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NONCLASSICAL CORRELATIONS VIA SHARMA–MITTAL QUANTUM DISCORD IN HEISENBERG XYZ MODEL WITH DZYALOSHINSKII–MORIYA INTERACTION

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Abstract

In this paper, we study quantum correlations in a Heisenberg XYZ model with the Dzyaloshinskii–Moriya interaction based on the Sharma–Mittal quantum discord and its limiting cases. The results obtained show that the Sharma–Mittal quantum discord is a faithful quantifier and that the Dzyaloshinskii–Moriya interaction significantly enhances quantum correlations within the bipartite system. In contrast, temperature has the opposite effect, leading to a reduction in the quantity of quantum correlations in the system.

Keywords: Sharma–Mittal quantum discord, Heisenberg model, Dzyaloshinskii–Moriya interaction.

1. Introduction

Quantum correlations, widely acknowledged as essential resources, significantly improve the performance of quantum protocols compared to classical ones [1–4]. These correlations play a pivotal role in various fundamental aspects of quantum information processing and have diverse applications, due to their unique properties [5–7]. These applications extend across various scientific fields, including quantum computing [8, 9], cryptography [10, 11], teleportation [12], sensing and metrology [13, 14], communication [15, 16], machine learning [17], and superdense coding [18].

From an applied standpoint, the utilization of quantum correlations is limited by decoherence effects resulting from the interaction between the quantum system and its environment [19]. To address this issue, several approaches have been suggested in the literature to either eliminate or minimize the impact of environmental couplings in the evolution of open quantum systems [20, 21]. In contrast, from a foundational standpoint, significant efforts have been made to discern the key characteristics that set classical correlations apart from nonclassical correlations. This endeavor is crucial for gaining a deeper understanding of the distinct properties of quantum correlations and their implications in various contexts. This has led to extensive research efforts focused on quantifying quantum correlations in quantum systems with two or more qubits. While pure states can exhibit both separable and entangled properties, mixed states demonstrate more intricate characteristics of nonclassical correlations. Consequently,

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numerous indicators for quantum correlations have been investigated over the last two decades. Each quantifier comes with its own advantages and limitations. For instance, the quantum discord, introduced to characterize quantum correlations beyond entanglement [1, 22–24], is challenging to compute for a general two-qubit state.

On the other hand, the geometric quantum discord in the literature is computationally feasible, but it can grow under local operations on unmeasured qubits [21], making it an unreliable indicator of quantum correlations [25, 26]. In recent years, there has indeed been a notable increase in the development of quantifiers aimed at quantifying various aspects of quantum information. Notably, the Renyi quantum discord [27] and Tsallis quantum discord [28] have been introduced in different contexts. Furthermore, the Sharma–Mittal quantum discord [29] presents a comprehensive framework that encompasses both the Renyi [27] and Tsallis quantum discord measures [30]. These quantifiers play a crucial role in facilitating the understanding and characterization of quantum correlations and find applications across diverse areas of quantum information science. They provide valuable tools for analyzing and harnessing quantum correlations in quantum systems, contributing to advancements in quantum communication, quantum cryptography, quantum computation, and other quantum technologies.

In this context, our research paper focuses on a comparative study of the Sharma–Mittal quantum discord and its borderline cases as a quantifier of discord-like correlations in a Heisenberg XYZ model with the Dzyaloshinskii–Moriya interaction [31–33]. We explore how the Sharma–Mittal quantum discord behaves under different conditions and how it can effectively capture nonclassical correlations in this model described below by Hamiltonian (6), by examining the interplay between the model's parameters and the quantifier. Therefore, we identify the Sharma–Mittal quantum discord as a more useful correlation quantifier with respect to its limit cases and other quantifiers existing in the literature [34, 35].

This paper is organized as follows.

We start by introducing the basic concept related to general formulas of Sharma–Mittal quantum discord. In Sec. 2, we present the physical model within the framework of two-spin Heisenberg model, but we consider spin–orbit coupling describing by the Dzyaloshinskii–Moriya (DM) interaction, so that this section is closed by results and discussions. Conclusions and remarks are given in Sec. 4.

2. Basic Concepts of Sharma–Mittal Quantum Discord

Then, we delve into the foundational aspects of Sharma–Mittal quantum discord, elucidating the intricate interplay among entropy, quantum correlation, and the novel framework of Sharma–Mittal discord. We embark on our exploration by revisiting the theoretical underpinnings that pave the way for the Sharma–Mittal quantum discord. We then delve into the nuances of Sharma-Mittal entropy and its connections to Renyi and Tsallis entropies, culminating in the elucidation of quantum discord and its implications [36].

2.1. Sharma–Mittal Entropy

We commence our analysis by introducing the Sharma–Mittal entropy, which is a pivotal component within the framework of Sharma-Mittal quantum discord. The Sharma-Mittal entropy of a random variable X reads [37,38]

$$H_{q,r}(X=x)=\frac{1}{1-r}\left\{\left[\sum_{X=x}(P(X=x))^q\right]^{(1-r)/(1-q)}-1\right\}, \tag{1}$$

where q and r are two real parameters, with $q>0$, $q\neq 1$, $r\neq 1$, and P is the probability function [39].

Moving forward, we examine the relationship to Renyi and Tsallis entropies within the context of the Sharma-Mittal entropy.

2.2. Relationship to Renyi and Tsallis Entropies

It is notable that the Sharma–Mittal entropy encapsulates several well-known entropy measures as special cases. By examining the limit conditions of the Sharma–Mittal entropy expression, we can discern its relationship to various other entropic measures as follows:

- As $r\to 1$, we recover the Renyi entropy,

$$H_q^R(X)=\lim_{r\to 1}H_{q,r}(X=x)=\frac{1}{1-q}\log\left[\sum_{X=x}(P(X=x))^q\right]. \tag{2}$$

- Similarly, when $r\to q$, in Eq. (1,) we obtain the Tsallis entropy,

$$H_q^T(X)=\lim_{r\to q}H_{q,r}(X=x)=\frac{1}{1-q}\left[\sum_{X=x}(P(X=x))^q-1\right], \tag{3}$$

where $q\geq 0$ and $q\neq 1$.

Next, we delve into the Sharma–Mittal entropy for density matrices, which extends the concept of Sharma–Mittal entropy to the quantum realm. This adaptation offers a new perspective, through which to analyze uncertainty and complexity within quantum systems.

2.3. Sharma–Mittal Entropy for Density Matrices

Extending the concept of Sharma–Mittal entropy to quantum mechanics, we define the Sharma–Mittal entropy for density matrices. Given a density matrix ρ, the Sharma–Mittal entropy $H_{q,r}(\rho)$ reads

$$H_{q,r}(\rho)=\frac{1}{1-r}\left\{\left[\sum_i(\lambda_i)^q\right]^{(1-r)/(1-q)}-1\right\}, \tag{4}$$

where q and r are two real parameters, $q>0$, $q\neq 1$, $r\neq 1$, and λ_i's are eigenvalues of ρ. Furthermore, in the realm of quantum mechanics, the concepts of Renyi and Tsallis entropies find their counterparts and can be generalized in a manner similar to classical statistics [40,41].

2.4. Sharma–Mittal Quantum Discord

The notion of quantum discord within the Sharma-Mittal framework is a crucial development. Quantum discord, denoted as $\mathcal{D}_{q,r}(\rho)$, quantifies the nonclassical correlations in a quantum state represented by the density matrix ρ in the composite Hilbert space $\mathcal{H}^{(a)}\otimes\,\mathcal{H}^{(b)}$; it is defined as follows [29]:

$$\mathcal{D}_{q,r}(\rho) = H_{q,r}(\rho_b) + \max_{(\Pi_i)}\left[\sum_i p_i H_{q,r}(\rho_a^{(i)})\right] - H_{q,r}(\rho). \tag{5}$$

By replacing $H_{q,r}(\rho)$ with $H_q^R(\rho)$ and $H_q^T(\rho)$, we can obtain analogs of the Renyi and Tsallis discords denoted by $\mathcal{D}_q^R(\rho)$ and $\mathcal{D}_q^T(\rho)$, respectively.

In the next section, we seamlessly combine theory and practice by studying insights from the Sharma–Mittal quantum discord to the Heisenberg model.

3. System Model

In this section, we consider two-spins anisotropic XYZ model, with take into account the DM interaction. So, the Hamiltonian of the model has the following form [42]:

$$\mathcal{H} = J_x\sigma_1^x\sigma_2^x + J_y\sigma_1^y\sigma_2^y + J_z\sigma_1^z\sigma_2^z + \mathbf{D}\cdot(\sigma_1\times\sigma_2)\,, \tag{6}$$

where J_l's $(l=x,y,z)$, $\vec{D}$ and σ_k^l's $(l=x,y,z)$ are, respectively, the spin–spin interaction coupling and the strength of DM interaction and Pauli matrices of Kth spin. In this work, we restrict our study to the situation, where the Dzyaloshinski–Moriya interaction exists only along the z-direction, i.e., $D_x=D_y=0$. Then, we can rewrite Eq. (6) as follows:

$$\mathcal{H} = J_x\sigma_1^x\sigma_2^x + J_y\sigma_1^y\sigma_2^y + J_z\sigma_1^z\sigma_2^z + D_z\left(\sigma_1^x\sigma_2^y - \sigma_1^y\sigma_2^x\right). \tag{7}$$

In computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, Eq. (6) can be represented in the following matrix form:

$$\mathcal{H} = \begin{pmatrix} J_z & 0 & 0 & J_z - J_y \\ 0 & -J_z & J_x + J_y + 2iD_z & 0 \\ 0 & J_x + J_y - 2iD_z & -J_z & 0 \\ J_z - J_y & 0 & 0 & J_z \end{pmatrix}. \tag{8}$$

The spectra of the Hamiltonian (6) are

$$E_{1,2} = \pm J_x \mp J_y + J_z, \qquad E_{3,4} = -J_z \pm \varkappa, \tag{9}$$

with

$$\varkappa = \sqrt{(J_x+J_y)^2 + 4D_z^2}. \tag{10}$$

The eigenstates of (6) are defined as

$$|\Phi_{1,2}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}, \qquad |\Phi_{3,4}\rangle = \frac{|01\rangle \pm e^{i\theta}|10\rangle}{\sqrt{2}}, \tag{11}$$

where

$$\cos\theta = \frac{J_x + J_y}{\sqrt{(J_x + J_y)^2 + 4D_z^2}}. \tag{12}$$

The density operator of this model can be described as

$$\rho(T) = Z^{-1} e^{-\beta\mathcal{H}} = Z^{-1} \sum_{i=1}^{4} e^{-\beta E_i} |\Phi_i\rangle\langle\Phi_i|, \tag{13}$$

where $Z = \mathrm{Tr}(e^{-\beta\mathcal{H}})$ is the partition function of the system and $\beta = 1/k_B T$, with k_B being the Boltzmann constant ($k_B = 1$). Thus, the density matrix of the system reads

$$\rho_z(T) = \begin{pmatrix} r & 0 & 0 & s \\ 0 & u & v & 0 \\ 0 & v^* & u & 0 \\ s & 0 & 0 & r \end{pmatrix}, \tag{14}$$

with the elements

$$\begin{gathered} r = \frac{e^{-J_z/T}}{Z} \cosh\left(\frac{J_z - J_y}{2}\right), \qquad u = \frac{e^{J_z/T}}{Z} \cosh\left(\frac{\varkappa}{T}\right), \\ v = \frac{e^{J_z/T}}{Z\varkappa} \sinh\left(\frac{\varkappa}{T}\right) (J_x + J_y + 2iD_z), \qquad s = \frac{e^{-J_z/T}}{Z} \sinh\left(\frac{J_z - J_y}{2}\right). \end{gathered} \tag{15}$$

The partition function (13) can be written as

$$Z = 2e^{-J_z/T} \cosh\left(\frac{J_z - J_y}{2}\right) + 2e^{J_z/T} \cosh\left(\frac{\varkappa}{T}\right). \tag{16}$$

Next, exploiting to the local unitary transform $v^* \longrightarrow |v^*| = |v|$, one can find that

$$\rho_z(T) \longrightarrow \widetilde{\rho}_z(T) = \begin{pmatrix} r & 0 & 0 & s \\ 0 & u & |v| & 0 \\ 0 & |v^*| = |v| & u & 0 \\ s & 0 & 0 & r \end{pmatrix}, \tag{17}$$

where $|v^*| = |v| = \dfrac{e^{J_z/T}}{Z} \sinh\left(\dfrac{\varkappa}{T}\right)$ for the case of two-qubit, assuming the DM interaction in the z direction D_z and the temperature T.

The eigenvalues of ρ in terms of α_1, α_2, and α_3 are

$$\begin{aligned} \lambda_0 &= \frac{1}{4}(1 - \alpha_1 - \alpha_2 - \alpha_3), \qquad & \lambda_1 &= \frac{1}{4}(1 - \alpha_1 + \alpha_2 + \alpha_3), \\ \lambda_2 &= \frac{1}{4}(1 + \alpha_1 - \alpha_2 + \alpha_3), \qquad & \lambda_3 &= \frac{1}{4}(1 + \alpha_1 + \alpha_2 - \alpha_3), \end{aligned} \tag{18}$$

with

$$\alpha_1 = \frac{e^{(2J_z)/T}\sinh\left(\frac{\varkappa}{T}\right) + \sinh\left[\frac{1}{2}(J_z - J_y)\right]}{e^{(2J_z)/T}\cosh\left(\frac{\varkappa}{T}\right) + \cosh\left[\frac{1}{2}(J_z - J_y)\right]}, \tag{19}$$

$$\alpha_2 = \frac{e^{(2J_z)/T}\sinh\left(\frac{\varkappa}{T}\right) + \sinh\left[\frac{1}{2}(J_y - J_z)\right]}{e^{(2J_z)/T}\cosh\left(\frac{\varkappa}{T}\right) + \cosh\left(\frac{1}{2}(J_z - J_y)\right)}, \tag{20}$$

and

$$\alpha_3 = \frac{2}{e^{(2J_z)/T}\cosh\left(\frac{\varkappa}{T}\right)\operatorname{sech}\left[\frac{1}{2}(J_z - J_y)\right] + 1} - 1. \tag{21}$$

Since ρ is a Hermitian matrix, all the eingenvalues are real. Also, ρ is positive semi-definite matrix; therefore, $\lambda_2 \geq 0$. As $\mathrm{Tr}(\rho) = 1$, we have $\lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 = 1$. As $\lambda_0 \geq 0$, we have $\alpha_1 + \alpha_2 + \alpha_3 \leq 1$. In addition, $\lambda_0 + \lambda_1 \geq 0$ indicates $\alpha_1 \leq 1$ and $\lambda_2 + \lambda_3 \geq 0$ indicates $\alpha_1 \geq -1$. Combining, we obtain $-1 \leq \alpha_1 \leq 1$. Similarly, $-1 \leq \alpha_2 \leq 1$ and $-1 \leq \alpha_3 \leq 1$.

In the following section, we take an alternative approach by directly demonstrating limit operations on $\mathcal{D}_{q,r}(\rho)$ instead of relying on entropy. This reasoning serves to establish that the traditional concept of quantum discord is essentially a result of the limit scenario within the framework of the Sharma–Mittal quantum discord, our results agree with the expressions deduced in the following analysis.

- **Sharma–Mittal quantum discord**:

In this section, we discuss $\mathcal{D}_{q,r}(\rho)$ analytic expression and its relationship with $\mathcal{D}_q^R(\rho)$ and $\mathcal{D}_q^T(\rho)$ measures, as they emerge in the context of limiting cases. Additionally, we perform a comprehensive calculation of the quantum discord within the framework of Sharma–Mittal quantum discord and elucidate its distinctive characteristics pertaining to our analyzed system.

Recall Eq. (5), upon reviewing the reduced density matrices of the subsystems, we can verify the quantum discord, as proposed by the Sharma–Mittal entropy, $\mathcal{H}^{(a)}$ and $\mathcal{H}^{(b)}$ are given by $\rho_a = \rho_b = I_2/2$, which is a density matrix, which eigenvalues are equal to 1/2.

The Sharma–Mittal entropy value of I_2 is now given by

$$H_{q,r}(\rho_a) = H_{q,r}(\rho_b) = H_{q,r}\left(\frac{I}{2}\right) = \frac{2^{1-r} - 1}{1 - r}. \tag{22}$$

We obtain the analytical expression of $H_{q,r}(\rho^{(0)})$

$$\max_\theta\left[H_{q,r}(\rho^{(0)})\right] = \max_\theta\left[H_{q,r}(\rho^{(1)})\right] = \frac{1}{1-r}\left[\left\{\frac{(1+\alpha)^q}{2^q} + \frac{(1-\alpha)^q}{2^q}\right\}^{\frac{1-r}{1-q}} - 1\right]. \tag{23}$$

After a straightforward calculation, we arrive at the Sharma–Mittal entropy; the analytical expression is

$$\begin{aligned} H_{q,r}(\rho) = \frac{1}{1-r}\Bigg[&\frac{1}{4^{\frac{q(1-r)}{(1-q)}}}\{(1-\alpha_1-\alpha_2-\alpha_3)^q + (1-\alpha_1+\alpha_2+\alpha_3)^q \\ &+ (1+\alpha_1-\alpha_2+\alpha_3)^q + (1+\alpha_1+\alpha_2-\alpha_3)^q\}^{\frac{1-r}{1-q}} - 1\Bigg]. \end{aligned} \tag{24}$$

By replacing Eqs. (22) and (23) in Eq. (5), we find the analytical expression for the Sharma–Mittal quantum discord; it reads

$$\mathcal{D}_{q,r}(\rho) = \frac{\left[\dfrac{(1-\alpha)^q}{2^q} + \dfrac{(\alpha+1)^q}{2^q}\right]^{\frac{1-r}{1-q}} - 1}{1-r} + \frac{2^{1-r}-1}{1-r} - H_{q,r}(\rho), \tag{25}$$

where $H_{q,r}(\rho)$ is given by Eq. (24) and $\alpha = \max(|\alpha_1|, |\alpha_2|, |\alpha_3|)$, as well as q and r are two real numbers, such that $q > 0$, $q \neq 1$, $r \neq 1$.

- **Renyi quantum discord**:

The Sharma–Mittal entropy is equivalent to the Renyi entropy for certain conditions.

If $r \to 1$, in Eq. (2) we obtain

$$H_q^{(R)}\left(\frac{I}{2}\right) = \frac{1}{1-q}\log\left(\frac{1}{2^q} + \frac{1}{2^q}\right) = 1. \tag{26}$$

Taking $r \to 1$ in Eq. (23), we arrive at

$$\max_\theta(H_{q,r}(\rho_0)) = \max_\theta(H_{q,r}(\rho_1)) = \frac{1}{1-q}\log\left[\left(\frac{1+\alpha}{2}\right)^q + \left(\frac{1-\alpha}{2}\right)^q\right]. \tag{27}$$

The analytical expression of $H_q^{(R)}(\rho)$ reads

$$\begin{aligned} H_q^{(R)}(\rho) = \quad & \frac{1}{1-q}\log\bigg(\frac{1}{4^q}\{(1-\alpha_1-\alpha_2-\alpha_3)^q + (1-\alpha_1+\alpha_2+\alpha_3)^q \\ & +(1+\alpha_1-\alpha_2+\alpha_3)^q + (1+\alpha_1+\alpha_2-\alpha_3)^q\}\bigg). \end{aligned} \tag{28}$$

By replacing Eqs. (26) and (27) in Eq. (5), we obtain the analytical expression for the Renyi quantum discord as follows:

$$\mathcal{D}_q^R(\rho) = 1 + \frac{\log\left[\left(\dfrac{1-\alpha}{2}\right)^q + \left(\dfrac{\alpha+1}{2}\right)^q\right]}{1-q} - H_q^{(R)}(\rho), \tag{29}$$

where $H_q^{(R)}(\rho)$ is given by Eq. (28).

- **Tsallis quantum discord**:

The Sharma–Mittal entropy corresponds to the Tsallis entropy under specific conditions.

If $r \to q$, in Eq. (3), we arrive at

$$H_q^{(T)}\left(\frac{I}{2}\right) = \frac{1}{1-q}\left(\frac{1}{2^q} + \frac{1}{2^q} - 1\right) = \frac{2^{1-q}-1}{1-q}. \tag{30}$$

Assuming $r \to q$ in Eq. (23), we obtain

$$\max_\theta\left[H_q^{(T)}(\rho^{(0)})\right] = \max_\theta\left[H_q^{(T)}(\rho^{(1)})\right] = \frac{1}{(1-q)}\left[\left\{\frac{(1+\alpha)^q}{2^q} + \frac{(1-\alpha)^q}{2^q}\right\} - 1\right]. \tag{31}$$

The analytical expression of $H_q^{(T)}(\rho)$ reads

$$H_q^{(T)}(\rho) = \frac{1}{1-q}\left[\frac{1}{4^q}\{(1-\alpha_1-\alpha_2-\alpha_3)^q + (1-\alpha_1+\alpha_2+\alpha_3)^q + (1+\alpha_1-\alpha_2+\alpha_3)^q + (1+\alpha_1+\alpha_2-\alpha_3)^q\} - 1\right]. \tag{32}$$

By replacing Eqs. (30) and (31) in Eq. (5), we obtain the analytical expression for the Tsallis quantum discord as follows:

$$\mathcal{D}_q^T(\rho) = \frac{\left[\dfrac{(1-\alpha)^q}{2^q} + \dfrac{(\alpha+1)^q}{2^q}\right] - 1}{1-q} + \frac{2^{1-q}-1}{1-q} - H_q^{(T)}(\rho), \tag{33}$$

where $H_q^{(T)}(\rho)$ is the Tsallis entropy given by (32).

In the next section, we move from the analytical results to the step of substantive discussions in the Heisenberg model.

4. Results and Discussions

In this section, we examine the behavior of Sharma–Mittal quantum discord, Renyi quantum discord, and Tsallis quantum discord as functions of both the DM interaction and T.

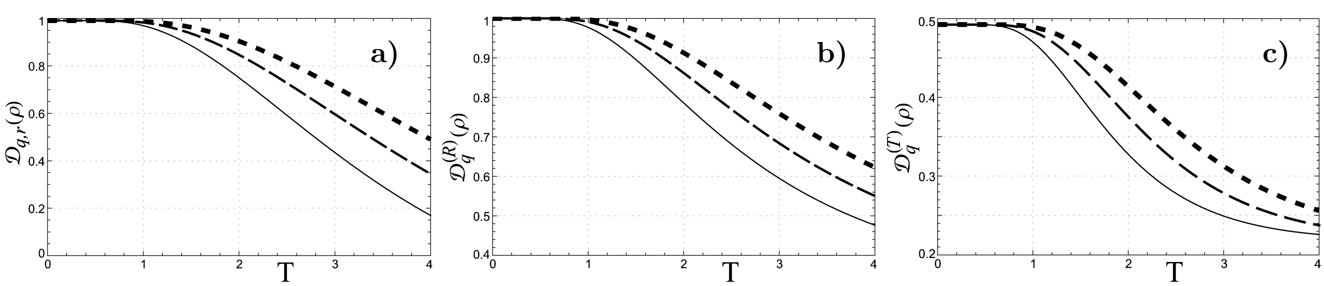

Fig. 1. The variations in the Sharma–Mittal discord (a), Renyi discord (b), and Tsallis discord (c) in terms of T for $D = 2$ (solid curves), $D = 2.5$ (dashed curves), and $D = 3$ (dotted curves); here, $J_x = 0.5$, $J_y = 0.75$, $J_z = 0.25$, $q = 2.05$, and $r = 0.2$.

In Fig. 1, we illustrate our analysis into the impact of the DM interaction on quantum correlations in a two-qubit Heisenberg XYZ model system at different values of temperature T. One can see that $\mathcal{D}_{q,r}(\rho)$, $\mathcal{D}_q^R(\rho)$, and $\mathcal{D}_q^T(\rho)$ behave in the same way, with respect to the parameter D. The first increase in the initial values depends on T, with D increasing to reach asymptotically the values 1 for the very sufficient Sharma–Mittal quantum discord and Renyi quantum discord, and the value 0.4926 for the Tsallis quantum discord, when D reaches a value very sufficient. Moreover, it is noticed that the influence of temperature diminishes entirely for large D values. This observation emphasizes the crucial role played by the antisymmetric contribution of spin–orbit coupling in the z-direction, as it enhances and sustains the quantum correlations within the two-spin Heisenberg XYZ model. Additionally, this characteristic renders the studied system more resilient against the decoherence phenomenon caused by fluctuations in temperature effects.

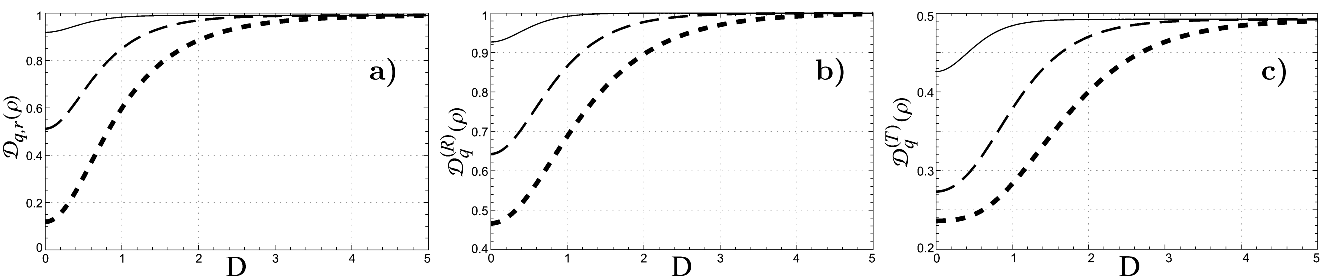

Fig. 2. The variations in the Sharma–Mittal discord (a), Renyi discord (b), and Tsallis discord (c) in terms of D for $T = 0.5$ (solid curves), $T = 1$ (dashed curves), and $T = 1.5$ (dotted curves); here, $J_x = 0.5$, $J_y = 0.75$, $J_z = 0.25$, $q = 2.05$, and $r = 0.2$.

Next, we investigate the impact of the DM interaction, acting antisymmetrically, due to spin–orbit coupling along the z axis DM, on quantum correlations under constant temperature conditions.

In Fig. 2, we observe that the Sharma–Mittal quantum discord, Renyi quantum discord, and Tsallis quantum discord exhibit almost very similar behavior with respect to temperature T. They all reach their maximum values as T approaches zero. The reason behind this behavior is that, at very low temperatures, the bipartite system exists in a state of maximum entanglement, which is a pure and fundamental state. However, as the temperature rises, thermal fluctuations start to influence the system, leading to an asymptotic decay of all three quantifiers. Remarkably, the thermal characteristics of $\mathcal{D}_{q,r}(\rho)$, $\mathcal{D}_q^R(\rho)$, and $\mathcal{D}_q^T(\rho)$ manifest a minimum quantum correlation, when the value of parameter D is relatively small. Likewise, this characteristic completely disappears as the values of D increase significantly. Therefore, higher values of D effectively eliminate the minimum for the three quantifiers.

Finally, our simulation results indicate that the Sharma–Mittal quantum discord is a faithful quantifier, when compared to the others in our system, regardless of the values of T and D.

5. Concluding Remarks

We summarize the key points developed in this paper.

We studied quantum correlations in a two-qubit Heisenberg XYZ model in the presence of the Dzyaloshinski–Moriya interaction. In our research, we used three quantifiers, namely, the Sharma–Mittal quantum discord, Renyi quantum discord, and Tsallis quantum discord to assess quantum correlations. We obtained analytic expresions for each of these quantifiers, specific to our bipartite system under study. A comparative study was carried out to analyze the quantum correlations presented in the examined model. To sum up, our results demonstrated that the Dzyaloshinskii–Moriya DM interaction enhances quantum correlations in the two-qubit Heisenberg XYZ model. Moreover, we found that the Sharma–Mittal quantum discord proved to be a reliable and efficient quantifier to capture the maximum information present in the system under study. This observation highlights the importance of the DM interaction in enriching quantum correlations and highlights the crucial role of the Sharma–Mittal quantum discord as a valuable tool to assess these correlations in this specific context. Thus, these discoveries contribute to deepening our understanding of quantum physics and open up new perspectives for future research in this field.

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