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ORIGINAL PAPER

Metallic and Superconducting Photonic Crystal

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Abstract We have investigated the transmittance for two kinds of photonic crystals; (i) One-dimensional metallic photonic crystals (1DMPCs) and (ii) One-dimensional superconducting photonic crystals (1DScPCs). The variance of the intensity and the bandwidth of the transmittance are strongly dependent on the thicknesses and frequencies. We have compared the transmittance spectra in 1DScPCs and 1DMPCs at low frequencies, and we present some details about the transmittance spectra by using Transfer Matrix Method (TMM) at the same conditions.

Keywords Sc · PBG · Metallic · Transmittance · TMM · Photonic crystal

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1 Introduction

Studies of photonic crystals (PCs) have attracted the attention of many researchers in recent times. Generally, PCs can be divided into 1D, 2D, and 3D structures depending on their dielectric function, which can be periodic in one, two, or three dimensions, respectively. It is worthy to mention that the one-dimensional photonic crystals (1DPCs) are the most studied and used due to its cheap cost and its variety applications [1–10]. The propagation of electromagnetic waves in periodic media has been a subject of interest because of its several useful applications [1–13]. One-dimensional MPCs have been shown to have high transmittance within a certain controllable spectral range [5–7] and an enhanced nonlinear optical response in one dimensional MPC has also been reported [9]. Moreover, the research results suggest that MPCs can be used for various photonic and electronic applications. The band structure for a dielectric–dielectric photonic crystal (DDPCs) displays that the photonic band gap (PBG) between the first and second bands widens goodly as the difference in dielectric permittivity is increased [14–17]. Moreover, the electromagnetic waves cannot propagate in such a system at frequencies lower than the so-called cutoff frequency, ω_c [18]. If the lattice constant of these structures lies in the millimeter and submillimeter ranges, their fabrication is relatively simple. However, the damping of electromagnetic waves in metals can suppress many potentially useful properties of metallic PCs. The possible solution of this problem is the replacement of the PC metallic component by a superconducting (SC) component [19, 20]. The dielectric function depends on superconducting gap [19, 20] and SC state can be varied by alteration of external parameters, such as temperature and external magnetic field which provide a method for a control of the optical properties of metallic PCs. All dielectrics (except vacuum) have two types of losses. One is a conduction loss, representing the flow of actual charge through the dielectric. The other is a dielectric loss due to movement or rotation of the atoms or molecules in an alternating electric field. The dielectric losses are reduced essentially below superconducting transition in SC State, and recent experiments

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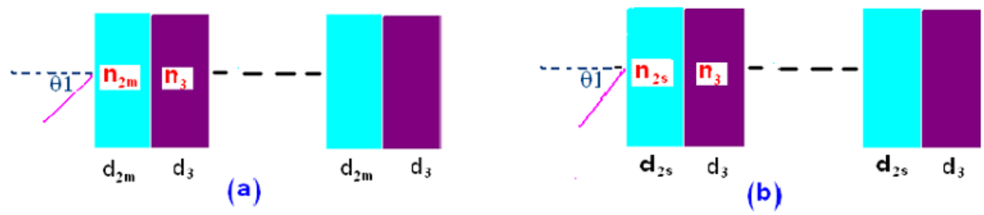

Fig. 1 (**a**) 1D-MPCs, where d_{2m} is the thickness and n_{2m} the index for normal metal layer and (**b**) 1D-ScPCs, where d_{2s} is the thickness and n_{2s} the index for superconductor layer

have shown that photonic band edges become sharper in SC metals [21].

Recently, there have been studies of photonic crystals consisting of a superconducting material and a dielectric [22, 24]. By changing the Ginzburg–Landau parameter and static magnetic field were investigated the electromagnetic properties of Abrkosov vortex lattice as a photonic crystal [22]. In addition to a low-frequency band gap below the first band, they [22, 24] also obtained the PBGs for a superconductor in the presence of vortices. A full band structure is a basic and important means for understanding the fundamental physics about electromagnetic wave propagation characteristics in a photonic crystal. This information is not only fundamental, but also of technical use for a superconducting material. Motivated by this, in our work, we shall follow the work of [23–25, 29]. We use the Abeles theory for a stratified media to calculate the frequency-dependent transmittance [26, 29]. Also, we are going to examine and compare the behaviors of the transmittance for transversal electrical (TE) mode at low frequencies in 1DScPCs and 1DMPCs structures based on the transfer matrix method.

2 Theoretical Treatment

We have investigated the transmittance for two kinds of photonic crystals: (i) 1DMPCs, composed of normal metals and dielectric layers. (ii) 1DScPCs, composed of superconductor and dielectric layers.

(i) 1D-MPCs: In photonic band structure calculations to 1DMPCS, the dielectric constant of the metallic layer simply takes the form of the Drude model, i.e.,

$$n_{2m}^2 = 1 - \frac{\omega_{\text{plasma}}^2}{\omega(\omega - i\gamma)}, \tag{1}$$

where ω is the frequency of the incidence radiation, ω_{plasma} and γ are frequency-independent parameters and the thickness of metallic layer is d_{2m}. The plasma frequency is defined by $\omega_{\text{plasma}}^2 = ne^2/\varepsilon_o m_{\text{eff}}$ where n is the electron density, m_{eff} is the effective mass of the electron, e is the electronic charge, and the permittivity of the vacuum is ε_o. For the metallic layer, the local wave vector should be imaginary for frequencies below ω_{plasma}. If the thickness of the metallic layer in the 1D MPC is close to or smaller than the relevant skin depth of the corresponding metal, some portion of the electromagnetic waves would traverse this metallic layer. With MPCs being periodic structures, multiple Bragg scatterings play an important role, contributing to the creation of photonic bands and PBGs in infinite PCs and to significant transmission for frequencies within photonic bands in finite PCs. These factors can give rise to an absorption enhancement for frequencies within photonic bands.

(ii) 1D-ScPCs: We use the two-fluid model to describe the electromagnetic response of a typical superconductor without an external magnetic field [26, 27]; this is because the superconductor is strongly sensitive to temperature and external magnetic fields [26, 27]. In the model, the electrons in the superconductor occupy one of two states, superconductor state, or normal state.

For the superconducting state, the temperature-dependent penetration depth is given by

$$\lambda_L, = \lambda_L(T) = \lambda_o/\sqrt{1 - f(T)}. \tag{2}$$

The Gorter–Casimir expression for $f(T)$ is given by

$$f(T) = (T/T_c)^4. \tag{3}$$

When the temperature is above 0.8 times the critical temperature, the London penetration depth increases rapidly and approaches infinity as the temperature is close to the critical temperature, T_c [26, 27]. Adjusting the temperature of superconductors can control the refractive indices of superconductors as well as the photonic band structures of PCs composed of superconductors. When $T \le T_c$ is satisfied, the dependence of the plasma frequency on the temperature is given as [26, 27]

$$\omega_p^s(T) = \omega_p^s(0)/\big(1 - \sqrt{f(T)}\big)^{0.5}. \tag{4}$$

We consider that a TE wave (see Fig. 1) is incident at an angle θ_1 from the free space with a refractive index, $n_1 = 1$. The index of refraction of the lossless dielectric is given by $n_3 = \sqrt{\varepsilon_3}$, where ε_3 is its relative permittivity and the thickness of dielectric layer is d_3. For the superconductor, the thickness layer is d_{2s} and the index of refraction, n_{2s};

can be described on the basis of the conventional two-fluid model [28, 29],

$$n_{2s} = \sqrt{\varepsilon_s} = \sqrt{1 - (c^2/\omega^2\lambda_L^2)}. \tag{5}$$

According to Abeles's theory [28, 29], we can calculate the transmittance and reflectance for a periodic multilayered structure.

The transmission coefficient can be determined and is given by [28–30]

$$t = 2p_1/(m'_{11} + m'_{12}p_\ell)p_1 + (m'_{21} + m'_{22}p_\ell). \tag{6}$$

Here, $p_i = \sqrt{\varepsilon_o/\mu_o}(n_i \cos\theta_i)$, $i = 1, \ell$ is the first and last medium. Both media here are to be free space.

The explicit expressions for matrix elements m'_{11}, m'_{12}, m'_{21}, m'_{22} [30]:

$$\begin{aligned} m'_{11} &= \left(\cos\beta_2\cos\beta_3 - \frac{p_3}{p_2}\sin\beta_2\sin\beta_3\right)U_{N-1}(\Psi) \\ &\quad - U_{N-2}(\Psi), \\ m'_{12} &= j\left(\frac{1}{p_3}\cos\beta_2\cos\beta_3 - \frac{1}{p_2}\sin\beta_2\cos\beta_3\right)U_{N-1}(\Psi), \\ m'_{21} &= j(p_2\sin\beta_2\cos\beta_3 - p_3\cos\beta_2\sin\beta_3)U_{N-1}(\Psi), \\ m'_{22} &= \left(\cos\beta_2\cos\beta_3 - \frac{p_2}{p_3}\sin\beta_2\sin\beta_3\right)U_{N-1}(\Psi) \\ &\quad - U_{N-2}(\Psi), \end{aligned} \tag{7}$$

where $\beta_i = (2\pi/\lambda_o)n_i d_i \cos\theta_i$, $p_i = \sqrt{\varepsilon_o/\mu_o}n_i\cos\theta_i$ $i = 2, 3$, it depends on the kind of layer and $\lambda_o = 2\pi/k_o = 2\pi c/\omega$ is the wavelength in free space. The angles θ_1 and θ_2, determined by Snell's law of refraction, are the ray angles in layers 2 and 3, respectively. The total characteristic matrix for an N-period structure can be obtained, that is, $\Psi = (m_{11} + m_{22})/2$ where $m_{11} = \cos\beta_2\cos\beta_3 - \frac{p_3}{p_2}\sin\beta_2\sin\beta_3$ and $m_{22} = \cos\beta_2\cos\beta_3 - \frac{p_2}{p_3}\sin\beta_2\cos\beta_3$. Where U_N are the Chebyshev polynomials of the second kind defined by: $U_N(\Psi) = \sin[(N+1)\cos^{-1}\Psi]/\sqrt{1-\Psi^2}$. The transmittance T, and is related by

$$T = \frac{p_\ell}{p_1}|t|^2. \tag{8}$$

3 Results and Discussions

By using Abeles's theory and transfer matrix method, we examined the transmittance spectra for two kinds of photonic crystals: 1DMPCs (see Figs. 2a, 3a, and 4a) and 1DScPCS (see Figs. 2b, 3b, and 4b). We have calculated the index of the metallic layer in 1DMPcs by using the Drude model, but we used two fluid models for the superconducting case because the superconductor is strong sensitive to the temperature and magnetic field. Also, for the case of 1ScPCs, the frequencies must be lie inside the superconducting gaps [19].

At low frequencies (0.2∼0.5 THz), we calculated the transmittance and found that the behavior of 1DMPCs is different from that of 1DScPCs.

In the 1DMPCs (Fig. 2a), the magnitude of the transmittance is very low (around 0.0002), which means the reflectance is high in 1DMPCs at low frequencies. The magnitude of transmittance peak becomes quite high at 650 periods, as is clearly found in Fig. 2a at 10 periods. In Figs. 2a

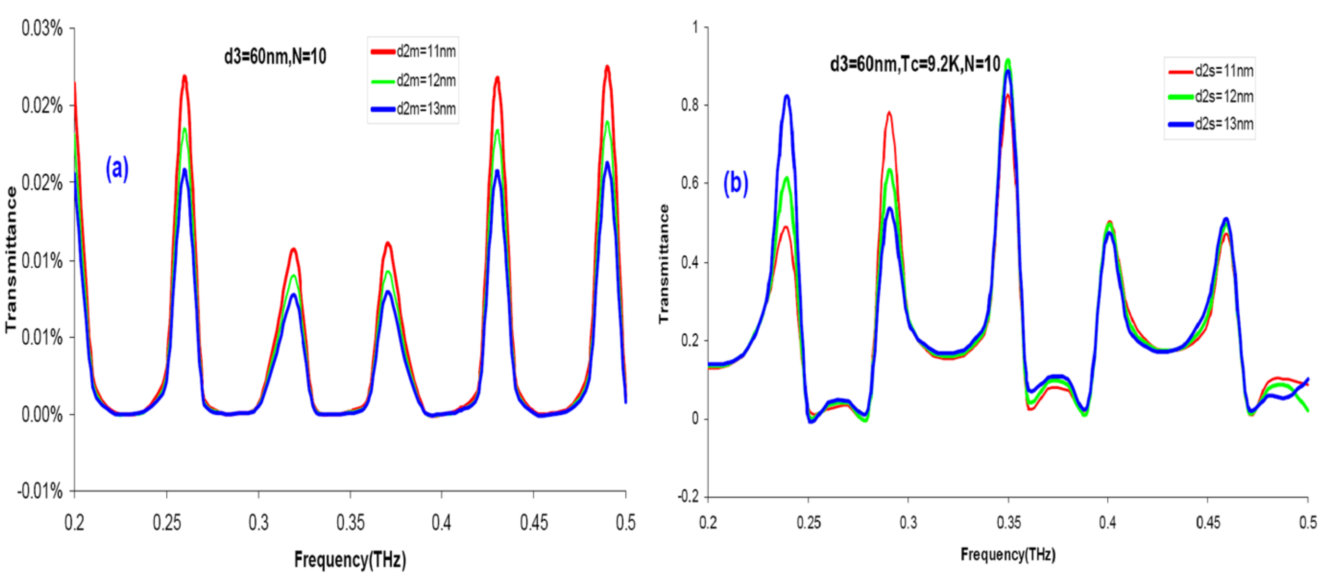

Fig. 2 Transmittance spectra dependence on the frequencies at different thicknesses, with $\varepsilon_3 = 15$; (**a**) 1D-MPCs, and (**b**) 1D-ScPCs

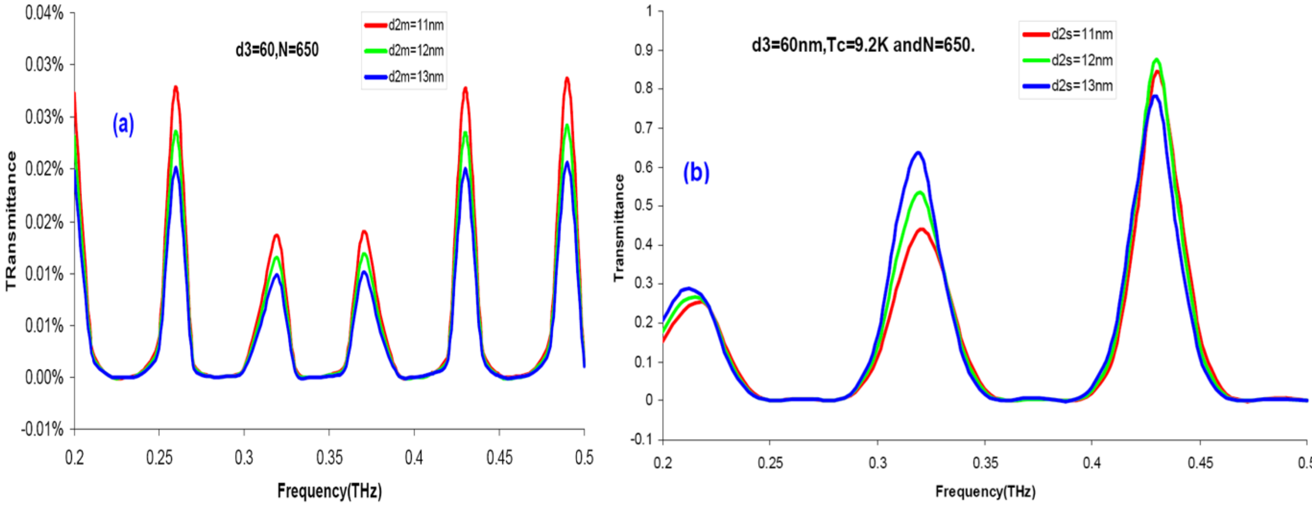

Fig. 3 Transmittance spectra dependence on the frequencies at different thicknesses, with $\varepsilon_3 = 15$; (**a**) 1D-MPCs, and (**b**) 1D-ScPCs

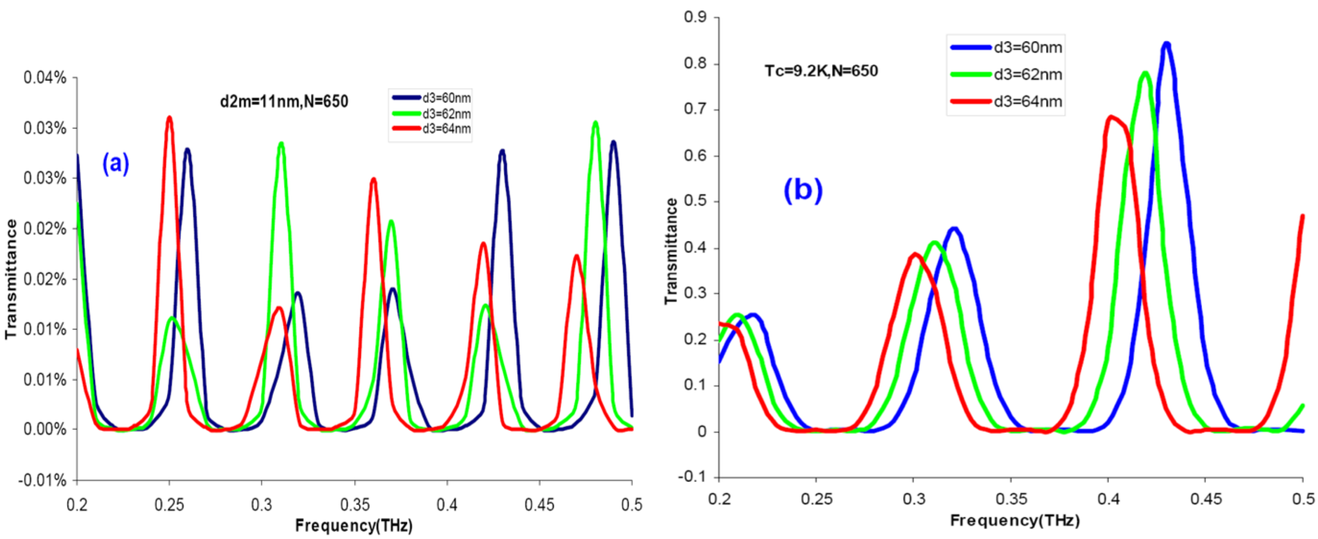

Fig. 4 Transmittance peaks are shifted due to the variance of dielectric thickness, d_3 with $\varepsilon_3 = 15$; (**a**) 1D-MPCs and (**b**) 1D-ScPCs

and 3a, the magnitude of the transmittance does not change a lot when we increase the periods from 10 to 650. That means the number of periods has not more significance to enhance the transmittance at low frequencies for the metallic photonic crystal behavior. In a metallic photonic made of a normal metal and a dielectric, it is, however, found that a low-frequency (or metallicity) gap may exist. Contrary to a PBG, this metallicity gap which does not depend on the periodicity, is of the order of the plasma frequency, and thus is regarded as a modified effective plasma frequency [15–17]. Also for 1DMPCs, we should notice that there is a kind of similarity around 0.3 THz to the transmittance behaviors at low frequencies. We have shown that the value of transmittance increases when the thickness of metallic layer decreases, thus we found the high value of transmittance at, $d_{2m} = 11$ nm, which indicates to the significance of the layer thickness.

For the 1DScPCs, at the low frequencies (0.2∼0.5 THz), there is a high transmittance which approaches to unity at 10 periods (Fig. 2b) and 650 periods (Fig. 3b). It could be one of ScPCs advantages that may not exist in other types of photonic crystals and is not valid in DDPCs or MPCs up on my knowledge. Therefore, we can say the ScPCs is the only one that presents the high transmittances at low frequencies. It is very important and can be used in the optical devices. Also, we can see the position of transmittance's peaks shifted by increasing the periods from 10 (Fig. 2b) to 650 (Fig. 3b).

We have examined the significance of the dielectric layer thickness within the ranges 60–62 nm at the both cases; 1DMPCs and 1DScPCs. The position of transmittance peaks can be shifted by changing the dielectric layer thickness, for 1DMPCs one (Fig. 4a) and 1DScPCs one (Fig. 4b) while we keep the same conditions; $d_{2m} = d_{2s} = 11$ nm and $\varepsilon_3 = 15$.

This result shows that dielectric thickness is an important parameter. Indeed, it is worthy to mention that the transmittance value at 1DScPCs depends on the thickness of dielectric layer as shown in (Fig. 4b), thus the value of transmittance is increasing when the thickness of dielectric layer decreases. The transmittance value at 60 nm (red line) is larger than the value at 62 nm (blue line); there is 0.1 unit difference between them. This is why the thickness of the layer is one of important parameters and should be highly concerned in designing photonic or any optical devices depends on photonic as well.

We have studied two kinds of photonic crystals, 1DMPCs and 1DScPCs, by using TMM and Abeles's theory. In 1DScPCs, the transmittance is high and near the unity at low frequencies. The high transmittance at low frequencies is one of ScPCs advantages that may not exist in other types of photonic crystals DDPCs or MPCs. In 1DMPS, the magnitude of transmittance does not change a lot when we increase the periods from 10 to 650. Moreover, the thickness of each layer plays a significant role in cases and should be paid high attention when we design photonic devices or any optical devices.

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