

# Microwave Transmission and Reflection for a Type-II Superconducting Superlattice in the Mixed State

Tsung-Wen Chang · Chien-Jang Wu

Received: 3 July 2010 / Accepted: 15 July 2010 / Published online: 5 August 2010  
© Springer Science+Business Media, LLC 2010

**Abstract** The microwave reflection and transmission for a type-II superconducting superlattice in the mixed state are theoretically calculated based on the vortex dynamical model together with the transfer matrix method in a layered medium. The superlattice is made of alternating layers of high-temperature superconductors and dielectric materials. We analyze the microwave reflection and transmission as functions of the static magnetic field, the number of periods, and the thickness of the dielectric layer. It is shown that the reflection decreases as the static field increases. In addition, the reflection will be enhanced by increasing the number of periods.

**Keywords** Type-II superconductors · Mixed state · Vortex dynamics · Abeles theory

## 1 Introduction

Vortex response to an electromagnetic microwave field has attracted much attention since the discovery of high-temperature superconductors. Microwave response for a type-II superconductor in the mixed state is generally studied through the calculation or the measurement of the surface impedance,  $Z_s = R_s + jX_s$ , where the real part,  $R_s$ , is the surface resistance, and the imaginary part,  $X_s$ , is known

as the surface reactance. It has been known that the insight into related physics of vortex dynamics, such as the flux flow, flux pinning and flux creep, can be gained from the knowledge of surface impedance. The theoretical calculation of surface impedance belongs to an attenuation-dominated problem and is related to the model of vortex dynamics. So far, there have been many reports on the high-frequency vortex response for the high-temperature superconductors, as can be found in some review articles [1, 2].

In addition to the surface impedance, microwave properties of a superconducting thin film in the mixed state can also be explored by the measurements of microwave transmission and reflection [3, 4]. The theory of microwave transmission and reflection for a type-II superconducting film on a dielectric substrate was first developed by Coffey and Clem based on their linear response model of the self-consistent treatment of vortex dynamics, which is now called the Coffey–Clem (CC) model [5, 6].

In recent years, a periodic multilayer has become the hot topic in optical and physical communities since the pioneering works of Yablonoitch and John [7, 8]. They combined the electromagnetics and solid state physics to describe the electromagnetic wave propagation in a periodic structure, and then a new era of photonic crystals (PCs) was quickly opened two decades after [9]. A PC is a periodic layered structure consisting of two or more different materials with different refractive indices (permittivities and/or permeabilities). A superlattice which can be, from the structural viewpoint, regarded as a kind of PC is also an important structure in the optical physics and the condensed matter physics for a long time. For a simple all-dielectric superlattice, it is commonly designed as a Bragg reflector (dielectric mirror) that has a very high reflectance for some selected spectral ranges [10]. A theoretical investigation of microwave propagation characteristics in a magnetic–dielectric super-

T.-W. Chang  
Graduate Institute of Electro-Optical Engineering, Chang Gung University, Tao-Yuan 333, Taiwan

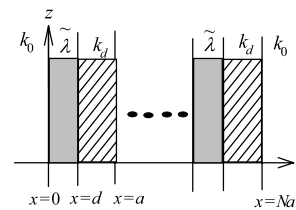
C.-J. Wu (✉)  
Institute of Electro-Optical Science and Technology, National Taiwan Normal University, Taipei 116, Taiwan  
e-mail: [jasperwu@ntnu.edu.tw](mailto:jasperwu@ntnu.edu.tw)

lattice was given by Camley and Mills [11]. Studies of periodic superconducting multilayer structures are also available in the literature. For instance, based on the two-mode electrodynamics, Tagantsev and Traito calculated the surface impedance of a superconductor superlattice made of alternating layers with weak pinning and layers with strong pinning [12]. They also investigated the surface impedance of a multilayer superconductor system consisting of type-II superconductors and normal metals [13]. The study of microwave propagation characteristics in a superconductor–dielectric layered waveguide structure in the mixed state was also reported previously [14]. Recently, the superconductor–ferromagnetic superlattice has been used to demonstrate the existence of a negative refractive index, a fundamental result of the so-called double negative material with permittivity and permeability both being negative [15]. In addition, a superconductor–dielectric superlattice with a spatial periodicity comparable to the wavelength of the electromagnetic wave can be treated as a superconducting photonic crystal (SPC). The photonic band structures for the SPCs have been reported in recent years [16–23].

This purpose of this paper is to study the microwave properties for a type-II superconducting superlattice in the mixed state. Microwave properties will be investigated based on the calculated transmission and reflection in the superlattice. We shall use the transfer matrix method [24] to calculate the mixed-state microwave reflectance and transmittance for superconductor–dielectric superlattice. It will be seen that our results can be reduced to the previous ones given in [3] in which the simple case of a single period was considered. That is, the theory of microwave reflection and transmission for a type-II superconducting film in the mixed state will be successfully extended to structure of a superlattice. Some numerical analyses will be made on the microwave transmission and reflection as a function of the static magnetic field, the number of periods, and the thickness of the dielectric layer.

## 2 Theory of Vortex Dynamics and Abeles Theory

The superconductor–dielectric superlattice is depicted in Fig. 1, where the total number of periods is  $N$  and the periodicity is denoted by  $a$ . Each period consists of a type-II superconductor layer of thickness  $d$  and a dielectric layer of thickness  $t = a - d$ . We assume that the superlattice is immersed in free space. A static magnetic field  $B \geq 2B_{c1}$  is applied along the  $x$ -axis to cause the superconductors to be in the mixed state, where  $B_{c1}$  is the lower critical field of the superconductor. A microwave wave with  $z$ -polarized magnetic field impinges normally on the plane boundary at  $x = 0$ . All rf fields are assumed to have an  $e^{j\omega t}$  time dependence. The electromagnetic wave propagation in each layer



**Fig. 1** A type-II superconducting superlattice under study, in which the periodicity is denoted by  $a$ , the superconductor slab has a thickness of  $d$ , and the thickness of the dielectric slab is  $t = a - d$ . A static magnetic field  $\mathbf{B} = \hat{x}B$  is applied to put all the superconductor films in the mixed state. A microwave field is launched normally into the layered medium at the plane boundary  $x = 0$ . The superconductor film is characterized by the complex penetration depth,  $\tilde{\lambda}$ . The dielectric slab is characterized by the wave number,  $k_d$

is governed by the Helmholtz-like equation from which the wave number can be defined. In free space, the wave number is given by  $k_0 = \omega\sqrt{\mu_0\epsilon_0} = \omega/c$ , where  $\mu_0$  and  $\epsilon_0$  are its permeability and permittivity, respectively, and  $c$  is the speed of light. In the dielectric layer (with permittivity  $\epsilon_d$ ), the wave number is  $k_d = \omega\sqrt{\mu_0\epsilon_d} = k_0n_d$ , where  $n_d = \sqrt{\epsilon_d/\epsilon_0}$  is the index of refraction of the dielectric layer.

As for the superconductor layer, the electromagnetic wave properties can be described by an effective complex index of refraction given by

$$n_s = -\frac{j}{k_0\tilde{\lambda}}, \quad (1)$$

where  $\tilde{\lambda} = \tilde{\lambda}(\omega, B, T)$  depends on the frequency, the static field, and the temperature. It is a complex penetration depth which represents the vortex response function to a microwave field. According to the CC model, it is expressible as [3, 5, 6]

$$\begin{aligned} \tilde{\lambda}(\omega, B, T) &= \lambda' - j\lambda'' \\ &= \sqrt{\frac{\lambda^2(B, T) - (j/2)\delta_{vc}^2(\omega, B, T)}{1 + 2j\lambda^2(B, T)\delta_{nf}^{-2}(\omega, B, T)}}, \end{aligned} \quad (2)$$

where  $\lambda = \lambda(B, T)$  is the static-field and temperature-dependent penetration depth given by

$$\lambda = \lambda(B, T) = \frac{\lambda(0, T)}{\sqrt{1 - \frac{B}{B_{c2}(T)}}}, \quad (3)$$

where the zero-field penetration length is  $\lambda(0, T) = \lambda_0/\sqrt{1 - (T/T_c)^4}$ , and the temperature-dependent upper critical field is given by  $B_{c2}(T) = B_{c2}(0)[1 - (T/T_c)^2][1 + (T/T_c)^2]^{-1}$ . The normal-fluid skin depth is  $\delta_{nf} = \delta_{nf}(\omega, B, T) = \sqrt{2\rho_{nf}/\omega\mu_0}$  with the normal-fluid resistivity given by  $\rho_{nf} = \rho_n/f(T, B)$ , where  $f(T, B) = 1 - [1 - (T/T_c)^4][1 - B/B_{c2}(T)]$ . In addition,  $\delta_{vc} = \delta_{vc}(\omega, B, T) = \sqrt{2\tilde{\rho}_v/\omega\mu_0}$  is the complex skin depth arising from both the vortex motion and flux creep, where the effective resistivity

is defined by  $\tilde{\rho}_v = \tilde{\rho}_v(\omega, B, T) = B\phi_0\tilde{\mu}_v(\omega, B, T)$ , where  $\phi_0$  is the flux quantum and the dynamic mobility is given by

$$\tilde{\mu}_v(\omega, B, T) = \frac{1}{\eta} \left[ 1 + \left[ \frac{j\omega\eta}{\alpha\kappa_p} + \frac{1}{I_0^2(\varpi) - 1} \right]^{-1} \right]^{-1}. \tag{4}$$

Here  $\eta$  is the viscous drag coefficient which is defined by  $\eta = B\phi_0/\rho_f$  with the flux flow resistivity being described by the Bardeen–Stephen theory, i.e.,  $\rho_f = \rho_n B/B_{c2}(T)$ .  $\kappa_p$  is the force constant of the pinning well which can be expressed as  $\kappa_p = \kappa_{p0}[1 - (T/T_{c2})^2]^2$ , where  $T_{c2}$  is the temperature at which  $B = B_{c2}(T)$ . The parameter  $\alpha$  is given by  $\alpha = I_1(\varpi)/I_0(\varpi)$ , where  $I_1$  and  $I_0$  are the modified Bessel functions of the first kind of order one and zero, respectively. The argument  $\varpi$  is given by  $\varpi = U_0(B, T)/2k_B T$ , where the barrier height of the periodic potential is  $U_0(B, T) = U[1 - (T/T_{c2})^{3/2}]/B$ .

Having defined the aforementioned indices of refraction for all layers, the microwave transmission and reflection problem for a multilayer system in Fig. 1 can be worked out by making use of the transfer matrix method called the Abeles theory [24]. Based on this theory, the characteristic matrix for a single period with a periodicity of  $a$  should be constructed and the result is

$$\mathbf{M}(a) = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \tag{5}$$

where the matrix elements are written by

$$m_{11} = \cos \beta_2 \cos \beta_3 - \frac{p_3}{p_2} \sin \beta_2 \sin \beta_3, \tag{6a}$$

$$m_{12} = \frac{j}{p_3} \cos \beta_2 \sin \beta_3 + \frac{j}{p_2} \sin \beta_2 \cos \beta_3, \tag{6b}$$

$$m_{21} = jp_2 \sin \beta_2 \cos \beta_3 + jp_3 \cos \beta_2 \sin \beta_3, \tag{6c}$$

$$m_{22} = \cos \beta_2 \cos \beta_3 - \frac{p_2}{p_3} \sin \beta_2 \sin \beta_3, \tag{6d}$$

where

$$\beta_2 = k_0 n_s d, \quad \beta_3 = k_0 n_d t, \tag{7}$$

and

$$p_2 = n_s Z_0^{-1}, \quad p_3 = n_d Z_0^{-1}, \tag{8}$$

where  $Z_0 = \sqrt{\mu_0/\epsilon_0} = 120\pi \Omega$  is the wave impedance of free space. With the matrix in (5), the total characteristic matrix for an  $N$ -period layered medium can be obtained to be

$$\begin{aligned} \mathbf{M}(Na) &= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = [\mathbf{M}(a)]^N \\ &= \begin{pmatrix} m_{11}U_{N-1}(\psi) - U_{N-2}(\psi) & m_{12}U_{N-1}(\psi) \\ m_{21}U_{N-1}(\psi) & m_{22}U_{N-1}(\psi) - U_{N-2}(\psi) \end{pmatrix}, \tag{9} \end{aligned}$$

where

$$\psi = \cos \beta_2 \cos \beta_3 - \frac{1}{2} \left( \frac{p_2}{p_3} + \frac{p_3}{p_2} \right) \sin \beta_2 \sin \beta_3, \tag{10}$$

and the function  $U_N$  defined as

$$U_N(\psi) = \frac{\sin[(N+1)\cos^{-1}\psi]}{\sqrt{1-\psi^2}}, \tag{11}$$

is known as the Chebyshev polynomials of the second kind.

Based on the matrix in (9), the reflection coefficient,  $\tilde{r}$ , and the transmission coefficient,  $\tilde{t}$ , can be determined, namely [24]

$$\tilde{r} = \frac{(M_{11} + M_{12}Z_0^{-1})Z_0^{-1} - (M_{21} + M_{22}Z_0^{-1})}{(M_{11} + M_{12}Z_0^{-1})Z_0^{-1} + (M_{21} + M_{22}Z_0^{-1})}, \tag{12}$$

$$\tilde{t} = \frac{2Z_0^{-1}}{(M_{11} + M_{12}Z_0^{-1})Z_0^{-1} + (M_{21} + M_{22}Z_0^{-1})}. \tag{13}$$

The corresponding reflectance,  $R$ , and transmittance,  $\Gamma$ , are consequently given by

$$R = |\tilde{r}|^2, \quad \Gamma = |\tilde{t}|^2. \tag{14}$$

Before presenting the numerical results, let us examine the above-derived formulae in some special case. For a simple single period structure, say,  $N = 1$ , we have  $M_{ij} = m_{ij}$ ,  $i, j = 1, 2$ . In this case, explicit expressions of the transmission coefficient and reflection coefficient can be obtained. A direct calculation for the transmission coefficient given in (13) leads to

$$\tilde{t} = \frac{2}{\tilde{D}}, \tag{15}$$

where

$$\begin{aligned} \tilde{D} &= \frac{-j}{2k_d\tilde{\lambda}} \left\{ e^{jk_d t} \left( 1 + \frac{k_d}{k_0} \right) \left[ (1 - k_0 k_d \tilde{\lambda}^2) \sinh\left(\frac{d}{\tilde{\lambda}}\right) \right. \right. \\ &\quad \left. \left. + j(k_d + k_0)\tilde{\lambda} \cosh\left(\frac{d}{\tilde{\lambda}}\right) \right] \right. \\ &\quad \left. - e^{-jk_d t} \left( 1 - \frac{k_d}{k_0} \right) \left[ (1 + k_0 k_d \tilde{\lambda}^2) \sinh\left(\frac{d}{\tilde{\lambda}}\right) \right. \right. \\ &\quad \left. \left. - j(k_d - k_0)\tilde{\lambda} \cosh\left(\frac{d}{\tilde{\lambda}}\right) \right] \right\}. \tag{16} \end{aligned}$$

Hence the transmittance is written as

$$\Gamma = |\tilde{t}|^2 = \frac{4}{|\tilde{D}|^2} = \frac{16|k_d|^2|\tilde{\lambda}|^2}{D}, \tag{17}$$

where

$$D = \left| e^{jk_d t} \left( 1 + \frac{k_d}{k_0} \right) \left[ (1 - k_0 k_d \tilde{\lambda}^2) \sinh \left( \frac{d}{\tilde{\lambda}} \right) + j(k_d + k_0) \tilde{\lambda} \cosh \left( \frac{d}{\tilde{\lambda}} \right) \right] - e^{-jk_d t} \left( 1 - \frac{k_d}{k_0} \right) \left[ (1 + k_0 k_d \tilde{\lambda}^2) \sinh \left( \frac{d}{\tilde{\lambda}} \right) - j(k_d - k_0) \tilde{\lambda} \cosh \left( \frac{d}{\tilde{\lambda}} \right) \right] \right|^2 \tag{18}$$

Similarly, the reflection coefficient in (12) can be calculated as

$$\tilde{r} = \frac{\tilde{N}}{\tilde{D}}, \tag{19}$$

where

$$\tilde{N} = \frac{j}{2k_d \tilde{\lambda}} \left\{ e^{jk_d t} \left( 1 + \frac{k_d}{k_0} \right) \left[ (1 + k_0 k_d \tilde{\lambda}^2) \sinh \left( \frac{d}{\tilde{\lambda}} \right) + j(k_d - k_0) \tilde{\lambda} \cosh \left( \frac{d}{\tilde{\lambda}} \right) \right] - e^{-jk_d t} \left( 1 - \frac{k_d}{k_0} \right) \left[ (1 - k_0 k_d \tilde{\lambda}^2) \sinh \left( \frac{d}{\tilde{\lambda}} \right) - j(k_d + k_0) \tilde{\lambda} \cosh \left( \frac{d}{\tilde{\lambda}} \right) \right] \right\} \tag{20}$$

Accordingly, the corresponding reflectance is given by

$$R = |\tilde{r}|^2 = \frac{|\tilde{N}|^2}{|\tilde{D}|^2} = \frac{N}{D}, \tag{21}$$

where

$$N = \left| e^{jk_d t} \left( 1 + \frac{k_d}{k_0} \right) \left[ (1 + k_0 k_d \tilde{\lambda}^2) \sinh \left( \frac{d}{\tilde{\lambda}} \right) + j(k_d - k_0) \tilde{\lambda} \cosh \left( \frac{d}{\tilde{\lambda}} \right) \right] - e^{-jk_d t} \left( 1 - \frac{k_d}{k_0} \right) \left[ (1 - k_0 k_d \tilde{\lambda}^2) \sinh \left( \frac{d}{\tilde{\lambda}} \right) - j(k_d + k_0) \tilde{\lambda} \cosh \left( \frac{d}{\tilde{\lambda}} \right) \right] \right|^2 \tag{22}$$

Equations (17), (18) and (21), (22) agree with the ones previously given by Coffey and Clem [5]. Note that if the microwave is incident backward on the plane boundary,  $x = Na$ , the reflection coefficient and transmission coefficient can be easily obtained by simply interchanging  $\beta_2 \leftrightarrow \beta_3$ , and  $p_2 \leftrightarrow p_3$ .

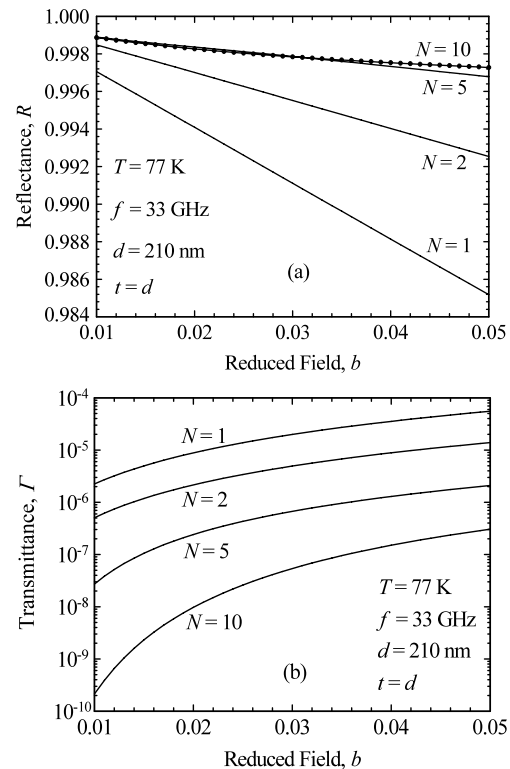
In addition, the Abeles theory is an elegant transfer matrix method for dealing with the wave propagation problem in a one-dimensional layered structure. However, if one is interested in the static field solution for a layered superconducting system without dielectric layers, then other transfer matrix method is also available [25]. In [25], Coffey has

used this method to solve the static field solution in a system containing a point dipole magnetic force microscopy (MFM) tip located above a multilayer superconductor in the Meissner state.

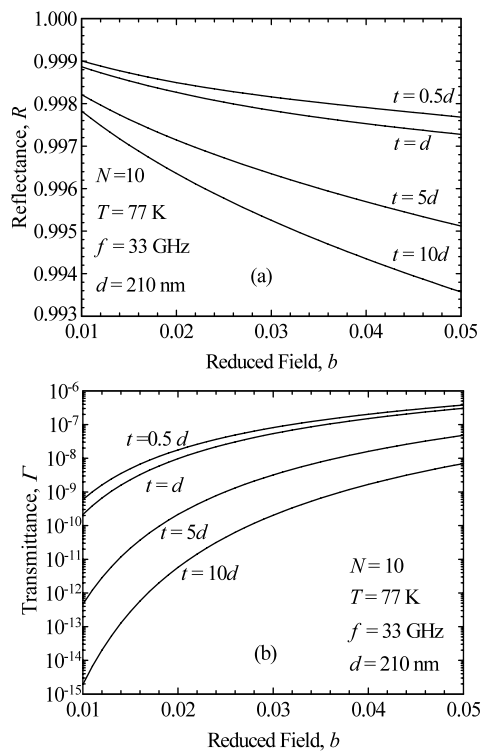
### 3 Numerical Results and Discussion

Let us now present some numerical results based on the previous derived equations. The dielectric layers in the superlattice are taken to be LaAlO<sub>3</sub> with an index of refraction,  $n_d = 4 - j10^{-4}$  [5], where the imaginary part indicates the dielectric loss. As for the superconductor layers, the material parameters are taken on the typical values of a high-temperature superconducting system, YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7-x</sub>, but not for a specific sample. These include the critical temperature  $T_c = 92$  K, the zero-temperature London penetration depth  $\lambda_0 = 140$  nm, the zero-temperature upper critical field  $B_{c2}(0) = 112$  T, the pinning potential constant  $U = 0.15$  eV T, and the temperature-dependent normal-state resistivity  $\rho_n(T) = 1.1 \times 10^{-8} T + 2 \times 10^{-7} \Omega \text{ m}$ . Also, the reduced field is introduced as usual and is defined by  $b = B/B_{c2}(0)$  [5].

In Fig. 2, the following has been plotted: (a) the reflectance, and (b) the transmittance as a function of the sta-



**Fig. 2** Calculated static-field dependence of microwave reflectance in (a) and transmittance in (b) for different numbers of periods,  $N = 1, 2, 5,$  and  $10$ . The conditions used are: frequency,  $f = 33$  GHz, temperature,  $T = 77$  K, and superconductor and dielectric slabs both having a thickness of  $210$  nm



**Fig. 3** Calculated static-field dependence of microwave reflectance in (a) and transmittance in (b) in a 10-period medium for different thicknesses of the dielectric slab,  $t = 0.5d$ ,  $d$ ,  $5d$ , and  $10d$ . The conditions used are: frequency,  $f = 33$  GHz, temperature,  $T = 77$  K, and superconductor slab with a thickness of 210 nm

tic magnetic field for different number of periods,  $N = 1, 2, 5$ , and  $10$ . The calculated conditions are  $T = 77$  K,  $f = 33$  GHz,  $d = 210$  nm, and  $t = d$ . For a single period,  $N = 1$ , the reflectance decreases as the static field increases. The decreasing feature is due to the more vortices produced by the increase in the static magnetic field. The more vortices in the superconductor will give rise to more losses when they move in response to the external microwave field. The decreasing behaviors become weak for  $N = 5$ , and  $10$ . The results indicate that a high-reflectance reflector at microwave can be obtained by piling more layers in the structure, i.e., by increasing the number of periods.

The effects of dielectric thickness on the field-dependent  $R$  and  $\Gamma$  are shown in Figs. 3(a) and 3(b), respectively. It is also seen that, at a fixed static magnetic field,  $R$  and  $\Gamma$  are lowered when the thickness of the dielectric slab is increased. The decreasing behavior in both  $R$  and  $\Gamma$  can be ascribed to the increasing loss in the dielectric layers when its thickness increases.

We have investigated the static-field dependence of  $R$  and  $\Gamma$  at different numbers of periods. It is thus expected that the experimental work previously done in [3] can be conducted for more than a single period. In addition, the recent experiment on the superconducting superlattice [15] can be

extended to be in the mixed state. The novel negative refractive index could be studied in the presence of vortices.

## 4 Summary

By using the Coffey–Clem model together with the Abeles theory in a layered medium, microwave transmission and reflection in a superconductor–dielectric superlattice in the mixed state has been theoretically investigated. Analytical formulae have been presented for the microwave transmission problem in a type-II superconducting film in a periodic structure. Explicit expressions for the microwave reflectance as well as transmittance in a single bilayer structure,  $N = 1$ , are also given. With these formulae, some numerical analyses have been made on the field-dependent microwave transmittance and reflectance. Results show that both reflectance and transmittance decrease with increasing the thickness of the dielectric slab. In addition, as the number of periods is increased, a high-reflectance reflector can be achieved in a layered medium. Because of the high potential in the application of layered medium in superconductor electronics, it is expected that the current work may be of fundamental and technical use in the microwave superconductivity.

**Acknowledgement** C.-J. Wu wishes to acknowledge the financial support from the National Science Council of the Republic of China (Taiwan) under grant No. NSC-97-2112-M-003-013-MY3.

## References

- Golosovsky, M., Tsindlekht, M., Davidov, D.: *Supercond. Sci. Tech.* **9**, 1 (1996)
- Porch, A., Lancaster, M.: *J. Supercond. Nov. Magn.* **20**, 1 (2007)
- Moser, E.K., Tomasch, W.J., McClorey, M.J., Furdyna, J.K., Coffey, M.W., Pettiette, C.L., Schwarzbeke, S.M.: *Phys. Rev. B* **49**, 4199 (1994)
- Wu, C.J.: *Physica C* **305**, 293 (1998)
- Coffey, M.W., Clem, J.R.: *Phys. Rev. B* **48**, 342 (1993)
- Coffey, M.W., Clem, J.R.: *Phys. Rev. Lett.* **67**, 386 (1991)
- Yablonovitch, E.: *Phys. Rev. Lett.* **58**, 2059 (1987)
- John, S.: *Phys. Rev. Lett.* **58**, 2486–2489 (1987)
- Joannopoulos, J.D., Meade, R.D., Winn, J.N.: *Photonic Crystals: Molding the Flow of Light*. Princeton University Press, Princeton (1995)
- Yeh, P.: *Optical Waves in Layered Media*. Wiley, Singapore (1991)
- Camley, R.E., Mills, D.L.: *J. Appl. Phys.* **82**, 3058 (1997)
- Tagantsev, A.K., Taito, K.B.: *Phys. Rev.* **47**, 2760 (1993)
- Tagantsev, A.K., Taito, K.B.: *Phys. Rev.* **48**, 10537 (1993)
- Wu, C.J.: *J. Appl. Phys.* **96**, 3348 (2004)
- Pimenov, A., Loidl, A., Przyslupski, P., Dabrowski, B.: *Phys. Rev. Lett.* **95**, 247009 (2005)
- Bermann, O.L., Lozovik, Y.E., Eiderman, S.L., Coalson, R.D.: *Phys. Rev. B* **74**, 092505 (2006)
- Poddubny, A.N., Ivchenko, E.L., Lozovik, Y.E.: *Solid State Commun.* **146**, 143 (2008)
- Takeda, H., Yoshino, K.: *Phys. Rev. B* **67**, 245109 (2005)

19. Wu, C.-J., Liu, C.-L., Yang, T.-J.: *J. Opt. Soc. Am. B* **26**, 2089 (2009)
20. Wu, C.-J., Wang, Z.-H., Yang, T.-J.: *J. Supercond. Nov. Magn.* Online First (2010). doi:[10.1007/s10948-010-0790-0](https://doi.org/10.1007/s10948-010-0790-0)
21. Wu, C.-J., Chen, M.-S., Yang, T.-J.: *Physica C* **432**, 133 (2005)
22. Chen, M.-S., Wu, C.-J., Yang, T.-J.: *Phys. Lett. A* **373**, 3594 (2009)
23. Wu, C.-J., Chen, Y.L., Yang, T.-J.: *J. Supercond. Nov. Magn.* **23**, 545 (2010)
24. Born, M., Wolf, E.: *Principles of Optics*. Cambridge, London (1999)
25. Coffey, M.W.: *Phys. Rev. B* **61**, 15361 (2000)