J Supercond Nov Magn (2009) 22: 471–477
DOI 10.1007/s10948-009-0442-4

ORIGINAL PAPER

Magnetic Interaction Force and a Couple on a Superconducting Sphere in an Arbitrary Dipole Field

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Received: 31 December 2008 / Accepted: 5 January 2009 / Published online: 24 January 2009

Abstract The calculation of the magnetostatic potential and levitation force due to a point magnetic dipole placed in front of a superconducting sphere in the Meissner state is readdressed. Closed-form analytical expression for the scalar potential function that yields the image system for an arbitrarily oriented magnetic dipole located in the vicinity of a superconducting sphere is given. Analytic expression for the lifting or levitation force acting on the sphere is extracted from the solution for a general dipole. A special case of our expression where the initial magnetic dipole makes an angle with the z-axis is derived. Our expression for the force in this particular case shows that a recently obtained result (J. Supercond. Nov. Magn. 21:93–96, 2008) for an arbitrary dipole is incorrect. A brief discussion of another erroneous result (J. Supercond. Nov. Magn. 15:257–262, 2002) for a transverse/tangential dipole–sphere configuration, corrected elsewhere recently, is reproduced. Correct expressions for the interaction energy with some limiting cases are also provided. The result derived here demonstrates that the value of the levitation force for a dipole that makes an angle with z-axis lies between the values for a radial dipole–sphere and transverse dipole–sphere configurations providing upper and lower bounds. It is found that for a magnetic dipole making an angle with z-axis, there exits a second force component along the negative y-direction, which influences a couple acting on the superconducting sphere. It is also shown that the couple is proportional to the second force component and that both the couple and second force components vanish for a radial dipole–sphere and transverse dipole–sphere configurations, respectively. These results appear to be new and have not had received due attention in the context of superconductivity.

Keywords Dipole · Images · Superconducting sphere · Levitation force · Couple

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1 Introduction

The magnetic interaction between a point dipole and a superconducting sphere in the Meissner state has been of great interest in recent studies. Several analytic results for the magnetic interaction energy and levitation force have been presented for various orientations of a magnetic point dipole located in the vicinity of a superconducting sphere. For a radial dipole–sphere configuration, Coffey [1] found closed-form expressions for the interaction energy and levitation force by the use of the so-called image theory. For a horizontally oriented dipole (transverse dipole) located in front of a superconducting sphere, Coffey [2, 3] derived analytic solutions by utilizing the standard techniques involving spherical harmonics. Based on his findings, Coffey concluded that the levitation force for a transverse dipole–sphere system is one half that for a radial magnetic dipole–sphere system in the Meissner state. However, Coffey's results for a transverse dipole–sphere configuration and his conclusion have been shown to be incorrect recently by Lin [4] and Palaniappan [5] independently. It was understood that the incorrect use of boundary conditions by Coffey led to erroneous expressions and conclusions.

More recently, Al-Khateeb et al. [6] (referred as AAAA here) claimed to have solved the problem of a magnetic dipole–sphere model for arbitrary orientation of the dipole using the image principle. In particular, AAAA arguably utilized the image dipole theory, as in [3] for a radial dipole, and arrived at the expressions for interaction energy and levitation force. Unfortunately, their results are also incorrect as shown in this article. Apparently, AAAA seem to be unaware of the work due to Coffey [2, 3] and the recent corrected versions of Coffey's results [4, 5]. Since these authors (AAAA) did not provide expressions for the scalar potential or magnetic induction, the error in their solution could not be traced. But the comparison of their results for the levitation force and magnetic energy with those due to Coffey shows that the use of inappropriate boundary conditions at the superconducting surface might have led to their erroneous results which are corrected in the present paper.

The calculation of the image solution for sources and dipoles in the presence of a superconducting sphere has been investigated by many authors in various contexts. It appears that the solution for a source–sphere problem was given a long time ago by Carl Neumann [7] in an appendix of his book in 1883. The solutions for sources and dipoles in the presence of a fixed sphere were rediscovered by Weiss [8] in both magnetostatic and non-viscous/inviscid fluid environments. Note that the Neumann problem for a superconducting surface is mathematically equivalent to the problem of ideal fluid flow around a bounding surface in hydrodynamics [5, 9]. The solutions for the magnetostatic boundary value problems with sources, dipoles, current loops, etc., located inside/outside a superconducting sphere (and for planar surfaces) are also discussed in [10–16], among others. The fact that there is a nice correspondence between magnetostatics and hydrodynamics makes it easier to translate results from the former to the latter, and vice versa. Therefore, our present results, derived in the context of magnetostatics, are also applicable in the inviscid fluid flow theory.

We should emphasize that although the exact analysis for a dipole–sphere system has appeared in the late nineteenth century, the results have not been recognized in some recent studies. Due recognition those results could have avoided the misrepresentation of the theoretical results presented in [2, 3, 6], for instance. Since the dipole–sphere problem has an immense value in practice in a variety of fields related to magnetostatics, especially in superconductivity, the accurate presentation of the analytical results is absolutely necessary. In view of this, the results for a dipole–sphere problem in the Meissner state is readdressed again in this paper. The exact image solution for a general dipole located in the vicinity of a superconducting sphere is presented in Sect. 2 with a brief demonstration of the image system. The analytical results for the levitation force and interaction energy are provided in Sect. 3. Our results correct the earlier erroneous results for the force and energy and also yield appropriate limiting cases for a semi-infinite, flat superconductor. Some numerical results for the levitation force for a particular type of magnets are provided. Additional new results for the second force component and the couple are also presented in this section (see Sect. 3.3). Finally, in the concluding Sect. 4, our main findings in this paper are given.

2 Image Solution for a Dipole–Sphere Problem

The problem of the field induced by a magnetic dipole placed in the vicinity of a superconducting surface in the Meissner state can be solved via magnetostatic scalar potential approach [1–5, 8, 14]. This requires the construction of solution to the magnetostatic potential due to a dipole located in front of a superconducting object. For a superconducting spherical surface of radius b, one seeks a field solution for the magnetostatic potential $\Phi(\mathbf{x})$ such that the magnetic field is given by $\mathbf{H} = -\nabla\Phi$ and the induction by $\mathbf{B} = \mu_0\mathbf{H}$. The mathematical problem in this case reduces to a Neumann boundary value problem for the magnetic scalar potential as follows:

$$\nabla^2\Phi = 0, \tag{1}$$

$$\frac{\partial\Phi}{\partial r} = 0 \quad \text{on } r = b. \tag{2}$$

Notice that condition (2) implies vanishing of the normal component of the magnetic field at the surface of the sphere $r = b$. Coffey [2, 3] used the condition that the magnetic field vector vanishes (that is, both tangential and normal components of the magnetic field vanish) at the spherical surface $r = b$ in the derivation of his solutions for point magnetic dipole–sphere interaction problems. However, Maxwell's equations do not restrict the tangential components of the magnetic field (see [17], for instance). Therefore, the boundary condition (2) is more suitable for a superconducting sphere. A recent article by Lin [4] gives an elaborate explanation of the use of the boundary condition (2) for a superconducting sphere. It appears that AAAA [6] have also assumed a similar condition as Coffey, although this point could not be checked since they did not give a complete solution.

The solution to any externally induced magnetic potential in the presence of a superconducting sphere can be obtained using the standard spherical harmonics series expansion method [1–4, 14] or using the general result given in [5, 8]. While the spherical harmonics scheme requires expansion of each initial potential in terms of an infinite series and then solves the required problem as explained in [14], the general result given in [5, 8] provides a unified approach for an arbitrary magnetostatic potential in the presence of a superconducting sphere. The exact solution for a general dipole–sphere problem obtained via the latter approach in [5] is reproduced below for the sake of completeness. The two locations of the dipole considered in the present paper are illustrated in Fig. 1(i) and (ii). For a general magnetic dipole of moment $\mathbf{M} = \mathbf{M}_\parallel + \mathbf{M}_\perp$ positioned at $\mathbf{a}$ outside the

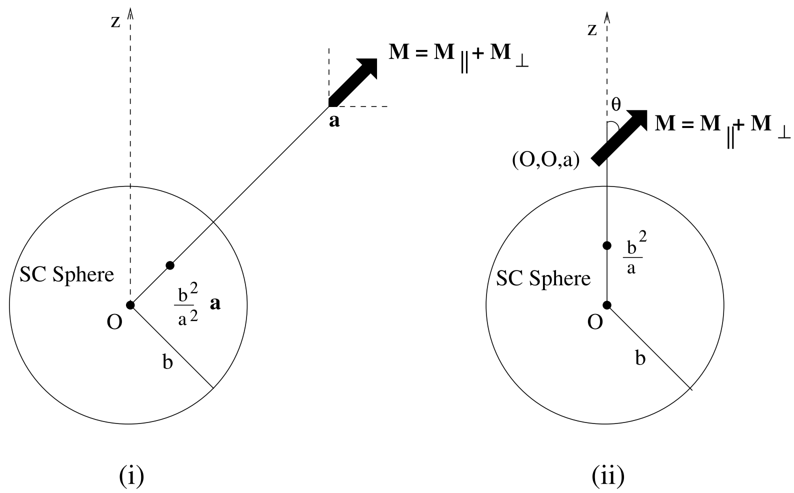

Fig. 1 Schematic illustration of the superconducting sphere–dipole configuration: (**i**) arbitrary orientation and location of the dipole; (**ii**) dipole located at $(0, 0, a)$ that makes an angle θ with the z-axis

sphere, as shown in Fig. 1(i), the solution for the magnetostatic potential is

$$\Phi = \frac{\mathbf{M}\cdot(\mathbf{r}-\mathbf{a})}{|\mathbf{r}-\mathbf{a}|^3} + \left(\frac{b}{a}\right)^3\left[\frac{\mathbf{M}\cdot(\mathbf{r}-\frac{r^2}{b^2}\mathbf{a})}{|\mathbf{r}-\frac{b^2}{a^2}\mathbf{a}|^3} - \int_0^1 \frac{\mathbf{M}\cdot(s\mathbf{r}-\frac{r^2}{b^2}\mathbf{a})}{|\mathbf{r}-\frac{sb^2}{a^2}\mathbf{a}|^3}\,ds\right], \tag{3}$$

where $\mathbf{r}$ is the position vector with $|\mathbf{r}| = r$. The integral in the above expression can be evaluated and the resulting magnetostatic potential now reads

$$\Phi = \frac{(\mathbf{M}_\perp + \mathbf{M}_\parallel)\cdot(\mathbf{r}-\mathbf{a})}{|\mathbf{r}-\mathbf{a}|^3} + \left(\frac{b}{a}\right)^3\left[\frac{(\mathbf{M}_\perp - \mathbf{M}_\parallel)\cdot(\mathbf{r}-\frac{b^2}{a^2}\mathbf{a})}{|\mathbf{r}-\frac{b^2}{a^2}\mathbf{a}|^3}\right] - \frac{\mathbf{M}_\perp\cdot\mathbf{r}}{ba\left(r^2-(\frac{\mathbf{a}\cdot\mathbf{r}}{a})^2\right)}\left[r - \frac{r^2-\frac{b^2}{a^2}(\mathbf{a}\cdot\mathbf{r})}{|\mathbf{r}-\frac{b^2}{a^2}\mathbf{a}|}\right]. \tag{4}$$

The expression for the complete potential given in (4) represents the general solution for an arbitrarily oriented magnetic point dipole in the presence of a superconducting sphere. The first term on the right-hand side of (4) (and (3)) represents the initial magnetic dipole located outside the sphere. The remaining terms in the closed-form expression (4) (and (3)) may be interpreted as images inside the sphere. The image system can be best understood by analyzing the radial (the vertical case in [1]) and tangential/transverse (the horizontal case in [2–5, 14]) initial dipole orientations separately.

- For the initial radial dipole ($\mathbf{M}_\parallel$) the image system consists of a single image dipole at the Kelvin's image point $\frac{b^2}{a^2}\mathbf{a}$ (see Fig. 1). The orientation of the image dipole in this case is just opposite to the initial dipole.
- For the tangentially oriented initial dipole ($\mathbf{M}_\perp$) the image system consists of an image dipole (the same orientation as the initial dipole) at the Kelvin's inverse point, plus a distribution of magnetic dipoles (with opposite orientation) from the origin to the Kelvin's image point [5, 8, 12, 13].
- It follows that for a general magnetic point dipole outside a sphere, the image system in a superconducting sphere comprises a point dipole at the Kelvin's inverse/image point, plus a distribution of dipoles from the center to the Kelvin's image point.

Thus, there is a fundamental difference in the image systems for a radial and tangential dipole–sphere configurations and also in the general case. This difference is not recognized in the incorrect representation of the image solution discussed by AAAA [6]. The variation in the image system leads to the variation in the interaction forces and energy, as will be seen in the next section. It should be pointed out that our image representation for point dipole–sphere configurations agrees with those already given in [11–14]. As said before, the image system for the radial dipole–sphere system has also been given in [3], which is the same as ours. In the following we give expressions for the force, interaction energy and torque and discuss the differences.

3 Discussion

The exact image solution for the dipole–superconducting sphere problem presented in the preceding section can be exploited to extract the significant physical quantities such as the interaction/levitation/lifting force and the interaction energy. The general expression for the force is already given in [5] for the arbitrary orientation of the initial dipole and we simply reproduce it here for the sake of comparison and to add new results. For an initial dipole located on the z-axis, it turns out that if the dipole is aligned parallel (radial dipole) or perpendicular (transverse dipole) to the z-axis, then the force has just one component. On the other hand, if the initial dipole makes an angle θ with the z-axis, then there is a second force component in addition to the vertical/lifting force. This additional force component appears to influence a torque/couple on the sphere and it is observed that the torque is proportional to the second force component. We provide here expressions for this extra force and the couple which do not seem to have received sufficient attention in the literature.

3.1 The Interaction Force

Now the expression for the force exerted by a general initial dipole $\mathbf{M}$ on the superconducting sphere extracted from (4) is given by [5]

$$\mathbf{F} = 4\pi\mu_0 \frac{b^3}{(a^2-b^2)^4} \times \left[\left(6 - 4\frac{b^2}{a^2} + \frac{b^4}{a^4}\right)\left(M^2 + \frac{(\mathbf{M}\cdot\mathbf{a})^2}{a^2}\right)\mathbf{a} - \frac{1}{2}\left(3 - \frac{b^2}{a^2}\right)\left(1 - \frac{b^2}{a^2}\right)((\mathbf{M}\cdot\mathbf{a})\mathbf{M} + 3M^2\mathbf{a})\right]. \quad (5)$$

The force for several special dipole orientations, including radial and transverse/tangential directions of the initial dipole (see [5]) for details), can be deduced from the general expression (5). Now for a magnetic dipole with moment $\mathbf{M} = \mathbf{M}_{\parallel} + \mathbf{M}_{\perp}$ located at $(0, 0, a)$, $(a > b)$ which makes an angle θ with z axis, as in Fig. 1(ii), (5) takes the form

$$F_z = 4\pi\mu_0 \frac{ab^3}{(a^2-b^2)^4}\left[6M_{\parallel}^2 + \left(\frac{3}{2} + 2\frac{b^2}{a^2} - \frac{1}{2}\frac{b^4}{a^4}\right)M_{\perp}^2\right]. \quad (6)$$

Notice that this is another particular dipole orientation chosen to correct the erroneous results by AAAA [6], and to add new results as well. The difference in the lifting force for the radial and transverse dipole–sphere configurations appearing in (6) is due to the difference in the respective image systems. It is seen from (6) that the levitation force due to radial dipole–sphere configuration is *not twice* the force due to a tangential dipole–sphere configuration, as claimed in [2, 3, 6]. It follows from Fig. 1(ii) that for this orientation, $\mathbf{M}_{\parallel} = \langle 0, 0, M\cos\theta\rangle$ and $\mathbf{M}_{\perp} = \langle 0, M\sin\theta, 0\rangle$. Choosing $M = \frac{m}{4\pi}$, the expression for the levitation force (6) for the present dipole–sphere configuration reduces to

$$F_z = \frac{\mu_0 m^2}{4\pi}\frac{ab^3}{(a^2-b^2)^4}\left[\left(\frac{3}{2} + 2\frac{b^2}{a^2} - \frac{1}{2}\frac{b^4}{a^4}\right) + \left(\frac{9}{2} - 2\frac{b^2}{a^2} + \frac{1}{2}\frac{b^4}{a^4}\right)\cos^2\theta\right]. \quad (7)$$

The notation for the dipole moment m in the present paper and μ used in [6] are related by $\mu^2 = \frac{\mu_0 m^2}{8\pi}$. We first observe that the functional form in (7) is much different from that derived by AAAA [6] and is due to the incorrect image system used by these authors. When $\theta = 0$, our result given above yields the levitation force for a radial dipole which agrees with [1, 2, 4, 5, 14] and also with that given by AAAA [6]. However, when $\theta = \frac{\pi}{2}$, it gives the expression for a transverse dipole in agreement with [4, 5, 14], but does not agree with the corresponding result in [6]. A similar incorrect result for a transverse/horizontal dipole–sphere system is given in [2, 3] which was recently corrected in [4, 5]. Also, for intermediate values of θ, our result disagrees with that given in [6].

In the limit $a^2 \gg b^2$, (7) yields

$$F_z = \frac{3\mu_0 m^2}{8\pi}\frac{b^3}{a^7}(1 + 3\cos^2\theta). \quad (8)$$

When $\theta = 0$, the force expression (8) gives the result for a radial dipole in agreement with those given in [1, 5]. When $\theta = \frac{\pi}{2}$, it reduces to the correct result for a transverse dipole given in [5]. This limiting case was not discussed by AAAA in [6], but reduction of their result (equation (4) in their paper) in this special case leads to an incorrect expression for the force which does yield the correct limiting case results.

For a close dipole–sphere separation, $a - b = d \ll b$ (7) reduces to

$$F_z = \frac{3\mu_0 m^2}{64\pi d^4}(1 + \cos^2\theta). \quad (9)$$

This is the result for a dipole–superconducting plane (semi-infinite superconductor) system. For the special cases, $\theta = 0$ and $\theta = \frac{\pi}{2}$, (7) reduces to the results presented in [5] for a semi-infinite superconductor. It is seen that in this special case the result is given in [6] and agrees with ours. It is worthwhile to point out that in this limiting case the force for a radial dipole is *twice* the force for a transverse dipole, indicating that this result is true only for a dipole–superconducting plane system.

We now turn our attention to the numerical results based on our expression for the force given in (7). To this end, we use the model description utilized by Yang [18] several years ago. Thus, if we model the dipole as a sphere with radius r_0

Table 1 Numerical values of the levitation force for a typical Nd-Fe-B magnet

θ	F_z
0°	4.096×10^{-12} N
30°	$3.462847737 \times 10^{-12}$ N
45°	$2.829695473 \times 10^{-12}$ N
60°	$2.19654321 \times 10^{-12}$ N
90°	$1.563390947 \times 10^{-12}$ N

and magnetization M, then (7) can be rewritten as

$$F_z = \frac{2\times 10^7}{3}(\mu_0 M)^2 r_0^2\left(\frac{r_0}{b}\right)^2\frac{x}{(x^2-1)^4}\times\left[\left(\frac{1}{4}+\frac{1}{3x^2}-\frac{1}{12x^4}\right)+\left(\frac{3}{4}-\frac{1}{3x^2}+\frac{1}{12x^4}\right)\cos^2\theta\right], \tag{10}$$

where $x = a/b > 1$ is the relative distance. For Nd-Fe-B magnets, the typical values, according to Yang [18], may be chosen as $\mu_0 M = 1$ T, $r_0 = 100$ nm, $r_0/b = 0.1$ and $x = 1.5$. For these input values, the computed numerical results for the force are provided in Table 1 in order to illustrate the angular (that is, θ) dependence of the levitation force. The table gives the magnitude of the lifting force for $\theta = 0°, 30°, 45°, 60°$ and $90°$. When $\theta = 0°$ the numerical value for the force is maximal and is the same as the one given in [18] for a radial dipole, as expected. We also see that the force decreases with increasing θ in the first quadrant. The force is minimal when $\theta = 90°$, and this corresponds to a transverse dipole–superconducting sphere configuration. We also observe that the force in the case of transverse dipole orientation is *not one half* the value of the force with radial dipole orientation, contradicting the claims in [2, 3, 6] for these type of magnets. Indeed, the computed values in Table 1 show that $F_{z_\parallel} = 2.62 \times F_{z_\perp}$ (approximately) for a typical Nd-Fe-B magnets, where $F_{z_\parallel}$ and $F_{z_\perp}$ are the force components for the radial and transverse dipole–sphere systems, respectively. As mentioned in [5] (a nicer inequality is presented there), the force due to radial and transverse dipole cases serves as the upper and lower bounds for the levitation force in the general case.

3.2 The Interaction Energy

For the dipole–sphere configuration shown in Fig. 1(ii) and discussed in the previous subsection, the expression for interaction energy can also be extracted from the analytical solution given in (4). The energy can be calculated either using the approach given in [1–3] or using the integral formula utilized in [14]. Omitting the details, the expression for the interaction energy is given by

$$U = \frac{4\pi\mu_0 b^3}{(a^2-b^2)^3}\left[M_\parallel^2+\left(\frac{1}{4}+\frac{b^2}{4a^2}\right)M_\perp^2\right] = \frac{\mu_0 m^2 b^3}{4\pi(a^2-b^2)^3}\left[\frac{1}{4}+\frac{b^2}{4a^2}+\left(\frac{3}{4}-\frac{b^2}{4a^2}\right)\cos^2\theta\right]. \tag{11}$$

Our expression (11) for the energy shows that the energy in the radial dipole case is *not twice* the energy in the transverse dipole case, again contradicting the claims in [6]. The above result for the interaction energy disagrees with that given by AAAA [6], but agrees with the one obtained by Beloozerov and Levin [11] many years ago. In the limiting case, when $a^2 \gg b^2$, (11) reduces to

$$U = \frac{\mu_0 m^2 b^3}{16\pi a^6}(1+3\cos^2\theta). \tag{12}$$

The corresponding deduction of the result by AAAA [6] (equation (3) in their paper) yields an incorrect expression for the energy in this limiting case. For a close dipole–sphere separation $a - b = d \ll b$, (11) yields

$$U = \frac{\mu_0 m^2}{64\pi d^3}(1+\cos^2\theta) \tag{13}$$

which is the interaction energy for a dipole–superconducting plane configuration. However, in this case, the reduction of AAAA expression for the interaction energy gives a result that coincides with our expression (13). As in the lifting force case, the energy for the radial dipole–plane configuration is *twice* the energy for the transverse dipole–plane configuration. This correct result for a semi-infinite, flat superconductor is also given in [19].

3.3 Expressions for the Second Force Component and the Couple

Now for a dipole with moment $\mathbf{M} = \mathbf{M}_\parallel + \mathbf{M}_\perp = \langle 0, \frac{m}{4\pi}\sin\theta, \frac{m}{4\pi}\cos\theta\rangle$ (discussed in the preceding subsections), in addition to the vertical interaction force given in (6), there is a horizontal (tangential) force component as evident from the general result for the force given in (5). The expression for this second force component, extracted from (5), is given by

$$F_y = -2\pi\mu_0\frac{b^3}{a(a^2-b^2)^3}\left(3-\frac{b^2}{a^2}\right)M_\parallel M_\perp = -\frac{\mu_0 m^2}{8\pi}\frac{(3a^2-b^2)b^3}{a^3(a^2-b^2)^3}\sin\theta\cos\theta. \tag{14}$$

As seen from the above expression, the additional force component acting in the negative y-direction vanishes for a radial ($\theta = 0$) and transverse ($\theta = \frac{\pi}{2}$) dipole orientations. Note that F_y depends significantly on the initial dipole location a and it decreases with an increase of a. Below, we discuss the relation between this additional force component and the torque/couple on the superconducting sphere.

For a general dipole with an arbitrary direction there is a couple/torque that acts on the sphere [8]. The couple $\mathbf{N} = \langle N_x, N_y, N_z \rangle$ acting on the superconducting sphere of radius b due to a magnetic dipole at $\mathbf{a}$ is given by

$$\mathbf{N} = -2\pi\mu_0 \frac{b^3}{a^2(a^2-b^2)^3}\left(3 - \frac{b^2}{a^2}\right)(\mathbf{M}\cdot\mathbf{a})(\mathbf{M}\times\mathbf{a}). \qquad (15)$$

For a special dipole located at $(0, 0, a)$ making an angle θ with z-axis (considered above and in the preceding subsections), an expression for the couple becomes

$$N_x = -2\pi\mu_0 \frac{b^3}{(a^2-b^2)^3}\left(3 - \frac{b^2}{a^2}\right) M_\parallel M_\perp$$

$$= -\frac{\mu_0 m^2}{8\pi}\frac{(3a^2-b^2)b^3}{a^2(a^2-b^2)^3}\sin\theta\cos\theta. \qquad (16)$$

Note that for this special dipole–sphere configuration, the couple acts along the negative x-direction with the component N_x given above. Comparison of (14) and (16) shows that the couple is proportional to the second component of the force and indeed one sees that $N_x = a \times F_y$. As in the force discussion, for a radial ($\theta = 0$) and transverse ($\theta = \frac{\pi}{2}$) dipole orientations, the couple acting on the sphere is zero. Other special dipole–sphere configuration results can also be deduced from the expression for the couple given in (15) and (16). For instance, when $a^2 \gg b^2$, the couple in the x-direction becomes

$$N_x = -\frac{3\mu_0 m^2}{8\pi}\frac{b^3}{a^6}\sin\theta\cos\theta, \qquad (17)$$

and for a close dipole–sphere separation, $a - b = d \ll b$, (16) reduces to

$$N_x = -\frac{\mu_0 m^2}{32\pi d^3}\sin\theta\cos\theta. \qquad (18)$$

The last equation yields the couple on a superconducting plane due to a dipole. The new results presented in this subsection have not been given due attention in the literature. We must point out that these theoretical results are yet to be recognized in practice as well.

4 Conclusion

Closed-form image solution for a magnetic point dipole located in the vicinity of a superconducting sphere in the Meissner state is given for an arbitrary orientation of the dipole. The levitation force acting on the sphere and the interaction energy are extracted from the analytical solution for a dipole–superconducting sphere configuration. Our exact expressions for the force and interaction energy:

- correct the recent erroneous results presented in [6] for an arbitrary dipole–sphere configuration;
- show that neither the levitation force nor the interaction energy for a radial dipole–sphere configuration is twice the corresponding force and energy for a transverse dipole–sphere configuration. Such a result is true only for a semi-infinite, flat superconductor;
- are utilized to compute numerical values for the interaction force for Nd-Fe-B type magnets. Our computed values yield the relation $F_{z\parallel} = 2.62 \times F_{z\perp}$ (approximately) between the radial and transverse orientations of the initial dipole, again disproving the conclusions for the force given in [2, 3, 6];
- illustrate the presence of a second force component in the direction tangent to the superconducting sphere;
- reveal the existence of torque on the sphere due to a magnetic dipole and its connection to the second force component.

The expression for the interaction force given here also demonstrates that the levitation force is maximal for a radial dipole–sphere combination and is minimal for a transverse dipole–sphere configuration providing upper and lower bounds for various dipole orientations. A more explicit inequality for the interaction force is provided in [5]. Further, our results for the second force component and torque appear to be new and do not seem to have been discussed earlier in the literature. It is noted that the second force and the torque vanish for the radial and transverse dipole–sphere configurations.

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