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INFLUENCE OF DETECTOR NOISE ON COMPRESSED SAMPLING SINGLE-PIXEL IMAGING

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Abstract

Single-pixel imaging allows to obtain images without the use of photosensors with spatial resolution. In this method, an image is calculated by measuring the image conformity to a given set of light patterns by a single-pixel detector. However, when implementing single-pixel imaging in practice, one has to deal with various imperfections, which lead to the difference between the experiment and the idealized theoretical model. In this work, we analyze the effect of detector noise on the ability to compute an image using a compressed sampling algorithm. By conducting computer simulations of single-pixel imaging, we investigate methods for suppressing the effects of detector noise and find optimum parameters of the measurement process. As a result, we demonstrate the ability to obtain images with a realistic model of the detector noise.

Keywords: compressed sampling, single-pixel imaging, computational imaging, noise, optimization.

1. Introduction

To obtain an image, standard imaging methods use light-sensitive matrices consisting of a large number of pixels. Single-pixel imaging provides an alternative to the standard imaging methods. The light-sensitive element in single-pixel imaging is just one pixel, that is, an element that does not have spatial resolution. In this method, the spatial structure of objects is obtained, using various intensity distributions (patterns) in the spatial modulation of light, illuminating the object, or light emitted by the object, and measuring its integral amount for a given set of patterns [1, 2]. How to obtain an image, using a similar method, was initially shown in the quantum case, where random patterns arose due to spontaneous parametric scattering of light [3]. To create an arbitrary set of patterns, one can use both quantum light control methods [4] and classical ones; for example, a digital micro-mirror spatial light modulator, similar to those used in consumer digital video projectors [5, 6].

An important advantage of single-pixel imaging is a much larger choice of single-pixel photosensitive elements; in this case, the choice of multi-pixel matrices significantly exceeds and provides new opportunities for obtaining images [7–10]. For example, it is possible to obtain images in single-photon regime in the near-infrared region at a wavelength of 1.5 μm [11], that opens new imaging possibilities for quantum information technologies. Also, a significant difference between the single-pixel imaging method and the standard multi-pixel imaging method is the use of computational resources and algorithms. Single-pixel imaging relies on a system of equations connecting the image, the patterns, and the measured signal from a single-pixel detector. One of the modern methods of image reconstruction is compressed sampling [12,13]. In this method, one calculates the most sparse solution of a system of equations corresponding to the process of sampling an image, using an arbitrary (either incomplete or over-complete) set of patterns.

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When implementing single-pixel imaging in practice, one has to deal with various imperfections, which lead to differences between the real experiment and the idealized theoretical model. In particular, the measured signal in reality is always noisy. Noise in the detector adds uncertainty to the solution of the system of equations and leads to deviation of the calculated image from the original one. Therefore, to implement single-pixel imaging in practice, it is necessary to study the effect of detector noise on the ability to reconstruct an image.

In this work, we investigate how the detector noise affects the quality of compressed sampling image reconstruction in single-pixel imaging. For this, we conduct computer simulations of the single-pixel imaging process, taking into account the detector noise. We consider various parameters for generating patterns, which are used to sample an image, and find conditions, under which the influence of noise can be significantly suppressed. In particular, we propose methods for image noise reduction by optimizing image sampling and reconstruction algorithms.

2. Concept of Single-Pixel Imaging

As a test image, we consider a 16×16 pixel 8 bit gray-scale "smile" icon; see Fig. 1. Although, we show results, using this test image as an example; the qualitative conclusions are also valid for other types of images, as we show below.

For a better understanding of the single-pixel imaging method, we illustrate the idea in Fig. 1. We consider object illumination by nonuniform light with a certain spatial structure (pattern), which we can control and change by our choice. An example of patterns is shown in Fig. 1 (the top row). We consider binary patterns, i.e., patterns consisting of 0 and 1, represented by black and white colors, respectively. When an object is illuminated by a pattern (image sampling), the pixels of the pattern and the pixels

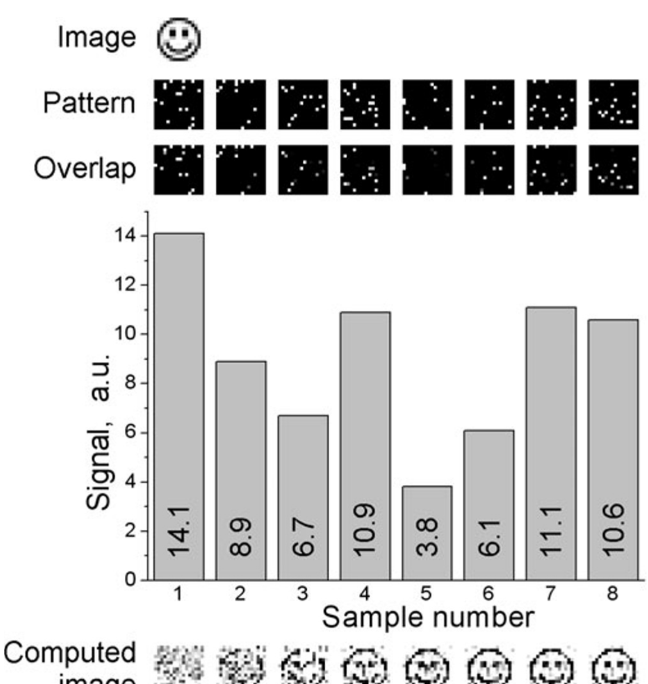

Fig. 1. Conception of single-pixel imaging. The original image ("smile") is overlapped with patterns, which have a random distribution of transparent and opaque parts (the first row). Depending on the correspondence of the pattern to the object, a certain part of the image is blocked by a given pattern (the second row). For each pattern, the total amount of light transmitted from the image of the object through this pattern is measured by a detector (signal from a single-pixel detector). Based on information about the set of patterns and the corresponding signal, the original image is calculated: the larger the data set, the closer the calculated image is to the original (the bottom row).

of the image are multiplied; see Fig. 1 (the middle row). Then, the single-pixel detector measures the total amount of light after pixel-by-pixel multiplication of the object and the pattern. The detector's signal for each pattern is proportional to the sum of the pattern pixels taken with the weight of the corresponding image pixel or, which gives the same thing, the sum of the image pixels taken with the weight of the corresponding pattern pixel; see the plot in Fig. 1. Thus, a value of 0 for a given pixel in a pattern means that the corresponding pixel in the image is not sampled, and 1 means, on the contrary, that it is sampled. After collecting data (measuring signals with a single-pixel detector), the main task is to calculate the image, which corresponds to the measured sequence of signals for a known sequence of patterns.

To assess the correspondence of the calculated image to the original (the accuracy of reconstruction), we use the metrics $L_1 = \langle |A - X| \rangle$ and $L_2 = \sqrt{\langle |A - X|^2 \rangle}$, that is, the average module of the pixel, the differences between the original image A and the calculated image X, and their standard deviation, respectively. Strictly speaking, the metric L_2 means the average squared deviation but, for ease of comparison, we take the square root of this number and further denote it by L_2.

3. Influence of Noise on Image Reconstruction

Under ideal conditions, in the absence of any noise and technical limitations, the sequence of detector signals S corresponding to a set of patterns P is expressed by the equation $PA = S$, where P is a matrix of $M \times N$, where each row of length N consists of a one-dimensional representation of one spatial pattern (one-dimensional row-by-row numbering of all pixels in the pattern), A is also a one-dimensional representation of the image of the object (column of N elements), and S is a set of signals from the detector (column of M elements).

However, in reality, the signal always has a noisy component. The physical source of noise in the signal can be, for example, noise from the detector, noise from the amplifier and other electronics, ambient light, radio interference on the electric wires, etc. Therefore, image calculation, in fact, is a problem of solving a system of equations with a noisy component δS in the signal S, namely,

$$PA = S + \delta S. \tag{1}$$

As a realistic model of noise, we consider additive Gaussian noise δS having the Gaussian probability density distribution with the variance proportional to the average signal value. Within the framework of this model, further we express noise as a percentage of the average signal. This model of noise is adequate for both dark and light images, since it is normalized to the average signal from the detector.

4. Image Reconstruction Method

The standard way to solve a system of linear equations (1) in matrix form can be represented as $X = P^{-1}S = A$. To perform it, one has to use sampling with a full set of patterns, that is, the number of patterns must be equal to the number of pixels in each pattern (or image). The noise in the signal from the detector leads to the noisy reconstructed image, $X = P^{-1}S + P^{-1}\delta S = A + P^{-1}\delta S$. This indicates that the noisy component, even in only one signal, can immediately affect the entire image, unless the matrix P is a unit matrix; in this case, noise in one signal affects only one corresponding pixel, and the problem of the complex influence of noise is greatly simplified [5, 6, 11].

Nevertheless, it is possible to obtain an exact solution with a smaller set of patterns, using the compressed sampling method. The essence of the method is to search for the most sparse solution X according to the norm L_1, which is ϵ-close to the original system of equations, according to the metric L_2; that is, satisfying the condition $||PX - S||_{L_2} < \epsilon$. By decreasing ϵ, one can obtain the exact solution $X = A$, under the condition that the solution A is sparse. To understand the efficiency of this method in practice, we use it for noisy signals $S + \delta S$.

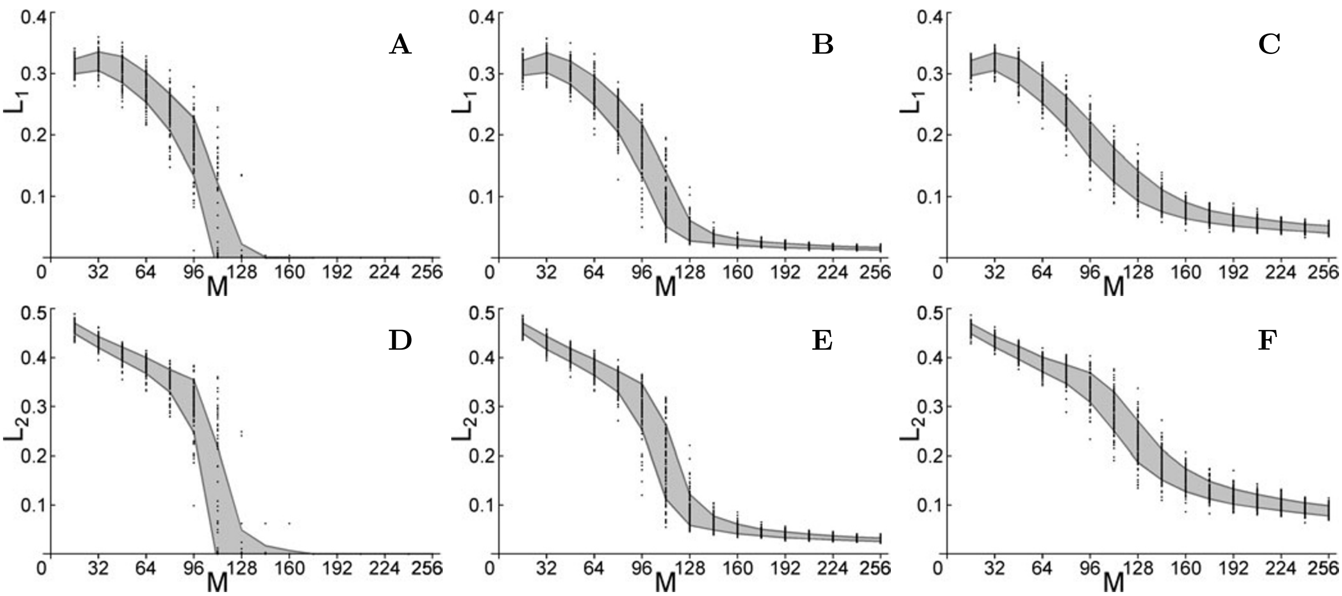

Fig. 2. The accuracy of image calculation expressed in its proximity to the original image, according to the metric L_1 (the top row: A, B, C) and L_2 (the bottom row: D, E, F), vs the number of image samples M. The signal noise level is 0% (the first column: A, D), 1% (the second column: B, E), and 3% (the third column: C, F) of the average signal value. Here, a value for one random set of patterns (dots) and statistical deviations (one standard deviation) for 100 random sets (error bars).

In Fig. 2, we show the dependence of the accuracy of image reconstruction for various noise level parameters. It is clear that the accuracy of reconstruction depends on the specific sampling of noise; thus, we collect statistics for different random sets of patterns, where we vary the percentage of units in each pattern. To understand the qualitative dependence, we consider 5% of filling each pattern with units, and the noise level equal to 0%, 1%, and 3%. The values of L_1 and L_2 for a specific realization of a set of patterns are shown by points, and one standard deviation of L_1 and L_2 from the corresponding average value is shown by error bars.

In the absence of noise; see Fig. 2 A, D, a specific feature of the compressed sampling method is shown, namely, the ability to accurately reconstruct an image with an incomplete set of patterns. As one can see, in some cases, the exact reconstruction can be realized with as few as 96 samples, that is, approximately one third of the full set. The sets of 112 patterns are divided into two groups: sets from the first group provide exact or almost exact image reconstruction, while the use of sets from the second group gives a significant inaccuracy of the reconstructed image. This effect leads to increase (while compared with the other number of patterns in the sample set) in the standard deviation of L_1 and L_2 from their average value around $96-112$ patterns. With a further increase in the set of patterns, the image is accurately reconstructed almost always. When the number of samples in the set is reduced to 96, the image is reconstructed with some deviations, and with a significantly incomplete set of pattern (smaller than 80); we see some plateau in both L_1 and L_2. This dependence of image reconstruction quality on the number of patterns used for sampling can be described as having a threshold. The plateau, in the case, where the metric L_1 is used, is more pronounced than, in the case, where the metric L_2 is used; however, a detailed study of the applicability of a particular metric for assessing the proximity of a reconstructed image to the original image is beyond the scope of this work.

The threshold-like feature of image reconstruction remains even in the presence of noise but, when the threshold number of samples is reached, the values of L_1 and L_2 do not fall strictly to zero, but to some non-zero value, and then gradually decrease; see Fig. 2 B, C, E, F.

When the percentage of units in each pattern increases above 5%, the qualitative behavior of obtained dependences remains approximately the same, but the image calculation time increases. As the percentage of units significantly decreases, below 5%, the quality of image reconstruction decreases, since some image pixels are rarely sampled and, accordingly, the influence of detector noise for them increases. Therefore, for practical reasons, it makes sense to form patterns with approximately 5% units.

For clarity, we show in Fig. 1 (the last row) specific examples of calculated images for the case of 5% noise and 5% units in each pattern, where the number of samples increases from 32 to 256 in increments of 32 (from left to the right).

As mentioned above, the results are obtained for a test image. which is a 16×16 pixel 8 bit gray-scale "smile" icon. The picture is pretty representative in several aspects. First, this is a gray-scale picture, i.e., it contains not only black or white pixels, but also intermediate shades around black lines, with the average pixel value around 0.74. This serves as an example of "something on a white background" which quite often occurs. Second, it is not very artificial in a sense that it lacks a clear pattern or extreme sparsity, in comparison to e.g. hand-written digits from the MNIST data set. Third, the results obtained for this image are not the best and not the worst compared to the results for other images that we tested. In Fig. 3, we show 10 other test images representing different types of pictures: mostly dark and mostly white, having more shades and less shades, sparse and not sparse (the initial "smile," "two cherries," "cup of tee," "duck," "ghost," "heart," "dancing man," "musical note," "smile with a tongue," and "yin and yang").

As one can see in Fig. 3, the qualitative dependence of the calculated image on the detector noise remains the same. For a fair comparison, we take a single random set of patterns without any optimization for a given image and run the same algorithm for all images, varying only the noise level (0%, 1%, and 3% of the maximum mean signal), similarly to the one shown in Fig. 2 for the "smile" icon. Comparing the visual pictures in Fig. 3 A, we can conclude that more sparse images allow for smaller number of samples to reconstruct (e.g., "musical note" is perfectly reconstructed with only a quarter of the full set of patterns), while images with an intense shades level ("heart" or "cup of tee") require more than a half of the full set of patterns. Thus, the initial "smile" is somewhere in between, and varying the set of patterns; see Fig. 2, is quite representative. After adding detector noise; see Fig. 3 B, C, the overall behavior of all calculated images reminds the results presented in Fig. 2, taking into account the fact that, in Fig. 3, we have a fixed set of patterns. Again, there are images that allow for better reconstruction (i.e., "dancing man," "duck," "musical note"), and there are images, which are more difficult to reconstruct.

We explicitly note, that both the test images and the calculated images are gray-scale, while the patterns are binary black-and-white. Our choice of binary patterns is due to their physical implementation with DMD spatial light modulator, where micro-mirrors can be set in either of two fixed positions. However, it is also possible to use gray-scale patterns and implement them with LCD spatial light

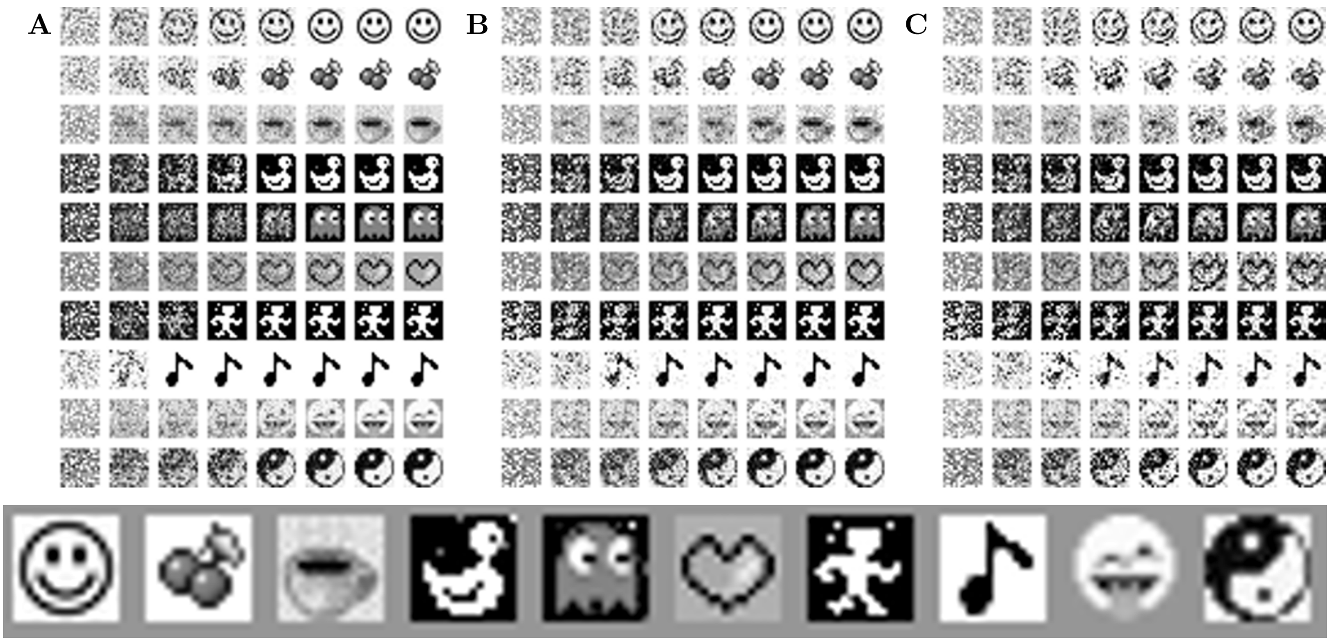

Fig. 3. Examples of calculated images; here, 10 ground truth 16×16 pixel images (the bottom row). Groups A, B, C are obtained for the noise level 0%, 1%, 3%, respectively. Each row in each group corresponds to the number of samples increasing from 32 to 256 in increments of 32 (from left to the right). The results are obtained for a random set of patterns with approximately 5% units, the same for all images and noise levels, without any optimization or post-selection.

modulators.

The other possible variation of the above-considered scenario is to reduce the calculated images to black-and-white instead of gray scale. For example, if we impose the target image to be a valid QR code, then we can add additional constrains on the space of possible calculated images. The standard way to get information out of the QR code is, first, to obtain a two-dimensional image of the QR code and, second, computationally process the obtained image (find the area of the QR code, rotate and reshape it, normalize the data). The QR code itself contains redundancy due to the error correction, which allows to extract information even if some part of the image is damaged or the read-out signal is noisy. In the above-considered computations, we do not set such constrains on the reconstructed images. Although it is possible to use a valid QR code as a test image for single-pixel imaging, the reconstruction algorithm has to be properly modified to deal with such scenario; otherwise, the reconstructed image can be an unreadable QR code.

5. Over-Complete Set of Patterns

One standard option to reduce the influence of noise is to average the signal over several samples. For this, we should measure the signal from the detector by repeating each pattern several times. With i.i.d Gaussian noise, the noise variance decreases after the averaging. When implementing this method, we increase the image sampling time and, thus, actually increase the number of elements in the set of patterns (proportionally to the number of repetition of each pattern). The system of equations in its original form (before averaging the signal over several samples) becomes over-complete, i.e., the image is over-sampled.

We consider the other way to reduce the influence of noise, also associated with over-sampling. The idea is to add new random patterns to the system of equations (1) instead of repeating each pattern several times. To compare the efficiency of this method with simple averaging, we fix the total number of image samples, i.e., the total number of equations in the system of equations (1). In the first case, we have 256 different equations, each of which is repeated several times, and the corresponding signals S are averaged. In the second case, we have an over-complete system, where all the equations are different.

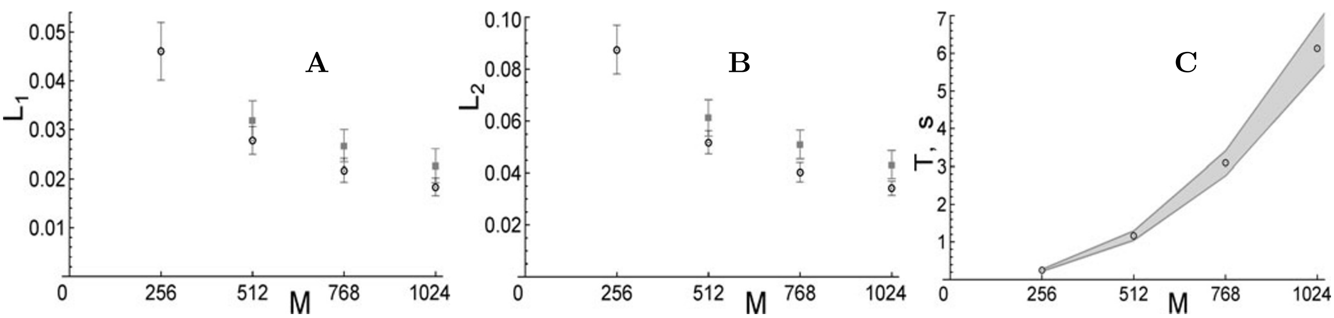

Fig. 4. The accuracy of image calculation, expressed in its proximity to the original image according to the metrics L_1 (A) and L_2 (B), as well as the time T for calculating one image (C) vs the number of image samples M. The signal noise level is equal to 3% of the average signal value. Number of units in one pattern is equal to 5%. Here, the results for averaging over several identical patterns (■) and the results for an over-complete set of patterns without repetition (⊙). A confidence interval of one standard deviation is calculated from 100 random sets of patterns.

To compare the accuracy of image reconstruction performed by both methods, we reconstruct the image by 100 random sets of patterns, and calculate the accuracy of the image reconstruction, using the metrics L_1 and L_2. As an example, we consider a realistic noise level of 3% from the average signal value and consider patterns with 5% units. The result of the calculations is expressed in the form of the average value of the metrics L_1 and L_2 and their standard deviation when over-sampling the image by 2, 3, and 4 times (the number of patterns in one set is 512, 768, and 1024, respectively); see Fig. 4.

The expected result is that the accuracy of the image calculation increases as the sample size increases. An unexpected result is the fact that, for a fixed number of patterns, the usage of over-sampling with different patterns leads to smaller noise in the calculated image in comparison to averaging the signal over each pattern. Considering the same result, from a different point of view, we can say that to obtain a given accuracy of the calculated image, the second method requires fewer patterns, i.e., the sampling time is reduced compared to simple signal averaging. This statement is true both on average for randomly selected patterns and in the case of choosing the best (that is, minimizing L_1 or L_2) random set of patterns; in the latter case, the difference in efficiency is even greater than the average for all sets.

A possible explanation for this effect can be the following.

The signal averaging apparently leads to the smaller noisy right hand side of the system of equations (1); however, a noise peak in a given signal component is distributed among few pixels, and constrained optimization algorithm forces them to be at the extreme values, i.e., to appear as more black or more white compared to the ground truth. When we increase the number of patterns and over-sample the image, the noise is higher (compared to the averaging), but it is distributed more evenly among more pixels. Thus, the constrained optimization is less likely to assign an extreme noisy value to a particular pixel.

To implement this method in an experiment, it is worth to mention two aspects.

First, the overall experimental arrangement remains the same. The only difference is an increased set of patterns. Usually the set of patterns is programmed in advance, e.g., by programming an electronic circuit driving a spatial light modulator. Increasing the set of patterns by a small factor does not make much difference in real implementation. Second, the practical difficulty is data processing and the required computational resources. The computation time for one image, when we use signal averaging for each pattern, obviously does not change with increasing over-sampling, since after averaging the number of equations in system of equations (1) remains the same. When we use the second method (over-sampling with different patterns), the computation time for one image increases, as the number of equations in the over-complete system of equations (1) increases. In our case shown in Fig. 4, the calculation time for the case of 4 times over-sampled system increases by an order of magnitude, and is equal to several seconds, when a standard laptop is used. Though, in real implementation, the computational time can be reduced by technical means (refining the algorithm, choosing faster programming language, using a more powerful computer, etc.), having in mind such scaling of computational time, a practical trade-off must be made between the desired image computation accuracy, the image sampling time, and the image computation time.

6. Conclusions

We showed that the single-pixel imaging method could reconstruct the original image in the presence of realistic detector noise level. As a natural noise model, we considered additive Gaussian noise equal to several percent of the average detector signal. Optimization of a set of patterns showed that, under such conditions, highly sparse random patterns were the best choice. As the number of patterns in the set increased, the accuracy of the reconstructed image nonlinearly increased in a threshold-like manner. After reaching a threshold number of patterns, the reconstructed image quite accurately matched the original image. A further increase in reconstruction accuracy could be achieved by sampling the image with an over-complete set of patterns. We found that this method provided higher quality of the reconstructed image compared to the standard averaging of the signal over several samples. However, the cost of this method consists in substantially increased computational time, provided otherwise the same experimental conditions. Our results showed the capabilities of different image noise reduction strategies in the single-pixel imaging method and gave an idea of their optimum practical implementation in an experiment.

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References

1. M. B. Wakin, J. N. Laska, M. F. Duarte, et al., "An Architecture for Compressive Imaging" in: 2006 International Conference on Image Processing, Atlanta, GA, USA (2006), pp.1273–1276; DOI: 10.1109/ICIP.2006.312577

2. M. F. Duarte, M. A. Davenport, D. Takhar, et al., *IEEE Signal Process. Mag.*, **25**, 83 (2008).
3. T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko, *Phys. Rev. A*, **52**, R3429 (1995).
4. D. Sych, V. Averchenko, and G. Leuchs, *Phys. Rev. A*, **96**, 053847 (2017).
5. M. D. Aksenov and D. V. Sych, *J. Russ. Laser Res.*, **39**, 492 (2018).
6. D. Sych and M. Aksenov, *AIP Conf. Proc.*, **1936**, 020016 (2018).
7. D. V. Strekalov, B. I. Erkmen, and N. Yu, *J. Phys. Conf. Series*, **414**, 012037 (2013).
8. D. Pelliccia, A. Rack, M. Scheel, et al., *Phys. Rev. Lett.*, **117**, 113902 (2016).
9. G. M. Gibson, B. Sun, M. P. Edgar, et al., *Opt. Express*, **25**, 2998 (2017).
10. R. A. Aguilar, N. Hermosa, and M. N. Soriano, *Am. J. Phys.*, **87**, 976 (2019).
11. M. Shcherbatenko, M. Elezov, N. Manova, et al., *Appl. Phys. Lett.*, **118**, 181103 (2021).
12. E. J. Candès, J. K. Romberg, and T. Tao, *Commun. Pure Appl. Math.*, **59**, 1207 (2006).
13. D. Donoho, *IEEE Trans. Inf. Theory*, **52**, 1289 (2006).