c_\psi a_{23} + c_\theta a_{33}\right|. \tag{24}$$

The rotation in the horizontal plane of the seismometer "*r*" with regard to the seismometer "*q*" can be estimated with the help of the Euler angles (see Fig. 2):

$$\alpha \approx -(\varphi + \psi \cos(\theta)). \tag{25}$$

Here, the angle α denotes the projection of the rotation of a seismological system onto the horizontal plane. The Euler angle θ gives information about the difference in the vertical inclination between both seismometers or in other words information about the angle between the horizontal planes of both seismometers.

The presented procedure is used to define the ratio of the generator constant G_3. A similar approach can also be used to calculate the values G_1 and G_2. The easiest way to calculate these values is a permutation of the elements in the vectors $\mathbf{x}^r$ and $\mathbf{x}^q$ (Eq. 1). For G_1 the indexes *i*, *j*, *k* are now *j*, *k*, *i*:

$$\begin{bmatrix} x_2^q \\ x_3^q \\ x_1^q \end{bmatrix} = \begin{bmatrix} a_{22} & a_{23} & a_{21} \\ a_{32} & a_{33} & a_{31} \\ a_{12} & a_{13} & a_{11} \end{bmatrix} \begin{bmatrix} x_2^r \\ x_3^r \\ x_1^r \end{bmatrix}. \tag{26}$$

In order to calculate the value G_1, the transformation matrix **A** in the Eq. (12) is now:

$$\mathbf{A} = \begin{bmatrix} \mathrm{a}_{22} & \mathrm{a}_{23} & \mathrm{a}_{21} \\ \mathrm{a}_{32} & \mathrm{a}_{33} & \mathrm{a}_{31} \\ \mathrm{a}_{12} & \mathrm{a}_{13} & \mathrm{a}_{11} \end{bmatrix}. \tag{27}$$

A similar procedure is performed to calculate the value G_2, where the indexes *i*,*j*,*k* from Eq. 1 are now k, i,j. Next, the transformation matrix **A** in Eq. (12) is rewritten as:

$$\mathbf{A} = \begin{bmatrix} \mathrm{a}_{33} & \mathrm{a}_{31} & \mathrm{a}_{32} \\ \mathrm{a}_{13} & \mathrm{a}_{11} & \mathrm{a}_{12} \\ \mathrm{a}_{23} & \mathrm{a}_{21} & \mathrm{a}_{22} \end{bmatrix}. \tag{28}$$

In both cases, the matrices **K** are upper diagonal matrices again and the new Euler angles need to be calculated for each ratio of the generator constants.

3 The determination of a seismometer's orthogonality deviation

When all the values G_1, G_2, and G_3 are known with respect to the definitions of the rotation of the seismometer "*r*", three additional angles can be calculated using the elements of the matrix K. These angles are δ_{23}, δ_{12}, and δ_{312} (Fig. 3). The angle between the axis of x_2^r of the seismometer "*r*" and the axis of x_2^q of the rotated seismometer "*q*", which represents the deviations in orthogonality in the 2–3 plane of the seismometer "*r*", is then:

$$\delta_{23} = \mathrm{atan2}[\mathrm{k}_{23}/\mathrm{G}_3, \mathrm{k}_{22}/\mathrm{G}_2]. \tag{29}$$

The angle between the axis of x_1^r of the seismometer "*r*" and the axis of x_3^q of the rotated seismometer

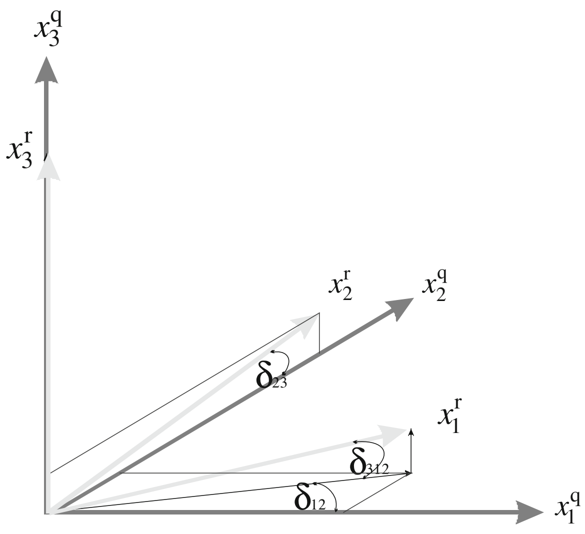

Fig. 3 Under the assumption that all three outputs of the reference seismometers (*blue*) are orthogonal to each other, the angle deviations δ_{23}, δ_{12}, and δ_{312} of the tested seismometer (*red*) can be calculated

"*q*" represents the deviations between these two components and is obtained from:

$$\delta_{312} = \mathrm{atan2}\left[\mathrm{k}_{13}/\mathrm{G}_3, \sqrt{(\mathrm{k}_{11}/\mathrm{G}_1)^2 + (\mathrm{k}_{12}/\mathrm{G}_2)^2}\right]. \quad (30)$$

The angle of the x_1^r component in the 1–2 plane of the seismometer "*r*" is simply:

$$\delta_{12} = \mathrm{atan2}(\mathrm{k}_{12}/\mathrm{G}_2, \mathrm{k}_{11}/\mathrm{G}_1). \quad (31)$$

We need to be aware that in real circumstances, the reference seismometer "*q*" also has some deviation in its orthogonality and because of this the calculated deviations between these two seismometers are relative.

As the deviation is expected to be small, it is useful to rewrite the upper diagonal matrix from (15) in the following form:

$$\mathrm{K}_G = \begin{bmatrix} \frac{k_{11}}{G_1} & \frac{k_{12}}{G_2} & \frac{k_{13}}{G_3} \\ 0 & \frac{k_{22}}{G_2} & \frac{k_{23}}{G_3} \\ 0 & 0 & \frac{k_{33}}{G_3} \end{bmatrix} \quad (32)$$

Using the expansion for small angles, where *tan*(*x*)~ x and *arctan*(*x*)~x, the following equations are obtained for δ_{23}, δ_{12}, and δ_{312}:

$$\delta_{23} \approx \mathbf{K}_G(2,3), \quad (33)$$

$$\delta_{12} \approx \mathbf{K}_G(1,2), \quad (34)$$

$$\delta_{312} \approx \mathbf{K}_G(1,3). \quad (35)$$

Because the ratio of the generator constants is given as the absolute value of k_{33} (Eq. (24)), the information about its sign is lost. But as we are interested in the relative deviation only, the equation for the deviation estimation can be left as it is.

4 Other Euler rotations

There is another possibility to use Euler angles. For example, the rotation [3,2,3], where the second rotation is not about the axis "*i*" but about the axis "*j*". Now, the three new Euler angels ψ_1, θ_1, and ϕ_1 using three rotation matrices $\mathbf{R}_{\psi 1}$, $\mathbf{R}_{\theta 1}$, and $\mathbf{R}_{\phi 1}$ can be calculated. The two rotation matrices $\mathbf{R}_{\psi 1}$ and $\mathbf{R}_{\phi 1}$ have the same form as the matrices in Eqs. (7) and (9), while the matrix $\mathbf{R}_{\theta 1}$ turns to:

$$\mathbf{R}_{\theta 1} = \begin{bmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) \\ 0 & 1 & 0 \\ \sin(\theta_1) & 0 & \cos(\theta_1) \end{bmatrix}. \quad (36)$$

The general rotation matrix $\mathbf{R}_1$ is $\mathbf{R}_1 = \mathbf{R}_{\phi 1}\mathbf{R}_{\theta 1}\mathbf{R}_{\psi 1}$. Equation (12) is now rewritten as $\mathbf{R}_1\mathbf{A} = \mathbf{K}_1$. In this case, the transformation matrix $\mathbf{K}_1$ is not an upper triangular matrix anymore, but becomes:

$$\mathbf{K}_1 = \begin{bmatrix} 0 & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ 0 & 0 & k_{33} \end{bmatrix}. \quad (37)$$

Simple mathematics leads to the final solutions for the Euler angels ψ_1, θ_1, and ϕ_1 by using elements of the general rotation matrix $\mathbf{R}_1$.

5 Tests, results, and discussion

The transformation matrices **A** were calculated using the procedure outlined in Tasič and Runovc (2011). Two main sources of error affect the calculation of the matrix **A**. The first one is the numerical procedure itself, where the output is affected by parameters used in the procedure, such as the frequency interval, in which the calculation is performed. Two conditions need to be fulfilled in order to properly define this frequency interval. The transfer functions of both seismometers need to be flat and the expected average self-noise needs to be as low as possible. Our experience shows that for seismometers having a bandwidth from 0.0083 to 50 Hz an optimal solution is obtained, when the transformation matrix **A** is evaluated in the frequency interval from 0.2 to 0.5 Hz. Additional information about the seismometer's self-noise (Ringler and Hutt 2009) can also be used to estimate the requested frequency range for different pairs of seismometers.

The second main source of error is the influence of an inaccurate installation of both seismometers, where strong non-seismic noise—which can be present at high frequencies—can affect the seismic detection, so the installation of the complete system needs to be carried out very precisely.

A large distance between the seismometers can be another possible source of error, because the computation is based on a calculation of the cross-correlation between two signals. But according to the paper of Juravlev et al. (1993), the correlation between two STS-2 seismometers is still high enough in the frequency interval from 0.2 to 0.5 Hz, even at a distance of 500 m, in order to allow the calculation of the transformation matrices.

In order to minimize these sources of errors, two transformation matrices are calculated. For each pair of seismometers, firstly the matrix $\mathbf{A}_{qr}$ is calculated, which actually represents the mapping of the data of seismometer "*r*" to the space of seismometer "*q*". Secondly, the matrix $\mathbf{A}_{rq}$ is calculated, representing the mapping of the data of seismometer "*q*" to the space of seismometer "*r*". In an ideal case, the next relation is applied: $\mathbf{A}_{qr}=inv(\mathbf{A}_{rq})$. But as the evaluation of the transformation matrices $\mathbf{A}_{qr}$ and $\mathbf{A}_{rq}$ involves a numerical procedure, some errors can be expected for any calculation and both matrices are used to compute an average value of the transformation matrix **A**:

$$\mathbf{A} = 0.5\left(\mathbf{A}_{qr} + \left(\mathbf{A}_{rq}\right)^{-1}\right). \tag{38}$$

These two matrices can also be used to estimate the error matrix $\Delta\mathbf{A}$

$$\Delta\mathbf{A} = \left|\mathbf{A} - \mathbf{A}_{qr}\right| \tag{39}$$

Elements of the matrix $\Delta\mathbf{A}$ are important in order to define the way of calculating the seismometer's orthogonality deviation. Because angles δ_{23}, δ_{12}, and δ_{312} are expected to be small we can use the small-angle approximation, tan (α)~α. If the order of magnitude of the angles is 10^{-4} *rad* (0.01°), then the error needs to be at least 1 order of magnitude lower so the elements of the matrix $\Delta\mathbf{A}$ need to be of an order of magnitude of 10^{-5} or lower.

The validity of transformation matrix **A** can be checked by another method as well. Let us define the transformation $\mathbf{y}=\mathbf{A}\mathbf{x}^r$. Then, the transformation matrix between $\mathbf{x}^q$ and **y** is calculated. If the transformation is an identity matrix **I**,

$$\mathbf{x}^q = \mathbf{I}\mathbf{y}, \tag{40}$$

then the transformation matrix **A** represents the correct mapping of the data of the seismometer "*r*" to the space of the seismometer "*q*" in a predefined frequency interval.

The following results are presented for two pairs of seismometers. The first pair consists of two STS-2 seismometers and the second pair consists of a CMG-3T and an STS-2 seismometer. All the seismometers have a bandwidth from 0.0083 to 50Hz. Some additional numerical tests and results are given for the second pair. Both tests were performed in the year 2011, the first pair being tested in April and the second pair being tested in December, both at the Observatory Golovec in Ljubljana, of the Slovenia Seismology Office (ARSO). A part of the observatory is a seismic room with a pier for the seismometers. The seismometers were installed 1 m apart, side by side. They were connected to a six-channel EarthData PR6 acquisition unit. The inputs in our tests were 6-h, finite length–time, seismic data segments, sampled at 200 samples per second, giving a total of 4,320,000 data points.

5.1 Two STS-2 seismometers

Figure 4 represents the matrices $\mathbf{A}_{qr}$, $inv(\mathbf{A}_{rq})$, **I**, and $\mathbf{K}_G$ in the [*i*,*j*,*k*] orientation (the vertical component is set to be oriented in the "*k*" axes) for a pair of two STS-2 seismometers. Both transformation matrices are

$$\mathbf{A}_{qr} = \begin{bmatrix} 1.00849 & 0.00134 & -0.00021 \\ -0.00171 & 1.01109 & 0.00068 \\ -0.00038 & -0.00049 & 1.00938 \end{bmatrix}; \mathbf{A}_{rq}^{-1} = \begin{bmatrix} 1.00849 & 0.00134 & -0.00021 \\ -0.00171 & 1.01109 & 0.00068 \\ -0.00038 & -0.00049 & 1.00938 \end{bmatrix}$$

$$\mathbf{I} = \begin{bmatrix} 1.00000 & 0.00000 & 0.00000 \\ 0.00000 & 1.00000 & 0.00000 \\ 0.00000 & 0.00000 & 1.00000 \end{bmatrix}; \quad \mathbf{K}_G = \begin{bmatrix} 1.00000 & -0.00037 & -0.00059 \\ 0.00000 & -1.00000 & -0.00019 \\ 0.00000 & 0.00000 & -1.00000 \end{bmatrix}$$

Fig. 4 Matrices $\mathbf{A}_{qr}$, $inv(\mathbf{A}_{rq})$, **I** and $\mathbf{K}_G$ in the [*i*,*j*,*k*] orientation, calculated for a pair of two STS-2 seismometers

$$\mathbf{A}_{qr} = \begin{bmatrix} 1.01264 & 0.01156 & 0.00829 \\ -0.01001 & 1.01651 & -0.00351 \\ -0.00162 & -0.00009 & 1.00821 \end{bmatrix}; \mathbf{A}_{rq}^{-1} = \begin{bmatrix} 1.01264 & 0.01156 & 0.00829 \\ -0.01001 & 1.01651 & -0.00351 \\ -0.00162 & -0.00009 & 1.00821 \end{bmatrix}$$

$$\mathbf{I} = \begin{bmatrix} 1.00000 & 0.00000 & 0.00000 \\ 0.00000 & 1.00000 & 0.00000 \\ 0.00000 & 0.00000 & 1.00000 \end{bmatrix}; \quad \mathbf{K}_G = \begin{bmatrix} 1.00002 & -0.00149 & -0.00666 \\ -0.00000 & 1.00001 & -0.00348 \\ -0.00000 & 0.00000 & 1.00000 \end{bmatrix}$$

Fig. 5 Matrices $\mathbf{A}_{qr}$, $inv(\mathbf{A}_{rq})$, **I** and $\mathbf{K}_G$ in the [*i,j,k*] orientation, calculated for a pair of CMG-3T and STS-2 seismometer

equal to the fifth order of the decimal. All the results for the angles will be presented for the *i*,*j*,*k* orientation only. The Euler angles are ψ=−141.64°, θ=179.96°, and ϕ=−141.74°. The rotation in the horizontal plane is α=−0.10° (Eq. (25)). The ratios of the generator constants are as follows: G_1=1.009, G_2=1.011, and G_3=1.009. The calculated orthogonal deviations are smaller than from the specification of STS2 and are $|\delta_{23}|$=0.01°, $|\delta_{312}|$=0.03°, and $|\delta_{12}|$=0.02°.

5.2 CMG-3T and STS-2 seismometer

Figure 5 represents the matrices $\mathbf{A}_{qr}$, $inv(\mathbf{A}_{rq})$, **I** and $\mathbf{K}_G$ for a pair consisting of a CMG-3T seismometer and an STS-2 seismometer. Again, both transformation matrices are equal. The Euler angles are ψ= −92.46°, θ=179.91°, and ϕ=−93.03°. The rotation in the horizontal plane is α=−0.57° (Eq. (25)). The ratios of the generator constants are: G_1=1.013, G_2=1.017, and G_3=1.008. It is clear that the $\mathbf{K}_G$ estimated from two collocated STS-2 seismometers has lower values compared to the $\mathbf{K}_G$ estimated in this experiment. This is also reflected in the calculated orthogonal deviations, which are $|\delta_{23}|$=0.20°, $|\delta_{312}|$=0.38°, and $|\delta_{12}|$= 0.09°. As we do not have any information about the deviations of the STS-2 seismometer, we cannot simply conclude that the CMG-3T is worse. But as one of the STS-2s was used in both experiments and the quality of the STS-2 seismometers is known to be high, we can say that the orthogonal deviations for the CMG-3T are worse than the ones obtained from the specification.

5.3 CMG-3T rotated by 30°

In this experiment, the same seismometers were used as in Section 5.2, but only the CMG-3T was rotated by 30°±0.3°. The calculated transformation matrices $\mathbf{A}_{qr}$ and $inv(\mathbf{A}_{rq})$ are now slightly different (Fig. 6), above all in the first row. Differences also exist in the second row, while the third row is the same. So the estimated values differ from the results in Section 5.2 and are less trustworthy. This is also reflected in the estimated ratios of the generator constants, which differ when compared to the data from Section 5.2 by approximately 0.13 % for horizontals (G_1= 1.014, G_2=1.015), while the vertical ratio (G_3= 1.008) is equal. The incorrectly calculated generator

$$\mathbf{A}_{qr} = \begin{bmatrix} 0.88277 & -0.49769 & 0.00804 \\ 0.49885 & 0.88495 & -0.00339 \\ 0.00075 & 0.00125 & 1.00808 \end{bmatrix}; \mathbf{A}_{rq}^{-1} = \begin{bmatrix} 0.88277 & -0.49770 & 0.00807 \\ 0.49885 & 0.88496 & -0.00340 \\ 0.00075 & 0.00125 & 1.00808 \end{bmatrix}$$

$$\mathbf{I} = \begin{bmatrix} 0.99999 & 0.00000 & -0.00001 \\ 0.00000 & 0.99999 & 0.00001 \\ 0.00000 & 0.00000 & 1.00000 \end{bmatrix}; \quad \mathbf{K}_G = \begin{bmatrix} 1.00002 & 0.00205 & 0.00604 \\ 0.00000 & 1.00002 & -0.00564 \\ 0.00000 & 0.00000 & 1.00000 \end{bmatrix}$$

Fig. 6 Matrices $\mathbf{A}_{qr}$, $inv(\mathbf{A}_{rq})$, **I** and $\mathbf{K}_G$ in the [*i,j,k*] orientation, calculated for a pair of CMG-3T and STS-2 seismometer, where CMG-3T is rotated by 30°, with regards to the previous orientation constant affects the calculation of the angles of orthogonal deviations: their values are $|\delta 23|=0.32°$, $|\delta 312|=0.35°$, and $|\delta 12|=0.12°$. The calculated Euler angles are now $\psi=1.47°$, $\theta=-0.08°$, and $\phi=30.94°$. From Eq. (25), we obtain $\alpha=29.47°$. The calculated rotation of a CMG-3T seismometer, taking its position from Section 5.2, is 30.04° (29.47°+0.57°), and is within the precision of manual rotation.

The transformation matrices $\mathbf{A}_{qr}$ and $inv(\mathbf{A}_{rq})$ need to be equal and matrix **I** needs to be an identity matrix, when the seismometer's orthogonal deviations and the ratio of the generator constants are the goal of the calculation.

6 Conclusions

A novel procedure is described that enables an evaluation of some of a seismometer's parameters using a reference seismometer. This approach offers improvements in terms of the quality control of seismometers, which can be performed by local seismic institutions.

The quality control of a scientific instrument is one of the most important tasks necessary to make its measurements reliable. This is also the case for seismometers, which are treated as scientific instruments because their outputs are used for additional scientific studies. Broadband seismometers usually come with a so-called "certificate of calibration" provided by the manufacturers, where parameters that are related to the transfer function, generator constant, self-noise, etc., are given. But because these calibration procedures are not standardized between producers, results cannot be compared between different types of seismometers without some concerns.

Also, some of parameters provided by producers, such as the deviation of orthogonality, are generic for the same type of seismometers. When seismometers are purchased by a particular institution, they are very rarely, if it at all, compared or calibrated using higher-level standards, but usually just installed at the seismic station. A high-quality broadband seismometer should have a known long-term stability of its transfer function (e.g., Wielandt 2004) and is usually permanently and precisely installed at a seismic station to detect seismic signals. All interruptions, like a temporary deinstallation, transportation to an institution where the calibration is performed, and a reinstallation at the seismic station, may cause more problems than are solved by a regular calibration. Because of this, they are just controlled with test signals, which are built into the acquisition units (which are often wrongly equated with the calibration signals). Only in cases when the response of a seismometer to the test signals is unusual is the seismometer returned to the manufacturer for verification.

We will address some additional improvements in our future work. We of course presume that the reference seismometer also has some unknown deviation in terms of orthogonality. Some other possible errors could originate from the deviation in orthogonality of the reference seismometer, from the influence of the distance between the seismometers, and from the influence of the nonlinearity of the seismometers. In particular, evaluations of the parameters where larger distances (vertical or horizontal) exist need to be investigated in more detail with a gyroscope in order to precisely define the azimuth of the tested systems.

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