Intensity attenuation in the U.K.

R.M.W. Musson

British Geological Survey, West Mains Road, Edinburgh EH9 3LA, U.K., e-mail: R.musson@bgs.ac.uk

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Abstract

Intensity attenuation is relatively little studied compared with the attenuation of peak ground acceleration, due to the fact that the PGA can be used for engineering design, while intensity cannot. However, intensity has other uses, including the estimation of effects (including damage) of future earthquakes, and hence, at least in a general way, the study of earthquake risk. Knowledge of intensity attenuation is also useful in calibrating hazard models against historical experience. In this study, the attenuation of intensity in the U.K. is thoroughly evaluated from a data set comprising 727 isoseismals from 326 British earthquakes, including both modern and historical events. Best results are obtained by restricting the data set to events contributing at least two isoseismals. The preferred equation is

I = 3.31 + 1.28ML − 1.22 ln*R*

where *I* is intensity (European Macroseismic Scale), ML is local magnitude, and *R* is hypocentral distance. The sigma (uncertainty) value is 0.46. Some sample applications of this formula are demonstrated.

Introduction

Although the literature on attenuation of strong ground motion is copious, that for the attenuation of intensity is comparatively small. The discrepancy is easily explained: the growth in seismic hazard studies for the purpose of deriving design coefficients for engineers has created a strong demand for better and more reliable equations expressing the attenuation of physical parameters of earthquake shaking, especially peak ground acceleration (PGA). Seismic hazard calculated in terms of intensity is less commonly encountered, but has its advantages. If the audience for the study is the public, or the planning or insurance communities, then the significance of hazard expressed as intensity is much easier to comprehend, in terms of its societal effects, than a PGA value. The well-known lack of correlation between PGA and damage further underlines this point.

Intensity attenuation has other uses as well. One that should be discounted is the practice, in lieu of having local data for strong ground motion, of computing intensity attenuation and converting it to PGA attenuation by means of an intensity/PGA correlation. As has frequently been pointed out, the correlation between intensity and PGA is so poor, and the scatter is so wide (see for example, Murphy and O'Brien, 1977) that such conversions are of doubtful value.

A better use is in the construction of site histories, where, for a given city or location, the observations of different intensities are plotted over time. These can be either intensities assessed from local data or estimated from attenuation in the absence of documentary records. Such site histories can be used as expressions of hazard in their own right (e.g. Azzaro et al., 1999) or used to calibrate conventional hazard probabilistic studies (e.g. Mucciarelli et al., 2000). This latter role is extremely valuable in view of the need to make hazard assessments more transparent and accountable.

It is also very useful to be able to estimate isoseismals for a potential future earthquake (for scenariobased planning purposes) or for an earthquake that has just occurred. In the hours immediately following a significant event, responses can be planned on the basis of the expected distribution of effects from an earthquake of given size and epicentre. In the U.K., such estimates have been used for planning macroseismic surveys following significant events, and the estimates have generally been found to be quite reliable.

The equation that has been used is one published first in Musson and Winter (1996) as follows:

$$
I = 3.33 + 1.44ML - 1.45 \ln R \tag{1}
$$

where *I* is intensity (EMS – European Macroseismic Scale), ML is local magnitude, and *R* is hypocentral distance. Equation (1) was calculated by the author in 1993 from a data set of 132 British earthquakes, but the workings were never published. This present paper updates the original study, using an improved and expanded data set, and publishes the calculations.

Forms of intensity attenuation and associated problems

Here some issues are discussed in a general way, not exclusive to the U.K. The form of equation (1) can be written as

$$
I = a + bM + c\ln R + dR \tag{2}
$$

where *a*, *b*, *c* and *d* are constants, representing scaling, energy release, geometric spreading and anelastic attenuation respectively. (In equation (1), *d* is equal to zero.) This form is familiar from studies of PGA attenuation, where the left hand side term *I* is replaced by ln PGA. Intensity is expressed as a function of magnitude and distance.

However, in many previous studies of intensity attenuation, especially in Europe, equations are derived in a form that expresses intensity as a function of epicentral intensity and distance. Common is the form derived by Kövesligethy (1906):

$$
I_0 - 1 = 3\log(R/h) + 3\alpha \log e (R - h)
$$
 (3)

where I_0 is the epicentral intensity, e the Euler's constant, *h* the depth and α a constant to be determined regionally, with typical values in the range 0.002–0.006 (Karník, 1969). Equation (3) is very useful for determining earthquake depth (Sponheuer, 1960; Musson,

2002; Musson and Cecić, 2002), but is less satisfactory for attenuation purposes.

This is because its use presupposes that I_0 can be used as a surrogate for magnitude. This is problematic for several reasons, as follows:

- 1. I_0 for small earthquakes is heavily influenced by depth, which is frequently not known. This is sometimes skated over by assuming that all earthquakes in a region are about the same depth; but this assumption becomes self-justifying and counterexamples are ignored.
- 2. I_0 for large earthquakes can be contaminated by effects that are due to fault rupture rather than shaking. In fact, the 1956 version of the Modified Mercalli Scale (Richter, 1958) is more or less unusable in its upper reaches because rupture effects are presented as intensity diagnostics.
- 3. I_0 for large earthquakes can be impossible to assess in cases, where the scale saturates because all buildings are destroyed at intensities as low as 8 (Ambraseys, 2001).
- 4. I_0 can be contaminated by soil amplification effects.
- 5. I_0 can be impossible to assess in cases where the epicentre is in an uninhabited area, or offshore.
- 6. I_0 is unlikely to be known immediately after an earthquake has occurred.

It is much more useful, therefore, to compile intensity attenuation equations in the form of equation (2) rather than equation (3).

Regressions of intensity data

Fitting equation (2) to a data set presents some particular problems because of the nature of intensity. Normal regression procedures handle continuous variables, but intensity has only integer values. When computing PGA attenuation, it is straightforward to use the recorded ground motion at a particular recording station as a value to be combined with the distance of the station from the epicentre, hypocentre, or rupture plane projection. This procedure will not work with intensity for several reasons. In the first case, any intensity value is a simple representation of a complex distribution. Thus, if intensity 5 is assigned to a town, it means that the earthquake effects in that town were predominantly those consistent with intensity 5, but effects may have been greater or less in some suburbs. Taking a survey of only one of the suburbs, effects may have still have been weaker or stronger in some streets than others. In any street, effects may have been weaker or stronger

in some houses than others. In any house, effects may have been weaker or stronger in some rooms than others. So whereas a single acceleration recording is an incontrovertible value attached to specific spatial coordinates, an intensity value is inherently complex, and more indeterminate.

Secondly, if, over an area, the intensity from an earthquake is predominantly 5, the single integer value 5 is associated with the whole of that area, and it does not shade from 5.1 at the outer edge to 5.9 at the inner edge. This means that the distribution of settlements (which is nothing to do with any property of the earthquake) influences the data. For one earthquake, it might be that there are many towns and villages near the inner edge of the intensity 5 area and few at the outer edge; for another earthquake the pattern may be reversed, but in both cases all these places are assigned intensity 5 (see Figure 1). This will affect the results in a quite undesirable way.

Thirdly, given that intensity drops in integer steps with distances from the epicentre, and that the appli-

Figure 1. Consider an earthquake with epicentre as shown by the star, and "true" isoseismals for intensity 5 and 6 as shown. The symbols indicate settlements in this imaginary case where the intensity has been assessed as 5. The distribution of settlements is controlled by topography, economics, etc. By chance, most of the data points on the west side of the felt area are in the inner part of the isoseismal, while on the east side they are in the outer part. Estimating attenuation from the data points instead of the isoseismals will give different results for the east side compared with the west side.

Figure 2. The "half intensity" problem. This figure shows a notional cross-section through an isoseismal map, considering the intensity field as a 3D surface. Should the attenuation equation represent the upper dashed line or the lower one?

cation of any equation will fit this stepped function by a line, a problem arises as to whether the line should intersect the forward edge or the middle of a step (see Figure 2). In an early discussion of this in a U.K. context, Soil Mechanics (1982) consider that the mean isoseismal radius for intensity *I* reflects an intensity value midway between I and $I - 1$. In other words, in the context of Figure 2, they consider the lower line a better fit to the intensity distribution. Actually, this ceases to be a problem if one thinks of the intensity attenuation equation not as

$$
I = f[M, R] \tag{4}
$$

but as

$$
I = \text{Int}(f[M, R]) \tag{5}
$$

where Int () represents integer truncation.

The methodological approach used has to reflect the use that will be made of the result. Clearly, it is better to use isoseismals rather than intensity data points in order to overcome the problems associated with population distribution. While it is true that, as often objected, the drawing of isoseismals is subjective and different workers will draw different contours, it is possible to overstate this problem. Differences between workers who are following the same principles can usually be resolved through discussion. Grossly different isoseismals for the same earthquake usually mean either that different data sets are being used, or different principles

are being followed. For example, in the comparative exercise presented in Cecić (1992) differences in making isoseismal maps of the same intensity data sets clearly reflect stylistic decisions about the amount of smoothing, with one person (an extreme case) deciding that all isoseismals should be near-circles. Problems arise when isoseismals are drawn in such a way that isoseismal *I* is constructed to include every single data point of intensity *I*, no matter how much it is an outlier. (A British example will be found in the highly exaggerated isoseismal maps of Tyrell, 1931).

In this study, isoseismals are used, which have been drawn so as to enclose areas where the intensity is predominantly equal to or greater than *I*, where *I* is the value of the isoseismal. This means that equation (2) needs to be set up so that when the predicted value of *I* is (for example) exactly 5.0, then the value for*r*(where *r* is the epicentral distance) is equal to the mean isoseismal 5 radius. That way, if synthetic isoseismals are constructed for an earthquake from the final attenuation equation, they will match the drawn isoseismals (which is what is needed). Thus, the prediction is, that for two values *r*1 and *r*2, such that for a particular earthquake the predicted intensities are 6.0 and 5.0, then in places between *r*1 and *r*2 from the epicentre the intensity will be predominantly 5.

Obviously, where isoseismals are strongly elliptical, converting them to equivalent circular areas introduces some error. Unless the ellipticality is consistent from earthquake to earthquake (which is not the case in the U.K.) this is unavoidable, and simply contributes to raising the degree of uncertainty when the resulting equation is put to use.

Intensity scales

All intensities referred to in this paper are EMS (in particular, EMS-98) unless otherwise stated (Grünthal, 1998). Many of the original data used in this study (Burton et al., 1984) were originally assessed using the MSK-81 scale (Ad hoc Panel, 1981), but the conversion is a one-to-one relationship, with the small difference that a number of values previously assigned 6–7 MSK can be more definitely assigned 6 EMS.

It may be objected that there are many textual differences between MSK and EMS and therefore such a simple correspondence cannot be taken for granted. However, one of the main rationales of the MSK to EMS revision process was to bring the text of the intensity scale into line with how it was being interpreted in best practice (Musson, 1990). There may be differences in the wording of the two scales, but in practice there is almost no difference between the values that will be obtained when both scales are used sensitively—except for the resolution of some "split" MSK intensities to single values.

Magnitude scales

The magnitude scale used in this study is local magnitude (ML). Since most recent PGA attenuation equations use surface-wave magnitudes (Ms) or moment magnitudes (Mw) some explanation is in order. ML is the preferred scale of most of the national monitoring agencies in NW Europe, firstly because it is easy to calculate, and secondly because it can be used smoothly from the largest events likely to be encountered (about 6ML) down to the smallest (in negative magnitudes). Despite the apparent limitations of a scale originally intended for use in California with a single instrument type (the Wood-Anderson), it has been demonstrated by Marrow (1992) that agreement between ML determinations for the same earthquakes by different national agencies in NW Europe is very consistent, with the exception of LDG, which tends to give values higher by about 0.2 units. Anyone involved with current earthquake monitoring in the region will have noticed from experience that this still holds true.

Therefore, since the BGS earthquake database (Walker et al., 2003) uses ML as the primary magnitude scale, as does the published U.K. earthquake catalogue (Musson, 1994), it is convenient to keep to this scale.

From data collected by Free et al. (1998), supplemented by additional U.K. data (Ritchie, 1999, personal communication), one can derive the equation

$$
Mw = 0.26 + 0.91ML \pm 0.25
$$
 (6)

Furthermore, there is no difference between the U.K. data from BGS and the larger data set collected by Free et al. (1998), mostly from the Swabian Jura, as seen in Figure 3.

(Note: After this paper was written, a regional ML to Mw conversion was published by Grünthal and Wahlström, 2003, using a quadratic formula. In the magnitude range 3–4ML, this formula gives Mw values that are around 0.3 units lower than equation (6)).

Figure 3. Regression of Mw and ML for NW Europe and the U.K. in particular.

Distance measures

In studies of PGA attenuation for large earthquakes, various different measures of distance have been used, according to whether site distance is measured to the epicentre, the hypocentre, the nearest point on the rupture plane, the surface projection of the nearest point on the rupture plane, or some other point (see, for example, Abrahamson and Shedlock, 1997). For the U.K., because earthquake sources are relatively small, the only distinction needed is that between epicentral and hypocentral distance. The typical rupture for a British earthquake is smaller than the uncertainty in location of the event, so the size of the rupture need not be taken into account. It is very clear from experience that focal depth is critically important for intensity distribution in the U.K. Therefore the use of hypocentral distance is required.

The data set

The data set available to this study consisted of 376 earthquakes ranging in date between 1382 and 2002 (at the time of writing there have been no significant felt earthquakes in the U.K. throughout 2003) and in magnitude from 2.0 to 6.1ML. Magnitudes for noninstrumental events were determined from macroseismic data as described in Musson (1996). The total num-

Figure 4. Number of isoseismals in the total data set, by intensity value.

ber of isoseismals is 727. Some events are present with only one isoseismal (usually for 3 EMS); the greatest number of isoseismals is seven, for the 1884 Colchester earthquake (intensities 2 to 8 EMS). As can be seen from Figure 4, the data set is heavily weighted towards lower intensities, as one might expect in a country of relatively low seismicity. There are 329 isoseismal 3 s and only 32 isoseismal 6 s.

The data are drawn from the BGS earthquake database. Much of it is published in Musson (1994), though some earthquakes have been revised since then, and significant events since 1993 have been added.

Two-stage or one-stage regression

In Joyner and Boore (1981) and Fukushima and Tanaka (1990) the case is made for the use of two-stage regression in attenuation studies, a practice adopted by them and followed by a number of authors since. The principle of two-stage regression is to derive the coefficients for magnitude and distance terms separately, in order to reduce the deleterious effects of the typical correlation between magnitude and distance in most strong motion data sets. In addition, it is often the case that, in a typical strong motion data set, a few earthquakes (typically larger ones) will contribute a large number of data points while others contribute very few, and the former events will have a disproportionate effect on the regression.

This approach was considered for the present study, but it seems to be unnecessary. Magnitude-distance correlation is a typical problem in strong motion data sets, because most data have been recorded from large earthquakes by accelerometers at middle-to-long distances. An intensity data set does not rely on instruments, and any earthquake will generally have a complete set of isoseismals, except in the case of offshore events. Also, the most number of data points contributed by a single earthquake is seven (out of 727 total). Brillinger and Preisler (1984, 1985) and Abrahamson and Youngs (1992) introduce a maximumlikelihood regression method in place of least-squares regression; again, it is not clear that methodological refinements introduced to deal with problems specifically in strong ground motion data sets confer significant benefits in studies using intensity data.

Results

As already discussed, the objective was to solve equation (2) such that when the predicted value of *I* is exactly equal to an integer value, then the corresponding value *r* is the mean isoseismal radius for the corresponding intensity. Values of *r* for each isoseismal were calculated by taking the isoseismal area and deriving the equivalent radius for a circle of the same area. This obviates any need to assign epicentral coordinates.

It was then necessary to convert r to R (epicentral to hypocentral distance). This was done using

$$
R = \sqrt{r^2 + h^2} \tag{7}
$$

where *h* is equal to the actual depth (if known) or h_0 otherwise, where h_0 is a notional depth optimised to reduce the residuals. The value obtained for h_0 was 12.8 km, which accords well with the typical depth for larger British earthquakes of between 10 and 15 km (Musson, 1996).

Equation (2) was solved for a , b , c , d and h_0 by a process of least squares regression, minimising the residuals between predicted *I* and the assumed intensity value at the edge of the isoseismal, i.e. exactly the intensity value of the isoseismal. The value of *d* was constrained to be negative or zero, since positive values are non-physical. The result was

$$
I = 3.28 + 1.41ML - 1.40lnR
$$
 (8)

which is very close to equation (1). As in the earlier study, *d* is found to be zero or negligible. The sigma value expressing the scatter of values, which can be used to model the aleatory uncertainty in attenuation in hazard studies, is calculated to be 0.50. Note that this uncertainty is normally distributed about the expected intensity value, not lognormally as is the case with PGA attenuation. This is discussed in more detail later.

Equation (8) is plotted in Figure 5, together with the supporting data points, which are grouped by magnitude, half a unit above and below the values for which the curves are plotted. Data for events smaller than 2.5ML are not plotted. Distances are hypocentral. There is a tendency for the curves to over-predict the radius of intensity 2, which is hardly surprising since the full extent of this isoseismal is usually not well reported, and many seismologists would be inclined not to attempt plotting this isoseismal at all. The fact that in the U.K. it is sometimes possible to do so is due to the fact that, earthquakes in Britain being a rare experience, low intensities are much better reported that they would normally be in other parts of the world. The effect of leaving out intensity 2 will be examined shortly.

The data for earthquakes in the magnitude range 5.5–6.4ML is rather dispersed, with one prominent outlier that has much smaller isoseismal radii than the rest. Probably the magnitude of this event (1926 Channel Islands) is overstated, being an instrumental determination from historical seismograms. The instrumental magnitude is 5.5ML (Neilson and Burton, 1988) while the macroseismic magnitude is only 5.1ML. Apart from this event, intensity 4 and 5, and to a lesser extent 6, are systematically under-predicted for the largest earthquakes in the data set, which are relatively few in number.

Some variations were explored using subsets of the total data set. The first of these was to restrict the calculations to modern data only, i.e. 1970 and after. This has the effect of removing any possible contamination of the results due to earthquake parameters (magnitude, depth) having been derived from macroseismic data in the first place. The data set is now reduced to 47 earthquakes and 137 isoseismals from intensity 2 to 6 (but with only three isoseismal 6 s). This data set is more internally consistent in that all the data are derived from questionnaires, whereas the full data set was heavily dependent on historical data from a variety of sources. The new equation is

I = 3.82 + 1.14ML − 1.24 ln *R* − 0.00058*R* (9)

Figure 5. Equation (8) plotted for magnitudes 3, 4, 5 and 6ML, with the supporting data, clustered by magnitude in steps of one unit. Intensity values have been displaced slightly above or below the exact value to make the graph easier to read.

The magnitude term has decreased, but is less well constrained, since the largest earthquake in the modern data set is 5.4ML (the Roermond earthquake was not included in the data set; although it was felt in the U.K. it was not considered to be a British earthquake for the purposes of the study). The sigma value increases to 0.54.

This is shown in Figure 6, in comparison with equation (8). The two equations are very similar for magnitude 5ML, but equation (9) is less satisfactory for larger earthquakes (not surprisingly).

The second variation on the total data set is to remove all data for intensity 2, since these isoseismals

Figure 6. Comparison of equation (8) (all data) with equation (9) (modern data) for magnitudes 3, 4, 5 and 6ML. Bolder lines are for equation (9).

Figure 7. Comparison of equation (8) (all data) with equation (10) (data >2) for magnitudes 3, 4, 5 and 6ML. Bolder lines are for equation (10).

are inevitably poorly constrained. The data set now has 641 isoseismals.

This yields equation (10) as follows:

$$
I = 3.11 + 1.35ML - 1.27lnR
$$
 (10)

The sigma value drops to 0.43. The reduction in sigma clearly reflects the fact that isoseismals 2 s are liable to be poorly determined. The other significant difference is that the distance parameter has decreased, so whereas equations (10) and (8) give similar results at distances less than 100 km, equation (10) predicts higher intensities at greater distances. This equation is shown in Figure 7.

It could be argued that including events with only one isoseismal (usually intensity 3) biases the data set too much towards smaller events and lower intensities. To check, the data set was reduced by removing all events with only one isoseismal. This left 514 isoseismals, and produced the following result:

$$
I = 3.32 + 1.27ML - 1.21lnR
$$
 (11)

The sigma value is 0.53.

It follows logically to recompute equation (11) without the data for intensity 2. This leaves 416 isoseismals. Some earthquakes were removed where they had only isoseismals for intensity 2 and one other intensity. This gives the result:

$$
I = 3.31 + 1.28ML - 1.22lnR
$$
 (12)

with a sigma value of 0.46. This is almost identical to equation (11), indicating that the intensity 2 data had more effect on the residuals than the parameters of the equation. The equation is shown in Figure 8. Note that for equations $(11-12)$ the h_0 value is irrelevant, as depths have been estimated for all earthquakes with more than one isoseismal (Musson, 1996). Ignoring determined depth and using h_0 throughout (on the grounds that some depth determinations may be rather uncertain) is not really a viable option, as in the U.K. situation there is a considerable difference in the effects of earthquakes occurring in the top 5 km, those occurring between 5 and 15 km in depth, and those greater than 15 km.

A further experiment, following from equations (9) and (12), was to restrict the data set entirely to events with instrumental magnitudes (including those from historical seismograms) for intensities 3 and above. This eliminates possible feedback from using macroseismic magnitude, while improving the spread of magnitudes and intensities from that used in equation (9). On the other hand, some of the historical instrumental magnitude values are not well constrained, and it is

Figure 8. Comparison of equation (8) (all data) with equation (12) (data above intensity 2 for events with more than one isoseismal) for magnitudes 3, 4, 5 and 6ML. Bolder lines are for equation (12).

debatable whether the use of macroseismic magnitude is really a problem. The number of isoseismals here was 206, including events with only one isoseismal.

In this case the equation obtained was:

$$
I = 3.61 + 0.99ML - 1.01lnR
$$
 (13)

The sigma value is 0.54, as it was for equation (9). This data set gives the lowest values for *b* and *c*, both being remarkably close to unity. The difference between equations (12) and (13) is shown in Figure 9, and is significant.

The fact that, as seen in Figure 5, the regressions seem to behave less well for larger earthquakes, suggests the use of a different magnitude term. In some PGA attenuation relations (e.g. Atkinson and Boore, 1997; Spudich et al., 1999) a quadratic form is used, as in

$$
Y = a + b_1(ML - 6) + b_2(ML - 6)^2
$$

+ c ln R + dR (14)

where *Y* is ln PGA or, in this case, *I*. The value of 6 is arbitrary; changing it affects the value of *a* (and obviously, b_1 and b_2), but has no effect on the fit. Because the U.K. data set comprises smaller events than those that would be used for most attenuation studies, a value of 4 can be used instead (the difference is cosmetic only). Using the same data set as was used for equation (12), the values obtained are:

$$
I = 8.25 + 1.25(ML - 4) + 0.17(ML - 4)^{2}
$$

- 1.20lnR - 0.00074R (15)

The sigma value is 0.44. This equation is plotted in Figure 10, along with the basic data, taken from Figure 5. The fit to the data from the larger earthquakes is now improved.

It may be considered surprising, or even unrealistic, that the coefficient of the quadratic term in equation (15) should be positive, when this is normally expected to be negative (see, for instance, Atkinson and Boore, 2003). A review of the subject by Fukushima (1996) found that empirical studies produced both negative and positive coefficients. He determined that the reason for this was most likely due to the magnitude scale employed. Studies using Mw found negative coefficients, and studies using ML found positive ones, and the difference is due to the scaling of magnitude with seismic moment. Since the present study is conducted using ML, the positive coefficient is in line with the findings of Fukushima (1996), although this does imply a leap from what is found for acceleration to what is found for intensity.

Figure 9. Comparison of equation (12) with equation (13) (instrumental data only) for magnitudes 3, 4, 5 and 6ML. Bolder lines are for equation (12).

Figure 10. Equation (15) superimposed on the complete data set (as in Figure 5).

The issue is complicated, however, firstly by the fact that beyond 6ML one is reaching the zone where the ML scale is liable to saturate, and secondly, the inherent rarity of such larger events means that modelling of the effects of larger events is not as well constrained as one would like. (Of the four largest events in the data set, all are offshore and two occurred before 1700,

which makes the data from these events not so dependable.) The lack of more representative data for larger magnitudes makes the use of the quadratic form more problematic.

In terms of the significance for hazard, the choice between equations (12) and (15) makes a difference to the effect of maximum magnitude on the hazard calculations. If equation (15) is used, the possible occurrence of earthquakes larger than any in the data set will generate strong intensities over substantial areas, and raising the maximum magnitude value used will have a noticeable impact on hazard calculations. It is considered by many that the maximum possible U.K. earthquake has probably occurred in the 1000 yr historical period (Ambraseys and Jackson, 1985; Bommer, 2002), in which case the extrapolation to magnitudes higher than 6ML (or its equivalent in other scales) would not be an issue. However, setting such a low maximum magnitude in hazard assessment (equivalent to about 5.5 Ms) would generally be perceived as unconservative.

In consequence, equations (12) and (15) are both worthy of note. The low sigma values of 0.46 and 0.44 respectively, are only bettered by equation (10), which is weighted more to smaller earthquakes. The extrapolation to magnitudes larger than those in the data set is debatable. On balance, equation (12) seems to be the best choice.

Discussion and conclusions

This study now puts the subject of intensity attenuation in the U.K. on a firmer footing. The equation originally published in Musson and Winter (1996) has now been updated using an expanded and improved data set, and the basis of the calculations set out.

The preferred equation is equation (12). The removal from the data set of: (a) poorly determined values for intensity 2; (b) a number of events contributing only an isoseismal 3 and nothing else, improves the applicability of this equation. Equation (15) incorporates a quadratic magnitude term, which seems to improve the fit to data from larger earthquakes, but this is not well constrained, and equation (12) is probably more reliable.

The sigma value for equation (12) is 0.46, which is quite low, and means that intensities predicted will usually be good to within half an intensity degree; however, this does not take into account deviations due either to ellipticality of isoseismals or local soil effects, both of which were removed from the basic data through the use of average isoseismal radii.

When computing hazard using intensity, it is important to realise that this sigma value follows a normal distribution. That this should be so follows naturally from the fact that intensity equations are generally written, as with equation (2), in terms of *I* and not ln *I*. However,

Figure 11. Normal probability plot for residuals from observations of intensity 4 in the entire data set and the values predicted by equation (8). The black line shows the fit for a perfectly normal distribution; the deviations are not statistically significant, showing that the scatter of observed values around the predicted value is normally distributed.

the matter is not completely straightforward, as intensity has a lower bound of 1 (at least in EMS). Thus in the case of a predicted intensity 2, the residual may credibly be $+1$ or $+2$, but cannot be lower than -1 . Residual populations for the lower intensities will therefore not be perfectly normally distributed. Figure 11 shows a Kolmogorov-Smirnov test for normality for the residuals for intensity 4 from the whole data set and equation (8) (to maximise the number of isoseismals to test). The *p*-value obtained was >0.15 , indicating that the residuals are not significantly different from a normal distribution.

Most seismic hazard software is designed with PGA or spectral acceleration in mind and expects attenuation uncertainty to follow a lognormal distribution, and this causes problems when calculating intensity hazard. The computation of intensity hazard requires modified or custom-written hazard software that correctly implements normal-distribution scatter capped at a minimum intensity value of 1. One such program is M3C (Musson, 2000), which implements a different procedure according to whether hazard is being computed as acceleration or intensity.

As an example of the application of equation (12), Table 1 presents a chart for estimating the radius of isoseismal 6 (effectively the radius of damage) for magnitudes up to 6.0ML and depths from 2 to 24 km. Similar charts can easily be drawn up for other intensities if desired.

Figure 12 shows two intensity hazard curves for the city of Cardiff (South Wales) computed using equation (12), and two different seismic source models. It also shows the intensity history of Cardiff, using either

Magnitude	Depth (km)											
	\overline{c}	$\overline{\mathcal{L}}$	6	8	10	12	14	16	18	20	22	24
3.0	$\,1$											
3.1	\overline{c}											
3.2	\overline{c}											
3.3	\overline{c}											
3.4	\mathfrak{Z}											
3.5	3	$\,1\,$										
3.6	$\overline{4}$	\overline{c}										
3.7	$\overline{4}$	3										
3.8	5	$\overline{4}$										
3.9	6	5	\overline{c}									
4.0	τ	$\sqrt{6}$	$\overline{\mathcal{L}}$									
4.1	$\overline{7}$	τ	5	$\,1$								
4.2	$\,$ 8 $\,$	8	6	$\overline{\mathcal{L}}$								
4.3	9	9	8	6	$\boldsymbol{0}$							
4.4	10	10	9	$\overline{7}$	$\overline{4}$							
4.5	12	11	10	9	7	\mathfrak{Z}						
4.6	13	13	12	11	9	6						
4.7	15	14	14	13	11	9	6					
4.8	16	16	15	14	13	11	9	5				
4.9	18	18	$17\,$	17	15	14	12	9	5			
5.0	20	20	20	19	18	$17\,$	15	13	10	6		
5.1	23	22	22	21	20	19	18	16	14	11	$\boldsymbol{7}$	
5.2	25	25	25	24	23	22	21	20	18	16	13	9
5.3	28	28	28	27	26	26	25	23	22	20	18	15
5.4	31	31	31	30	30	29	28	27	26	24	23	20
5.5	35	35	34	34	33	33	32	31	30	29	27	25
5.6	39	39	38	38	37	37	36	35	34	33	32	31
5.7	43	43	43	42	42	41	41	40	39	38	37	36
5.8	48	48	48	47	47	46	46	45	44	44	43	42
5.9	53	53	53	53	52	52	51	51	50	49	49	48
6.0	59	59	59	59	58	58	58	57	56	56	55	54

Table 1. Ready-reckoner for expected radius of isoseismal 6 EMS (radius of damage) for any combination of magnitude (ML) and focal depth

observed data where possible, or estimated data where accounts are lacking. It is assumed that the record for intensity 3 is complete for 200 yr, for intensity 4 for 250 yr, for intensity 5 for 300 yr and for intensity 6 for 450 yr. These assumptions are based on historical considerations. Intensities resulting from historical earthquakes less than 4ML are excluded, as 4ML was the minimum magnitude used in the hazard calculations. Figure 10 allows one to compare the two models against reality.

The model represented by the dotted line (model 2) gives a better fit to the historical observations, which is perhaps not surprising, as it was a site-specific model for Cardiff, whereas model 1 was a generic U.K. model intended for hazard mapping. However, even model 1 gives a reasonable fit.

This ability to evaluate seismic hazard studies is very useful (e.g. Mucciarelli et al., 2000). In the past, the tendency has been to evaluate studies by peer

Figure 12. Intensity hazard curves for the city of Cardiff computed using equation (12) and two different source models, compared with historical intensity observations.

review, a process that tends to focus on the theoretical basis for modelling decisions, rather than what the actual effect of those decisions is on the results (and therefore whether the decisions are truly realistic).

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