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COMPARATIVE STUDY OF TWO DEFINITIONS OF THE TURBULENT DISTANCE FOR LASER BEAMS PROPAGATING THROUGH NON-KOLMOGOROV ATMOSPHERIC TURBULENCE

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Abstract

Using two definitions of the turbulent distance to characterize the laser beam propagation through atmospheric turbulence, we derive a general analytical expression for the beam spread η depending on the turbulence parameter $T(\alpha)$ with the generalized exponent α and on the initial second-order beam moments in the $z = 0$ plane. Larger values of η correspond to a larger influence of atmospheric turbulence on the laser beam. We subsequently apply the analytical expression of η to a partially coherent Hermite–Gaussian beam propagating through non-Kolmogorov turbulence and illustrate the properties of η by numerical examples. The results show that the η values first increase, reach their maximum for a generalized exponent $\alpha \approx 3.11$, and then decrease with increase in α. Also η decreases with increasing beam order and wavelength, as well as with increasing values of the generalized refractive-index structural turbulence parameter, beam waist width, and coherence parameter.

Keywords: two definitions of the turbulence distance, non-Kolmogorov turbulence, partially-coherent Hermite–Gaussian (PCHG) beams.

1. Introduction

In the process of the laser beam propagation through the atmosphere, atmospheric turbulence causes beam spreading, scintillation, and wander, with a resulting degraded beam quality generally detrimental to practical applications [1–6]. However, within a spatial range known as the turbulent distance, the influence of turbulence on laser beams is small enough to be neglected [7–12]. Moreover, the influence of atmospheric turbulence on the beam-propagation properties is inversely proportional to the turbulent distance. Two definitions of the turbulent distance exist in the literature [7–12]. The first definition for turbulent distance only considers the influence of turbulence on the beam spread w (denoted hereafter η), with $\dfrac{w^2(z_T)_{\text{turb}} - w^2(z_T)_{\text{free}}}{w^2(z_T)_{\text{turb}}} = \eta$, where z_T is the characteristic turbulent distance for partially coherent beams [7–9]. The second definition relies on the beam quality factor; hereafter, M^2 factor introduced in [10,11] as $\mathrm{M}_r^2\,[z_M(\alpha)] = \sqrt{2}$, where α is the generalized exponent, also being dependent on the turbulent distance z_M. Variations of the M^2 factor in turbulence comprehensively characterize the influence of

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turbulence on the properties of the laser beam propagation, because the M^2 factor is invariant in free space [10, 11]. The general analytical expression of the turbulent distance z_M was established in [12].

In the first definition, the turbulent distance appears as closely related to the beam spread parameter η [7–9]. The typical range of η values stated as $10\% - 95\%$ in [9].

The questions arise: What is the appropriate value of η? What factors η determine?

If z_T and z_M represent distinct estimates of the same physical quantity, then equating both definitions can allow to express the relationship between these estimates. Therefore, the main purpose of this study is to better estimate η by deriving its general analytical expression and determining the factors influencing its value. Using a partially-coherent Hermite–Gaussian (PCHG) beam as an example, we investigate the η values and influencing factors by theoretical analysis and numerical simulation. Finally, we summarize the main results of this study.

2. Expression of η from Two Definitions of the Turbulent Distance

The second-order moments of partially coherent beams propagating through non-Kolmogorov turbulence along the x direction can be expressed as [4, 12]

$$\langle x^2 \rangle = \langle x^2 \rangle_0 + \langle \theta_x^2 \rangle_0 z^2 + T(\alpha) z^3, \tag{1}$$

where subscript 0 indicates the second-order moments in the $z = 0$ plane, and

$$T(\alpha) = \frac{2\pi^2}{3} \int_0^\infty \Phi_n(\varkappa, \alpha) \varkappa^3 d\varkappa \tag{2}$$

is the turbulence parameter characterizing the influence of atmospheric turbulence on the laser beam [4, 13, 14], with α being the generalized exponent parameter.

In [9, 10], the turbulent distance z_T is introduced to quantify the influence of turbulence on η; it reads

$$\eta = \frac{w^2(z_T)\Big|_{\text{turb}} - w^2(z_T)\Big|_{\text{free}}}{w^2(z_T)\Big|_{\text{turb}}}. \tag{3}$$

In view of Eq. (1), the squared beam width is [12]

$$w^2(z) = \langle x^2 \rangle = \langle x^2 \rangle_0 + \langle \theta_x^2 \rangle_0 z^2 + T(\alpha) z^3. \tag{4}$$

Substituting Eq. (4) into Eq. (3), we finally derive the general analytical expression for η, it reads

$$\eta = \frac{T(\alpha) z_T^3}{T(\alpha) z_T^3 + \langle \theta_x^2 \rangle_0 z_T^2 + \langle x^2 \rangle_0}. \tag{5}$$

In [12], the turbulent distance for laser beams propagating through atmospheric turbulence is defined as follows:

$$z_M(\alpha) = M_1 + \frac{1}{2}(4M_1^2 + Q)^{1/2} + \frac{1}{2}(8M_1^2 - Q + M_2)^{1/2}, \tag{6}$$

where

$$M_1 = -\frac{\langle\theta_x^2\rangle_0}{3T(\alpha)}, \qquad M_2 = \frac{16\langle\theta_x^2\rangle_0^3 + 216\langle x^2\rangle_0 T^2(\alpha)}{27T^3(\alpha)[4M_1^2+Q]^{1/2}}, \qquad Q = -\frac{32\sqrt[3]{2}\langle x^2\rangle_0\langle\theta_x^2\rangle_0}{3T^2(\alpha)P} - \frac{P}{\sqrt[3]{54}},$$
$$P = \left\{16S + \left[\frac{2^{17}\langle x^2\rangle_0^3\langle\theta_x^2\rangle_0^3}{T^6(\alpha)} + 256S^2\right]^{1/2}\right\}^{1/3}, \qquad S = \frac{27\langle x^2\rangle_0^2}{T^2(\alpha)} - \frac{4\langle\theta_x^2\rangle_0^3\langle x^2\rangle_0}{T^4(\alpha)}. \tag{7}$$

If z_T and z_M represent the same turbulent distance, z_M can be substituted for z_T in Eq. (5), and the analytical expression for η becomes

$$\eta = \frac{T(\alpha)\left[M_1 + \frac{1}{2}(4M_1^2+Q)^{\frac{1}{2}} + \frac{1}{2}(8M_1^2 - Q + M_2)^{\frac{1}{2}}\right]^3}{T(\alpha)\left[M_1 + \frac{1}{2}(4M_1^2+Q)^{\frac{1}{2}} + \frac{1}{2}(8M_1^2 - Q + M_2)^{\frac{1}{2}}\right]^3 + \langle\theta_x^2\rangle_0\left[M_1 + \frac{1}{2}(4M_1^2+Q)^{\frac{1}{2}} + \frac{1}{2}(8M_1^2 - Q + M_2)^{\frac{1}{2}}\right]^2 + \langle x^2\rangle_0}. \tag{8}$$

Equation (8) is the main result of this study. It indicates that η depends on the initial beam parameters, i.e., the second-order moments in the $z = 0$ plane, and on the turbulence parameter $T(\alpha)$. For any type of laser beams, Eq. (8) allows for direct calculation of η, if the initial second-order moments in the $z = 0$ plane and the turbulence parameter $T(\alpha)$ are known. Previous studies demonstrated that larger η values correspond to a larger influence of turbulence on the laser beam [7, 11]. For $\alpha = 11/3$, $\eta\,(11/3)$ directly represents the beam spread in Kolmogorov turbulence; see below.

3. Numerical Calculation Results and Analysis

In this section, we apply the analytical expression of η to a PCHG beam as an example, and then we conduct numerical simulations to evaluate the dependence of η on several parameters, i.e., the beam parameters m, λ, β, and w_0 or turbulent parameters α and $\tilde{C}_n^2$). Typical results are illustrated in Figs. 1–5.

In [15], the second-order moments for PCHG beams in the $z = 0$ plane were defined as follows:

$$\langle x^2\rangle_0 = \frac{w_0^2}{4}(2m+1), \tag{9}$$

$$\langle\theta_x^2\rangle_0 = \frac{\lambda^2}{4\pi^2 w_0^2}(\beta^{-2} + 2m + 1), \tag{10}$$

where λ is the wavelength, w_0 is the beam waist width, and the β is the coherence parameter of the PCHG beam.

In Eq. (2), $\Phi_n(\varkappa, \alpha)$ is the spatial power spectrum of refractive-index fluctuations in a non-Kolmogorov turbulent atmosphere expressed as [16]

$$\Phi_n(\varkappa, \alpha) = A(\alpha)\tilde{C}_n^2 \frac{\exp[-(\varkappa^2/\varkappa_m^2)]}{(\varkappa^2 + \varkappa_0^2)^{\alpha/2}}, \qquad 0 \le \varkappa < \infty, \qquad 3 < \alpha < 4, \tag{11}$$

where $\tilde{C}_n^2$ is a generalized refractive-index structural parameter in units [$\mathrm{m}^{3-\alpha}$] and the other parameters are

$$\varkappa_m = c(\alpha)/l_0 \qquad [l_0 \ \text{ is inner scale of the atmospheric turbulence}], \tag{12}$$

$$c(\alpha) = \left[\frac{2\pi}{3}\Gamma\left(\frac{5-\alpha}{2}\right)A(\alpha)\right]^{1/(\alpha-5)} \qquad [\Gamma(x) \ \text{ is the Gamma function}], \tag{13}$$

$$A(\alpha) = \Gamma(\alpha-1)\cos(\alpha\pi/2)/4\pi^2, \tag{14}$$

$$\varkappa_0 = 2\pi/L_0 \qquad [L_0 \ \text{ is outer scale of the atmospheric turbulence}]. \tag{15}$$

If $\alpha = 11/3$, $A(11/3) = 0.033$, $\tilde{C}_n^2 = C_n^2$, and $\Phi_n(\varkappa,\alpha)$, the conventional Kolmogorov spectrum takes place.

After substituting Eq. (11) into Eq. (2), we obtain $T(\alpha)$; it reads

$$T(\alpha) = \frac{2}{3}\pi^2\int\limits_0^\infty \Phi_n(\varkappa)\varkappa^3\,d\varkappa = \frac{A(\alpha)\pi^2\tilde{C}_n^2}{3(\alpha-2)}\left\{\left[\frac{c(\alpha)}{l_0}\right]^{2-\alpha}\left[\frac{8\pi^2}{L_0^2}+(\alpha-2)\frac{c^2(\alpha)}{l_0^2}\right]\right.$$
$$\left.\times\exp\left(\frac{4l_0^2\pi^2}{L_0^2c^2(\alpha)}\right)\Gamma\left[2-\frac{\alpha}{2},\frac{4\pi^2l_0^2}{c^2(\alpha)L_0^2}\right]-2\left(\frac{2\pi}{L_0}\right)^{4-\alpha}\right\}. \tag{16}$$

Equation (16) indicates that $T(\alpha)$ depends on α, inner scale l_0, and outer scale L_0 of the turbulence.

Finally, numerical calculations are conducted after substituting Eqs. (9), (10), and (16) into Eq. (8). In Fig. 1, we illustrate the dependence of η on the beam order m as a function of α, with the following parameter values: $\tilde{C}_n^2 = 10^{-14}\ \mathrm{m}^{3-\alpha}$, $w_0 = 0.05$ m, $\lambda = 1060$ nm, $L_0 = 100$ m, $l_0 = 0.001$ m, and $\beta = 0.5$. In Fig. 1, we see that η varies nonmonotonically with increasing α; first it increases with increasing α values until a maximum at $\alpha \approx 3.11$, then it monotonically descreases, indicating distinct beam spread variations for ideal Kolmogorov turbulence and non-Kolmogorov turbulence. For PCHG beams propagating in non-Kolmogorov turbulence, η sharply decreases for $3.11 < \alpha < 3.6$ but, in contrast, it is nearly invariant with increase in m for $\alpha > 3.6$.

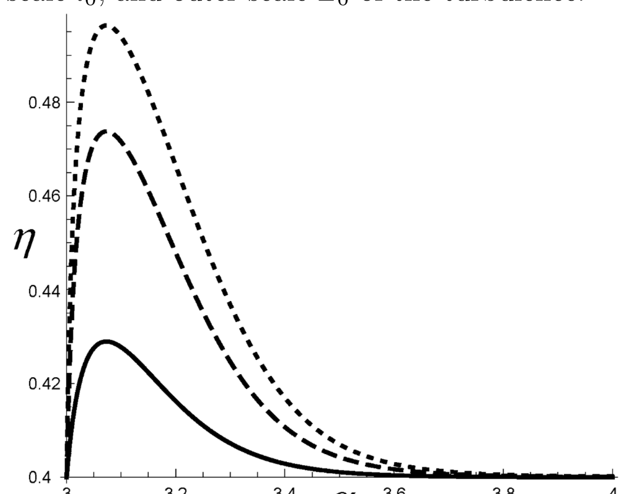

Fig. 1. The value of η versus the generalized exponent parameter α for the beam order $m = 10$ (the solid curve), $m = 3$ (the dashed curve), and $m = 0$ (the dotted curve).

The dependence of η on α as a function of the generalized refractive-index structural parameter $\tilde{C}_n^2$ is shown in Fig. 2, with $m = 10$ and the same values as in Fig. 1 for the other parameters. In Fig. 2, we illustrate a monotonic increase in η with increasing $\tilde{C}_n^2$, indicating gradually stronger influence of turbulence on the laser beam with increasing η values. Moreover, η exhibits a strong dependence on the value of α, whereas the dependence of η on $\tilde{C}_n^2$ is markedly smaller for laser beams propagating through ideal Kolmogorov turbulence than through non-Kolmogorov turbulence.

In Fig. 3, we illustrate the dependence of η on α as a function of the beam waist width w_0, with $\tilde{C}_n^2 = 10^{-14}\ \mathrm{m}^{3-\alpha}$, $m = 10$, and other parameter values identical to those in Fig. 1. Figure 3 clearly

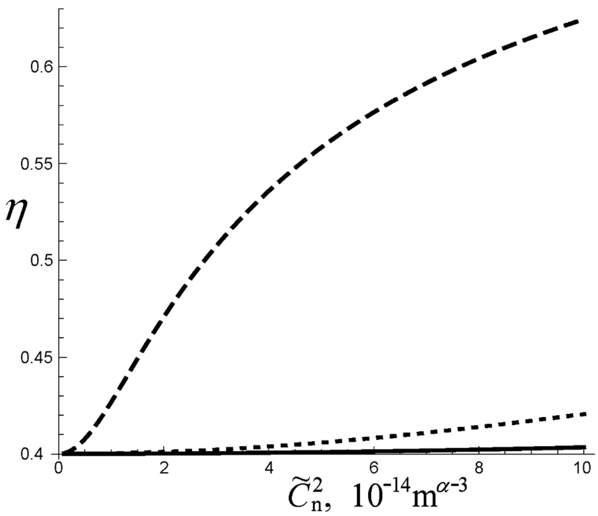

Fig. 2. The value of η versus generalized refractive-index structure parameter $\tilde{C}_n^2$ for the generalized exponent parameter $\alpha = 3.9$ (the solid curve), $\alpha = 3.11$ (the dashed curve), and $\alpha = 11/3$ (the dotted curve).

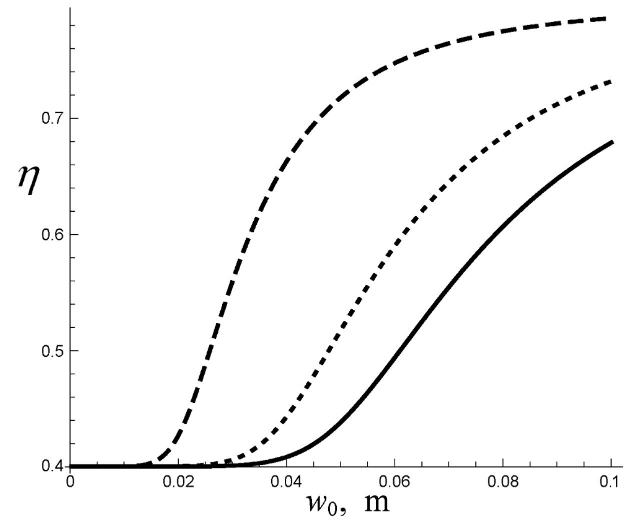

Fig. 3. The values of η versus the beam width w_0 for the generalized exponent parameter $\alpha = 3.9$ (the solid curve), $\alpha = 3.11$ (the dashed curve), and $\alpha = 11/3$ (the dotted curve).

indicates a notable increase in η with increasing w_0, even for laser beams propagating in Kolmogorov turbulence. Thus, we obtain a clear influence of w_0 variations on η. Conversely, for $w_0 < 0.02$ m, η is smaller for all α values, indicating that beams with a waist width smaller than 0.02 m are less sensitive to atmospheric turbulence. This is consistent with previous results [15].

In Fig. 4, we illustrate the dependence of η on α as a function of the wavelength λ, with all calculation parameters identical to those in Fig. 3. The η values decrease with increasing λ, with a strong sensitivity to α; see Fig. 4. For example, if $\alpha = 11/3$, η decreases sharply for $\lambda < 800$ nm, but remains nearly constant for $\lambda > 800$ nm, indicating that the larger λ, the smaller the value of η, and the laser beam propagation is less affected by turbulence.

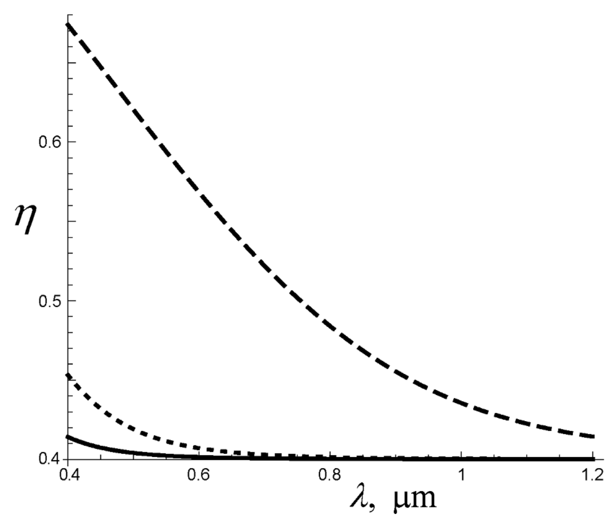

Fig. 4. The value of η versus the wavelength λ for the generalized exponent parameter $\alpha = 3.9$ (the solid curve), $\alpha = 3.11$ (the dashed curve), and $\alpha = 11/3$ (the dotted curve).

In Fig. 5, we show the dependence of η on the coherence parameter β as a function of α, with $m = 10$ and other parameter values identical to those in Fig. 1. In Fig. 5, we observe that η increases with increasing values of β for $\alpha < 3.6$, but exhibits negligible variations with increasing β for $\alpha > 3.6$. This indicates that, for $\alpha > 3.6$, including the ideal Kolmogorov turbulence case, the beam coherence hardly influences η values. Additionally, η values for partially coherent beams are demonstrably smaller than for fully coherent beams, indicating that partially coherent beams are less sensitive to turbulence. This is also consistent with the findings of a previous study [15].

In summary, in Figs. 1–5, we demonstrate that η values are markedly affected by the beam waist width w_0 independently of α values, but are less sensitive to other beam parameters, such as the wavelength, beam order, coherence parameter, and the generalized refractive-index structural turbulence parameter for $3.6 < \alpha < 4$. When combining these results with those from a previous study (Fig. 1 b in [17]), one can see that, in the region of $3.6 < \alpha < 4$, $T(\alpha)$ becomes relatively small, indicating weaker influence of atmospheric turbulence on η and negligible sensitivity of η values to other beam or turbulence parameters.

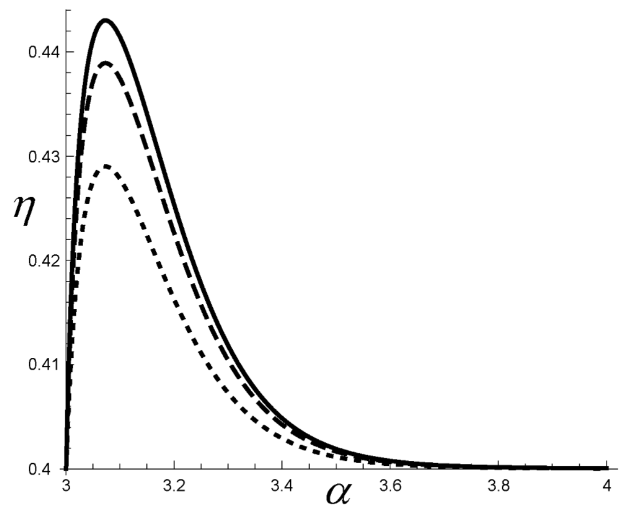

Fig. 5. The value of η versus the generalized exponent parameter α for the coherence parameter $\beta \to \infty$ (the solid curve), $\beta = 1$ (the dashed curve), and $\beta = 0.5$ (the dotted curve).

4. Summary

In this study, we used two definitions of the turbulent distance for laser beams propagating through atmospheric turbulence and derived the general analytical expression of η, which depended on the turbulence parameter $T(\alpha)$ and on the initial second-order beam moments in the $z = 0$ plane. Larger values of η indicated more influence of turbulence on the laser beam. To illustrate our findings, we applied the analytical expression of η to a PCHG beam propagating through non-Kolmogorov turbulence and evaluated the results for different values of the beam or turbulence parameters. We observed a clear dependence of η values on α and w_0: first, a monotonic increase with increasing α to a maximum (reached for $\alpha \approx 3.11$) followed by a continuous decrease with increasing α; second, decreasing η values with increasing m or λ, but increasing values with increasing w_0, β, or $\tilde{C}_n^2$ in the case $3 < \alpha < 3.6$. The η values were consistently sensitive to w_0 independently of α, but markedly less influenced by other beam parameters for the case $3.6 < \alpha < 4$. Our results demonstrate that η values, calculated for the beam propagation through atmospheric turbulence, accurately reflect the degree of influence of turbulence on the laser beam. They further indicate that this influence can be minimized by optimizing parameter selection. This result is potentially significant for application of laser technology to atmospheric optical communication.

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References

1. M. Salem, T. Shirai, A. Dogariu, and E. Wolf, *Opt. Commun.*, **216**, 261 (2003).
2. Y. P. Huang, A. P. Zeng, Z. H. Gao, and B. Zhang, *Opt. Lett.*, **40**, 1619 (2015).
3. H. T. Eyyuboglu, Y. Baykal, and Y. Cai, *Appl. Phys. B*, **91**, 265 (2008).

4. Y. Q. Dan and B. Zhang, *Opt. Lett.*, **34**, 563 (2009).
5. Y. P. Huang, Z. H. Gao, and B. Zhang, *Opt. Laser Technol.*, **50**, 125 (2013).
6. X. Ji, H. T. Eyyuboglu, and Y. Baykal, *Opt. Express*, **18**, 6922 (2010).
7. G. Gbur and E. Wolf, *J. Opt. Soc. Am. A*, **19**, 1592 (2002).
8. X. Li, X. Ji, H. T. Eyyuboglu, and Y. Baykal, *Appl. Phys. B*, **98**, 557 (2010).
9. J. H. Li, A. L. Yang, and B. D. Lü, *J. Opt. Soc. Am. A*, **25**, 2670 (2008).
10. Y. Q. Dan, S. G. Zeng, B. Y. Hao, and B. Zhang, *J. Opt. Soc. Am. A*, **27**, 426 (2010).
11. Y. L. Ai and Y. Q. Dan, *Opt. Commun.*, **284**, 3216 (2011).
12. Y. P. Huang and B. Zhang, *J. Opt. Soc. Am. A*, **30**, 2339 (2013).
13. Y. P. Huang, F. H. Wang, Z. H. Gao, and B. Zhang, *Opt. Express*, **23**, 1088 (2015).
14. Y. P. Huang and A. P. Zeng, *Opt. Commun.*, **436**, 63 (2019).
15. Y. P. Huang, B. Zhang, Y. Q. Dan, et al., *Opt. Lasers Eng.*, **49**, 514 (2011).
16. I. Toselli, L. C. Andrews, R. L. Phillips, and V. Ferrero, *Proc. SPIE*, **6551**, 65510E-1 (2007).
17. Y. P. Huang, G. P. Zhao, D. He, and Z. C. Duan, *J. Mod. Opt.*, **58**, 912 (2011).