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DIFFRACTION LOSSES OF A DIELECTRIC OPEN RESONATOR

Dedicated to the 100th Anniversary
of the Birth of Nikolai G. Basov
and to the History of Lasers in Russia

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Abstract

This paper was stimulated by the experimental studies of solid-state lasers initiated by N. G. Basov* and carried out at the Laboratory of Luminescence of the P. N. Lebedev Physical Institute under the direction of M. D. Galanin and A. M. Leontovich in the 1960-ies. Here, the classical parabolic equation method is extended in order to calculate complex eigenfrequencies of optical oscillations in a dielectric-filled open resonator. Accurate estimates confirm a high quality factor of ruby lasers. The developed approach can be used to find complex eigenfrequencies of other dielectric optical objects in laser systems of current interest.

Keywords: open resonator, dielectric core, trapped modes, diffraction losses, parabolic equation method.

1. Introduction

In the late 1950-ies, after the invention of the maser [1,2], the idea of a "maser at optical frequencies" was vigorously discussed in the radio-physical and optical communities of the United States of America and the USSR [3–5], and the laser race began. Despite the realization of the ruby maser [6], ruby was not considered promising for making lasers [7]. However, in April 1960, T. H. Maiman reported a change in the ground-state population of Cr^{3+} ions in ruby under optical pumping; the corresponding article in the Physical Review Letters was published in June 1960 [8]. In April 1960, N. G. Basov addressed M. D. Galanin from the Luminescence Laboratory of the P. N. Lebedev Physical Institute of the USSR Academy of Sciences in Moscow with a proposal to create a ruby laser. On May 16, 1960, T. H. Maiman succeeded to obtain the laser action in ruby. On July 7, he spoke about this event at the press release in New York, and the next day, eye-catching headlines appeared in many newspapers [7]. Then, after Physical Review Letters refused to publish "one more paper on masers," a short note "Stimulated Optical Radiation in Ruby" appeared in Nature on August 6, 1960 [9]. In the USSR, the first ruby laser was launched on June 12, 1961 at the S. I. Vavilov Optical State Institute in Leningrad, but this was

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not reported in open sources and no further research was conducted [10]. On September 18, 1961, M. D. Galanin, A. M. Leontovich, and Z. A. Chizhikova obtained the laser generation at the Luminescence Laboratory of the Lebedev Physical Institute and started comprehensive experimental studies of laser radiation properties [11, 12]. One of the authors (M.N.P) participated in these studies as a student and, then, as a PhD student of the Moscow Institute of Physics and Technology [13, 14]. The second author (A.V.P.), studying computational aspects of wave theory, provided an analytical tool for an accurate estimation of the quality factor of a laser resonator.

At that time, although many works were devoted to the theoretical study of electromagnetic oscillations in resonators; see, e.g., [15], open resonators containing a dielectric core have not been sufficiently studied. Usually, calculations of geometrical optics were performed, but they could not give diffraction losses for high-Q modes, which owed their existence to the effect of total internal reflection. This applies even to the simplest resonator in the form of a rectangular prism with ideally reflecting mirrors at the ends. Apparently, there was the only published work [16] at that time, where the diffraction losses of "locked" or "trapped" oscillations in such resonator were calculated and a dielectric prism or a cylinder with refractive index value n close to unity was considered. In our work [17], the eigenfrequencies and damping of optical oscillations in a dielectric open cavity were calculated, using the parabolic equation method proposed earlier by M. A. Leontovich [18] and developed by G. D. Malyuzhinets and V. A. Fock for the problems of underwater acoustics and radio wave propagation [19, 20]. This method was used by L. A. Vainshtein in order to calculate the complex eigenfrequencies of electromagnetic oscillations in an empty open resonator [21].

In our work, briefly reported at the International Symposium on Electromagnetic Wave Theory (Tbilisi, the USSR, 1971) [22] and marked by the internationally renowned expert J. B. Keller, the parabolic equation method was generalized to describe the transverse diffusion of an inhomogeneous wave propagating along the boundary of the dielectric core. The condition justifying transition to the parabolic equation is the smallness of the wavelength, compared with the dimensions of the resonator. Only the natural condition that the system has resonance properties is imposed on the value of the refractive index. Under this assumption, approximate formulas were derived for the complex frequencies and the spatial distribution of the eigenmodes of the resonator. In this paper, we outline the idea of the derivation, present the final expressions, and analyze the effect of the ruby core on the quality factor of an open resonator. We hope that the approach presented here will be useful in the study of other modern dielectric optical systems.

2. Methods

This analytical study was stimulated by the experimental work of A. M. Leontovich's group, which confirmed the previously predicted high quality factor of the ruby laser. Physical considerations explain small radiation losses in the solid-state laser by the effect of total internal reflection concentrating the optical wave field in the resonator core. Our goal was to turn these qualitative considerations into reliable quantitative estimates.

From the mathematical point of view, this work was a modest part of the ambitious international program, declared, among others, by J. B. Keller and L. B. Felsen in the USA [23, 24] and by V. A. Fock, S. L. Sobolev, and G. D. Malyuzhinets in the USSR [19, 20, 25] and aimed at the development of efficient computational methods for the problems of diffraction and wave propagation in open regions and natural environments. One of the authors (A.V.P.), being a postgraduate student of Prof. Malyuzhinets, contributed to the development of a numerical implementation of the parabolic wave equation [26, 27]. Further development included numerous applications to the problems of optics, quantum mechanics, and radio wave propagation [28–33]. The application of open resonators in microwave communication and optical sensing techniques is beyond the scope of this publication, we cite only a few references [34–37].

3. Results and Discussion

The problem of calculating the electromagnetic eigenmodes requires the solution of Maxwell's equations and is generally very difficult. Considering a dielectric open resonator, we introduce a translational symmetry in a transverse direction along the z axis; see Fig. 1. This reduces the vector problem to the solution of two independent scalar Helmholtz equations for the transverse components E_z and H_z of the electric and magnetic fields, respectively. This simplification, together with the assumption of a quasi-longitudinal vector $\mathbf{k}$, makes it possible to obtain analytical expressions for the eigenfrequencies and oscillation damping. The solution of this model problem gives a correct understanding of the diffraction losses in a dielectric open resonator.

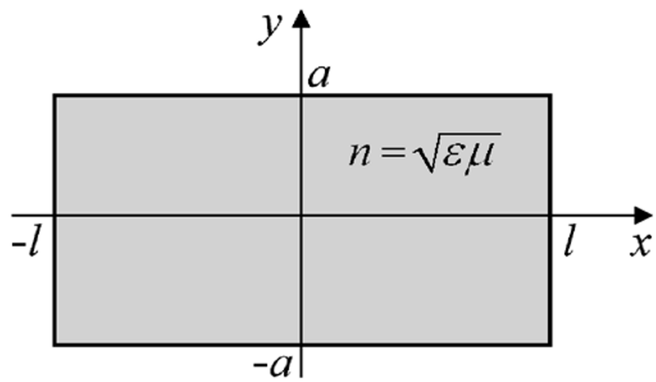

Fig. 1. Geometry of the dielectric open resonator.

Let an infinite prism $|x| < l$ and $|y| < a$ be filled with a dielectric with permeabilities ε and μ such that $n = \sqrt{\varepsilon\mu} \geq 1$. The ends $|x| = l$ are covered with perfectly reflecting mirrors, and the faces $|y| = a$ directly border with vacuum; see Fig. 1. We look for two-dimensional (independent of z coordinate) electromagnetic oscillations with a complex frequency $\omega = kc$ in the dielectric and the surrounding space. Two types of oscillations are possible; for the first one,

$$\vec{E} = (0, 0, E), \quad \vec{H} = \left(\frac{1}{ik\mu}\frac{\partial E}{\partial y}, -\frac{1}{ik\mu}\frac{\partial E}{\partial x}, 0\right), \quad \vec{E} = (0, 0, E_0), \quad \vec{H} = \left(\frac{1}{ik}\frac{\partial E_0}{\partial y}, -\frac{1}{ik}\frac{\partial E_0}{\partial x}, 0\right) \tag{1}$$

inside the dielectric core and in the outer space, respectively.

Similarly, for the second mode,

$$\vec{H} = (0, 0, H), \quad \vec{E} = \left(-\frac{1}{ik\varepsilon}\frac{\partial H}{\partial y}, \frac{1}{ik\varepsilon}\frac{\partial H}{\partial x}, 0\right), \quad \vec{H} = (0, 0, H_0), \quad \vec{E} = \left(-\frac{1}{ik}\frac{\partial H_0}{\partial y}, \frac{1}{ik}\frac{\partial H_0}{\partial x}, 0\right). \tag{2}$$

The functions $E(x, y)$ and $E_0(x, y)$ are the solutions of the scalar wave equations

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + k^2 n^2 E = 0, \quad \frac{\partial^2 E_0}{\partial x^2} + \frac{\partial^2 E_0}{\partial y^2} + k^2 E_0 = 0 \tag{3}$$

satisfying the Malyuzhinets "extinguishing principle" [38] (boundedness for $\operatorname{Im} k > 0$) and the boundary conditions

$$\begin{gathered} E = E_0, \quad \frac{1}{\mu}\frac{\partial E}{\partial y} = \frac{\partial E_0}{\partial y} \quad \text{for} \quad |y| = a, \quad |x| < l, \\ E = 0 \quad \text{for} \quad |x| = l, \quad |y| < a. \end{gathered} \tag{4}$$

The functions $H(x,y)$ and $H_0(x,y)$ satisfy the following equations, similar to Eqs. (3), with the boundary conditions

$$\begin{aligned} H = H_0, \quad \frac{1}{\varepsilon}\frac{\partial H}{\partial y} = \frac{\partial H_0}{\partial y} \quad \text{for} \quad |y| = a, \quad |x| < l, \\ \frac{\partial H}{\partial x} = \frac{\partial H_0}{\partial x} = 0 \quad \text{for} \quad |x| = l, \quad |y| < a. \end{aligned} \tag{5}$$

Our task consists in determining the complex values of the wave number k, for which one of the problems (4) or (5) has a nontrivial solution[†].

Consider oscillations of the first type. The highest Q factor have those of them that are propagating at small angles to the x axis inside the dielectric core; therefore, in the Ansatz,

$$E(x,y) = \exp(iknx)\, u(x,y) \pm \exp(-iknx)\, u(-x,y), \tag{6}$$

the wave amplitude $u(x,y)$ is a slowly varying function of the variable x. The electric field in vacuum, near the faces $|y| = a$, is close to inhomogeneous plane waves with the same propagation velocity. Separating out rapidly oscillating factors, we represent it in the form

$$E_0(x,y) = \exp(iknx)\, u_0(x,y) \pm \exp(-iknx)\, u_0(-x,y). \tag{7}$$

We substitute Egs. (6) and (7) into the wave equations (3), neglect the second derivatives in the x variable, and obtain for $u(x,y)$ the standard parabolic equation [18]

$$2\,ikn\,\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad x > -l, \qquad |y| < a, \tag{8}$$

while for $u_0(x,y)$, a modified parabolic equation

$$2\,ikn\,\frac{\partial u_0}{\partial x} + \frac{\partial^2 u_0}{\partial y^2} - k^2\,(n^2-1)\,u_0 = 0, \qquad x > -l, \qquad |y| > a, \tag{9}$$

describing the transversal diffusion [19] on the background of exponential extinguishing of the EM wave in the direction of the y axis. The boundary conditions (4) are transferred to the wave amplitudes,

$$u(x,\pm a) = u_0(x,\pm a), \qquad \frac{1}{\mu}\frac{\partial u}{\partial y}(x,\pm a) = \frac{\partial u_0}{\partial y}(x,\pm a), \tag{10}$$

and from the "total reflection" condition, it follows

$$u(l,y) = \mp\exp(-2iknx)u(-l,y). \tag{11}$$

The order of signs here is the same as in Eq. (6). The condition of absence of a field source outside the resonator yields an "initial" condition $u_0(-l,y) = 0$ and the requirement of the wave field boundedness for $\operatorname{Im} k \geq 0$, cf. [19].

[†]It would be more natural to consider k to be real, and the refractive index n to be complex. The advantage of our formulation lies in the possibility of a formal transition to the case of empty open cavity.

We look for the function $u(x,y)$ in the form of a Fourier series,

$$u(x,y)=\exp(-iknx)\sum_{s=-\infty}^{\infty}A_s w_s(y)\exp\left(i\frac{\pi s x}{2l}\right). \tag{12}$$

The condition (11) is satisfied, if the summation for the upper sign is carried out over odd powers of s, and for the lower one over even powers of s. Substitution in Eq. (8) yields for $w_s(y)$ an ordinary differential equation,

$$w_s''+2kn\left(kn-\frac{\pi s}{2l}\right)w_s=0, \tag{13}$$

with elementary solutions

$$w_s(y)=\begin{matrix}\cos\\ \sin\end{matrix}\,\lambda_s y,\qquad \lambda_s^2=2kn\left(kn-\frac{\pi s}{2l}\right). \tag{14}$$

An analytical solution of Eq. (9) bounded for $\operatorname{Im}k\geq 0$, coinciding at $y=\pm a$ with the boundary values $u(x,\pm a)$ and satisfying the initial condition $u_0(-l,y)=0$, can be found by the Laplace transform in the form

$$\begin{aligned}u_0(x,y) \;=\; & \frac{1}{2}\sum_{s=-\infty}^{\infty}A_s w_s(\pm a)\exp\left(-i\frac{\lambda_s^2}{2kn}x\right)\\ &\times\left\{\exp\left[(|y|-a)\sqrt{2kn\sigma_s}\right]\Phi\left[(|y|-a)\sqrt{\frac{kn}{2(x+l)}}+i\sqrt{\sigma_s(x+l)}\right]\right.\\ &\left.+\exp\left[(a-|y|)\sqrt{2kn\sigma_s}\right]\Phi\left[(|y|-a)\sqrt{\frac{kn}{2(x+l)}}-i\sqrt{\sigma_s(x+l)}\right]\right\},\end{aligned} \tag{15}$$

where $\sigma_s=\dfrac{k(n^2-1)}{2n}-\dfrac{\lambda_s^2}{2kn}$ and $\Phi(z)$ is Fresnel integral $\Phi(z)=\dfrac{2}{\sqrt{\pi}}\exp(-i\pi/4)\displaystyle\int_z^{\infty}\exp(i\alpha^2)\,d\alpha.$

By calculating the boundary value of the normal derivative

$$\begin{aligned}&\frac{\partial u_0}{\partial y}(x,\pm a)=\mp\frac{2}{\sqrt{\pi}}\exp(-i\pi/4)\sum_{-\infty}^{\infty}A_s w_s(\pm a)\exp\left(-i\frac{\lambda_s^2}{2kn}x\right)\\ &\times\left\{i\sqrt{2kn\sigma_s}\int_0^{\sqrt{\sigma_s(x+l)}}\exp(-i\alpha^2)d\alpha+\sqrt{\frac{kn}{2(x+l)}}\exp[-i\alpha^2\sigma_s(x+l)]\right\}\end{aligned} \tag{16}$$

equal, in virtue of the boundary condition (10), to

$$\frac{1}{\mu}\frac{\partial u}{\partial y}(x,\pm a)=\sum_{s=-\infty}^{\infty}A_s w_s'(\pm a)\exp\left(-i\frac{\lambda_s^2}{2kn}x\right), \tag{17}$$

we obtain an infinite set of algebraic equations for coefficients A_s,

$$A_s\left\{\frac{\exp(-i\pi/4)}{\mu}\sqrt{\frac{\pi l}{kn}}w_s'(a)+w_s(a)\left[\exp(-2i\sigma_s l)+\frac{1+4i\sigma_s l}{\sqrt{2\sigma_s l}}\int_0^{\sqrt{2\sigma_s l}}\exp(-i\alpha^2)d\alpha\right]\right\}$$
$$=-\sum_{p\neq s}A_p\frac{w_p(a)}{(\sigma_p-\sigma_s)l}\left[\sqrt{2\sigma_p l}\int_0^{\sqrt{2\sigma_p l}}\exp(-i\alpha^2)d\alpha-\sqrt{2\sigma_s l}\int_0^{\sqrt{2\sigma_s l}}\exp(-i\alpha^2)d\alpha\right]. \quad (18)$$

A nontrivial solution of these equations determines the eigenfunctions of our boundary problem, and the condition of its re-solvability serves as a characteristic equation for the eigenvalues k.

If one of the coefficients (marked below as A_q) is large compared to the others, then, to the first approximation, Eqs. (18) split into two separate equations as follows:

$$\frac{\exp(-i\pi/4)}{\mu}\sqrt{\frac{\pi l}{kn}}w_s'(a)+w_s(a)\left[\exp(-2i\sigma_s l)+\frac{1+4i\sigma_s l}{\sqrt{2\sigma_s l}}\int_0^{\sqrt{2\sigma_s l}}\exp(-i\alpha^2)\,d\alpha\right]=0, \quad (19)$$

$$\frac{A_s}{A_q}=-w_q(a)\frac{\sqrt{2\sigma_q l}\int_0^{\sqrt{2\sigma_q l}}\exp(-i\alpha^2)\,d\alpha-\sqrt{2\sigma_s l}\int_0^{\sqrt{2\sigma_s l}}\exp(-i\alpha^2)\,d\alpha}{(\sigma_q-\sigma_s)l\left\{\frac{\exp(-i\pi/4)}{\mu}\sqrt{\frac{\pi l}{kn}}w_s'(a)+w_s(a)\left[\exp(-2i\sigma_s l)+\frac{1+4i\sigma_s l}{\sqrt{2\sigma_s l}}\int_0^{\sqrt{2\sigma_s l}}\exp(-i\alpha^2)\,d\alpha\right]\right\}} \quad (20)$$

determining the eigenvalues σ_q and the coefficients A_s.

Let us find an approximate solution of the characteristic equation (19). Of our interest are just those oscillations, for which the value of $\lambda_q a$ is of the order of unity – only under this condition the use of the parabolic equation (8) is justified. In our case $k\alpha \gg 1$, it is possible, if

$$k=k_q+\varkappa, \qquad k_q=\pi q/2nl, \qquad \varkappa=\lambda_q^2/2kn^2\simeq\lambda_q^2/2k_q n^2. \quad (21)$$

Here, q is a large integer number, and the correction $\varkappa a \ll 1$. Under this condition, it follows that $\sigma_q \simeq k_q(n^2-1)/2n$, and Eqs. (14) and (19) yield

$$\begin{matrix}\cos\\ \sin\end{matrix}(\lambda_q a)=\pm\frac{B(\tau)}{\mu M}\lambda_q a\begin{matrix}\sin\\ \cos\end{matrix}(\lambda_q a), \quad (22)$$

where

$$B(\tau)=\frac{\sqrt{2\sigma_q l}\int_0^{\sqrt{2\sigma_q l}}\exp(-i\alpha^2)\;d\alpha-\sqrt{2\sigma_s l}\int_0^{\sqrt{2\sigma_s l}}\exp(-i\alpha^2)\;d\alpha}{\exp(-i\tau^2)+\frac{1+2i\tau^2}{\tau}\int_0^{\tau}\exp(-i\alpha^2)\;d\alpha}, \quad (23)$$

$M = M_q = \sqrt{2k_q na^2/l} \gg 1$, and $\tau = \tau_q = \sqrt{2\sigma_q l} \simeq \sqrt{k_q l(n^2-1)/n}$. As quantity M is a large parameter and function $B(\tau)$ is bounded, an approximate formula for λ_q reads

$$\lambda_{q,m} = \frac{\pi m}{2a}(1-\delta), \qquad \delta = \delta_{q,m} = \frac{B(\tau)}{\mu M} \ll 1. \tag{24}$$

In the first case, m is an odd number, and in the second case, it is an even integer number.

From these calculations, Eq. (21) provides a complex-valued correction to the wave number,

$$k = k_q + \varkappa_{q,m}, \qquad k_q = \frac{\pi q}{2nl}, \qquad \varkappa_{q,m} \simeq \frac{1}{8\,k_q}\left(\frac{\pi m}{na}\right)^2\left[1 - 2\frac{B(\tau_q)}{\mu M_q}\right]. \tag{25}$$

The oscillations having different polarization can be studied in a similar way. If we look for the solution to the boundary problem (9)–(10) in the following form:

$$H(x,y) = \exp(iknx)\,v(x,y) \mp \exp(-iknx)\,v(-x,y), \tag{26}$$

then for the slowly varying magnetic field amplitude $v(x,y)$, we obtain the same formulas as for $u(x,y)$, just substituting ε instead of μ. The derived formulas show that the properties of a dielectric open resonator essentially depend on the dimensionless parameter $\tau \sim \tau_q = \sqrt{k_q l(n^2-1)/n}$.

For $\tau \ll 1$, the function $B(\tau)$ is close to the limiting value $B(0) = (1+i)\sqrt{\pi}/2$, and Eq. (26) takes the form

$$\varkappa_{q,m} \simeq \frac{1}{8k_q}\left(\frac{\pi m}{a}\right)^2\left(1 - e^{i\pi/4}\sqrt{\frac{\pi l}{k_q a^2}}\right) \tag{27}$$

equivalent to the Vainshtein formula for an empty open resonator [15, 21] obtained by the method of parabolic equation.

For $\tau \gg 1$, after substituting the Fresnel integrals with their asymptotic values in (25), we arrive at

$$\varkappa_{q,m} \simeq \frac{1}{8k_q}\left(\frac{\pi m}{na}\right)^2\left(1 - \frac{2}{\mu k_q a\sqrt{n^2-1}} - i\frac{\pi}{\mu k_q^2 al(n^2-1)^{3/2}}\right). \tag{28}$$

In the intermedium region, one should use the exact formula (23).

In Fig. 2, we present the real and imaginary parts of the function $B(\tau)$ allowing one to calculate the eigenfrequencies and oscillation damping for $\tau \sim 1$.

The analysis of Eq. (20) shows that, for $s \neq q$, the coefficients A_s are small compared to A_q. Nevertheless, neglecting them (corresponding to ignoring the effect of mutual transformation of the normal waves reflecting from the end face) is not entirely correct. The comparison with more accurate results of Vainshtein [15, 21], obtained by rigorous solution of Maxwell's equations, shows that formula (28) gives a somewhat overestimated diffraction loss. These results can be refined by substituting the approximate values of A_s into Eqs. (18); however, it would be difficult to obtain explicit formulas for the eigenvalues in this way.

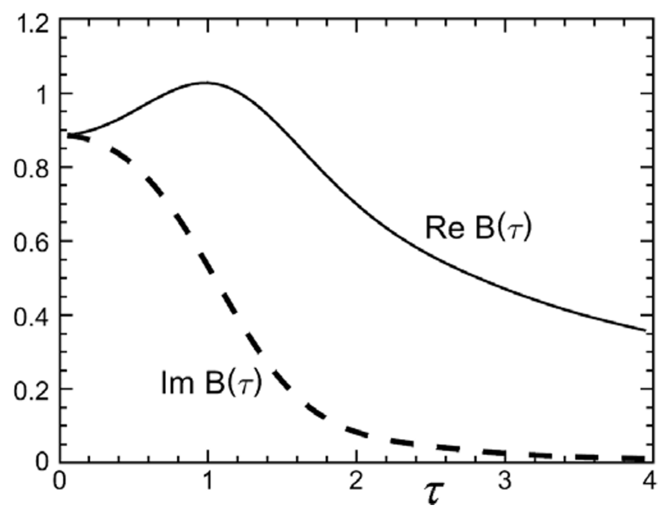

Fig. 2. Complex admittance $B(\tau)$ as a function of parameter $\tau \simeq \sqrt{kl\dfrac{n^2-1}{n}}$.

The characteristic value $l_0 \simeq n/k(n^2-1)$ corresponding to $\tau = 1$ can be interpreted as the distance at which, due to the effect of total internal reflection, a propagation mode concentrated inside the dielectric core is formed. If $l < l_0$, the presence of dielectric does not essentially influence the diffraction phenomena in the resonator. For $l > l_0$, a sharp decrease of diffraction loss takes place. For $\tau \gg 1$, the quality factor of the dielectric resonator reads

$$Q = -\frac{|k|}{\operatorname{Im} k} = \frac{4k_q^2 a^3 l\,(n^2-1)^{3/2}}{\pi^2 m^2} = \sqrt{\pi/2}\,\tau^3 Q_0, \tag{29}$$

where Q_0 is the quality of the equivalent empty resonator, with the same dimensions $2nl \times 2na$.

Consider a practical example.

In a typical case for small solid-state lasers, the dimensions of the resonator are as follows: half-length $l = 5$ cm and core radius $a = 0.5$ cm; see Fig. 1; ruby refraction index $n = 1.7$. With these parameters, the fundamental mode numbers are $m = 1$ and $q \sim 5 \cdot 10^5$; also, the quality factor reaches the values of the order of $Q = 10^{19}$, which is by eight orders of magnitude greater then $Q_0 = 10^{11}$ characteristic for an empty resonator. Equation (29) agrees with the early results obtained by Buts and Kurilko [16] under the additional assumption $n^2 - 1 \ll 1$. It gives a qualitative estimate of the ruby laser quality factor observed in the first experiments performed at the P. N. Lebedev Physical Institute [12,13]. To obtain a precise quantitative estimate of the Q factor, the factors such as material absorption, mirror leakage, and pump excitation should be taken into account. This can be done by adding small complex corrections to the mirror reflectivity and core dielectric constant [15].

4. Conclusions

The accurate analysis presented here formally confirms the experimentally proven advantage of solid-core open resonators, in the sense of small diffraction losses. The developed analytical approach can be applied to a number of realistic open structures with rotational symmetry, such as a cylindrical dielectric rod with circular translucent mirrors.

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