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KURTOSIS PARAMETER AND COUPLING COEFFICIENT OF PARTIALLY-COHERENT TWISTED GAUSSIAN BEAM PROPAGATING THROUGH NON-KOLMOGOROV ATMOSPHERIC TURBULENCE

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Abstract

We derive analytical expressions of the kurtosis (K) parameter and the coupling (C) coefficient of the partially-coherent twisted Gaussian (PCTG) beam propagating through non-Kolmogorov atmospheric turbulence, which can be given by the initial second-order moments and fourth-order moments of the Wigner distribution function for the PCTG beam at the source plane. Numerical results indicate that for the smaller generalized structure parameter, larger generalized exponent parameter, inner scale and outer scale, the K parameter and the C coefficient of the PCTG beam are less affected by the turbulence. It also can be found that the PCTG beam with larger twisted factor and smaller waist width and initial coherent lengths can resist the effect of turbulence more effectively. We also find that the coupling properties of the PCTG beam in turbulence are only caused by the turbulence. Therefore, our results can provide reference for detecting the related parameters of atmospheric turbulence.

Keywords: K parameter, C coefficient, PCTG beam, non-Kolmogorov atmospheric turbulence.

1. Introduction

Since the 1980s, Wolf et al. have studied the influence of spatial coherence of the beam on statistical characteristics in the light field, and the optical coherence theory has been well developed in the past few decades [1–3]. It was found that the partially-coherent (PC) beam exhibits some novel physical properties in comparison with the traditional fully-coherent beam. For example, PC beams can effectively overcome the speckle effect in laser nuclear fusion [4, 5], also can improve the signal-to-noise ratio and reduce the bit error rate in free-space optical communications [6, 7], and can improve the conversion efficiency of the nonlinear optical process [8,9]. In addition, PC beams have potential value in the fields of atomic cooling, particle capture, image processing, and beam shaping [10–12]. On the other hand, the studies show that the twisted phase can restrain the degeneration of PC beams in turbulence since the twisted phase was first proposed by Simon in 1993 [13–16]. It also is found that partially-coherent twisted Gaussian (PCTG) beams have great advantages in resisting the beam drift and intensity flicker caused by turbulence [15–18].

It is well known that the degree of flatness (or sharpness) of the beam intensity distribution is described by the kurtosis (K) parameter, which is related to the second-order moments and fourth-order moments [19–25]. The C coefficient is defined by the method of intensity moments in order to study the coupling characteristics of beams propagating through atmospheric turbulence [23–25]. Recently, much attention has been paid to the K parameter and the C coefficient of various beams. Analytical formulas

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of the K parameter and the C coefficient for arbitrary electromagnetic (AE) beams, Hermite–Gauss (HG) beams, Laguerre–Gauss (LG) beams, Airy beams, vortex array beams, etc. have been derived [19–27]. Moreover, lots of literatures indicate that the power spectrum of laser beam in atmospheric turbulence has the complex turbulence-spectrum models, including Kolmogorov spectrum, Tatarskii spectrum, von Karman spectrum, and non-Kolmogorov spectrum. It is found that the non-Kolmogorov spectrum is the most important spectrum model among them, because its structure parameter of the refractive index and the corresponding power spectrum are assumed to satisfy the arbitrary power laws [28–30].

Up to now, to the best of our knowledge, the K parameter and the C coefficient of a PCTG beam in non-Kolmogorov atmospheric turbulence have not been reported. Therefore, we take the PCTG beam as an illustration example, and discuss the K parameter and the C coefficient of the PCTG beam in turbulence in detail. Our results may be useful to determine the parameters of PCTB beams or to detect the parameters of atmospheric turbulence for some practical applications.

2. Kurtosis Parameter and Coupling Coefficient of the PCTG Beam

The K parameter describes the degree of flatness or sharpness of beams. The expression of the K parameter in the cylindrical coordinate system can be given by [23–25]

$$K = \frac{\langle \mathbf{r}^4 \rangle}{\left(\langle \mathbf{r}^2 \rangle\right)^2}, \tag{1}$$

where $\langle \mathbf{r}^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle$ and $\langle \mathbf{r}^4 \rangle = \langle x^4 \rangle + \langle y^4 \rangle + 2\langle x^2 y^2 \rangle$ are the second-order and fourth-order intensity moments at the received plane, respectively, [19, 23–25].

The cross-spectral density function (CSDF) of beams in non-Kolmogorov atmospheric turbulence at the received plane can be given by [23–25]

$$W(\mathbf{r}, \mathbf{r}_d; z) = \left(\frac{k}{2\pi z}\right)^2 \iint W(\mathbf{r}', \mathbf{r}'_d; 0) \exp\left\{\frac{ik}{z}[(\mathbf{r} - \mathbf{r}') \cdot (\mathbf{r}_d - \mathbf{r}'_d)] - H(\mathbf{r}_d, \mathbf{r}'_d; z)\right\} d^2\mathbf{r}'\, d^2\mathbf{r}'_d, \tag{2}$$

where $\mathbf{r}'_1 = (x'_1, y'_1)$ and $\mathbf{r}'_2 = (x'_2, y'_2)$ are the traverse vectors of arbitrary two points at the initial plane, $\mathbf{r}_1 = (x_1, y_1)$ and $\mathbf{r}_2 = (x_2, y_2)$ are the traverse vectors of arbitrary two points at the received plane. It should be noted that we have used the central abscissa coordinate systems in Eq. (2), i.e., $\mathbf{r}' = (\mathbf{r}'_1 + \mathbf{r}'_2)/2$, $\mathbf{r}'_d = \mathbf{r}'_1 - \mathbf{r}'_2$, $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, $\mathbf{r}_d = \mathbf{r}_1 - \mathbf{r}_2$, and $\mathbf{r}' = (x', y')$, $\mathbf{r}'_d = (x'_d, y'_d)$, $\mathbf{r} = (x, y)$, $\mathbf{r}_d = (x_d, y_d)$. Also z denotes the propagation distance and $k = 2\pi/\lambda$ is the wave number, with λ being the wavelength. Here, $W(\mathbf{r}', \mathbf{r}'_d; 0)$ is the CSDF of beams in the source plane ($z = 0$). The term $\exp[-H(\mathbf{r}_d, \mathbf{r}'_d; z)]$ denotes the effect of the turbulence, and H is expressed as [5, 19, 23–25]

$$H(\mathbf{r}_d, \mathbf{r}'_d; z) = 4\pi^2 k^2 z \int_0^1 d\xi \int_0^\infty \left\{1 - J_0\left[\varkappa |\mathbf{r}'_d \xi + (1 - \xi)\mathbf{r}_d|\right]\right\} \Phi_n(\varkappa) \varkappa\, d\varkappa, \tag{3}$$

where $J_0(\cdot)$ is the Bessel function of the first kind and zero order, $\Phi_n(\kappa)$ is the spatial power spectrum of the refractive index fluctuations of the turbulent atmosphere, and $\varkappa$ is the magnitude of the spatial wave number.

The form of the non-Kolmogorov atmospheric turbulence spectrum can be expressed as follows [27–30]:

$$\Phi_n(\varkappa) = \frac{A(\alpha)\tilde{C}_n^2}{(\varkappa^2+\varkappa_0^2)^{\alpha/2}} \exp\left(-\frac{\varkappa^2}{\varkappa_m^2}\right), \qquad 0 \le \varkappa < \infty, \qquad 3<\alpha<4, \tag{4}$$

where α denotes the generalized exponent parameter, $\tilde{C}_n^2$ is the generalized structure parameter with units $\mathrm{m}^{3-\alpha}$, and $\kappa_m = c(\alpha)/l_0$ and $\varkappa_0 = 2\pi/L_0$, with L_0 and l_0 being the outer and inner scales of turbulence, respectively. Also, $A(\alpha)$ and $c(\alpha)$ are given by [27–30]

$$A(\alpha) = \frac{1}{4\pi^2}\Gamma(\alpha-1)\cos(\alpha\pi/2), \qquad c(\alpha) = \left[\frac{2\pi}{3}A(\alpha)\Gamma\left(\frac{5-\alpha}{2}\right)\right]^{1/(\alpha-5)}, \tag{5}$$

where $\Gamma(\cdot)$ is the Gamma function. When $\alpha = 11/3$, we can obtain $A(11/3) = 0.033$, $c(11/3) = 5.91$, $\varkappa_m = 5.91/l_0$, and $L_0 = \infty$ then Eq. (4) reduces to the von Karman spectrum. Furthermore, the spectrum in Eq. (4) will reduce to the classical Kolmogorov spectrum when $\alpha = 11/3$, $L_0 = \infty$, and $l_0 = 0$.

According to the Fourier transform, the Wigner distribution function (WDF) of the PCTG beam propagating through non-Kolmogorov atmospheric turbulence reads [19, 23–25]

$$h(\mathbf{r},\boldsymbol{\theta};z) = \left(\frac{k}{2\pi}\right)^2 \iint W(\mathbf{r},\mathbf{r}_d;z)\exp(-ik\boldsymbol{\theta}\cdot\mathbf{r}_d)d^2\mathbf{r}_d, \tag{6}$$

where vectors $\boldsymbol{\theta} = (\theta_x,\theta_y)$, $k\theta_x$ and $k\theta_y$ are the wave vector components along the x axis and y axis, respectively. The intensity moments of the order $n_1+n_2+m_1+m_2$ of WDF for three-dimensional beams are [5, 19, 23–25]

$$\langle x^{n_1}y^{n_2}\theta_x^{m_1}\theta_y^{m_2}\rangle = \frac{1}{P}\iint x^{n_1}y^{n_2}\theta_x^{m_1}\theta_y^{m_2}h(\mathbf{r},\boldsymbol{\theta};z)\,d^2\mathbf{r}\,d^2\boldsymbol{\theta}, \tag{7}$$

where $P = \iint h(\mathbf{r},\boldsymbol{\theta};z)d^2\mathbf{r}d^2\boldsymbol{\theta}$ is the total irradiance of the PCTG beam.

Substituting Eqs. (2)–(6) into Eq. (7), we can obtain the second-order moments $\langle x^2\rangle$ and $\langle y^2\rangle$ and the fourth-order moments $\langle x^4\rangle$, $\langle y^4\rangle$, and $\langle x^2y^2\rangle$; they read

$$\langle\eta^2\rangle = \langle\eta'^2\rangle_0 + 2z\langle\eta'\theta'_\eta\rangle_0 + z^2\langle\theta'^2_\eta\rangle_0 + \frac{2}{3}z^3T_1, \tag{8}$$

$$\langle\eta^4\rangle = \left\langle(\eta'+z\theta'_\eta)^4\right\rangle_0 + 4z^3T_1\left\langle(\eta'+z\theta'_\eta)^2\right\rangle_0 + \frac{4}{3}z^6T_1^2 + \frac{3}{10k^2}z^5T_2, \tag{9}$$

$$\langle x^2y^2\rangle = \left\langle(x'+z\theta'_x)^2(y'+z\theta'_y)^2\right\rangle_0 + \frac{2}{3}z^3T_1\left[\left\langle(x'+z\theta'_x)^2\right\rangle_0 + \left\langle(y'+z\theta'_y)^2\right\rangle_0\right] + \frac{4}{9}z^6T_1^2 + \frac{1}{10k^2}z^5T_2, \tag{10}$$

where $\eta = x$ or y and $\eta' = x'$ or y'. Also T_1 and T_2 are expressed as follows [23–27] :

$$T_s = \pi^2\int_0^\infty \varkappa^{2s+1}\Phi_n(\varkappa)\,d\varkappa, \qquad s = 1, 2. \tag{11}$$

Substituting Eq. (4) into Eq. (11), we can arrive at [23–27]

$$T_1 = \frac{\pi^2A(\alpha)\tilde{C}_n^2}{2(\alpha-2)}\left\{[2\varkappa_0^2\varkappa_m^{2-\alpha} + (\alpha-2)\varkappa_m^{4-\alpha}]\exp(\varkappa_0^2/\varkappa_m^2)\Gamma'\left(2-\frac{\alpha}{2},\frac{\varkappa_0^2}{\varkappa_m^2}\right) - 2\varkappa_0^{4-\alpha}\right\}, \tag{12}$$

$$T_2 = \frac{\pi^2 A(\alpha)\tilde{C}_n^2}{2}\left\{\varkappa_0^{4-\alpha}\varkappa_m^2 - \frac{2}{2-\alpha}\varkappa_0^{6-\alpha} + \left[\left(2-\frac{\alpha}{2}\right)\varkappa_m^4 - 2\varkappa_0^2\varkappa_m^2 + \frac{2}{2-\alpha}\varkappa_0^4\right]\right.$$
$$\left. \times\, \varkappa_m^{2-\alpha}\exp\left(\frac{\varkappa_0^2}{\varkappa_m^2}\right)\Gamma'\left(2-\frac{\alpha}{2},\frac{\varkappa_0^2}{\varkappa_m^2}\right)\right\}, \tag{13}$$

where Γ' is the incomplete Gamma function.

Substituting Eqs. (8)–(10) into Eq. (1), we obtain the expression for K parameter [23–27]; it reads

$$K = \frac{1}{(\langle x^2\rangle + \langle y^2\rangle)^2}\left\{\langle x'^4\rangle_0 + \langle y^4\rangle_0 + 4z\left[\langle x'^3\theta'_x\rangle_0 + \langle y'^3\theta'_y\rangle_0\right] + 6z^2\left[\langle x'^2\theta'^2_x\rangle_0 + \langle y'^2\theta'^2_y\rangle_0\right]\right.$$
$$+4z^3\left[\langle x'\theta'^3_x\rangle_0 + \langle y'\theta'^3_y\rangle_0\right] + z^4\left[\langle\theta'^4_x\rangle_0 + \langle\theta'^4_y\rangle_0\right] + 2\left[\langle x'^2y'^2\rangle_0 + 2z\langle x'^2y'\theta'_y\rangle_0 + z^2\langle x'^2\theta'^2_y\rangle_0\right.$$
$$\left.+2z\langle x'y'^2\theta_x{}'\rangle_0 + 4z^2\langle x'y'\theta_x\theta_y\rangle_0 + 2z^3\langle x'\theta'_x\theta'^2_y\rangle_0 + z^2\langle y'^2\theta'^2_x\rangle_0 + 2z^3\langle y'\theta'^2_x\theta'_y\rangle_0 + z^4\langle\theta'^2_x\theta'^2_y\rangle_0\right]$$
$$\left.+\frac{16}{3}z^3T_1\left[\langle x'^2\rangle_0 + \langle y'^2\rangle_0 + 2z\left(\langle x'\theta'_x\rangle_0 + \langle y'\theta'_y\rangle_0\right) + z^2\left(\langle\theta'^2_x\rangle_0 + \langle\theta'^2_y\rangle_0\right)\right] + \frac{32}{9}z^6T_1^2 + \frac{4}{5k^2}z^5T_2\right\}, \tag{14}$$

where $\langle x^2\rangle$ and $\langle y^2\rangle$ are given by Eq. (8).

Equation (14) provides K parameter of arbitrary beams in non-Kolmogorov atmospheric turbulence; it depend on the initial second-order moments and fourth-order moments and turbulence quantities T_1 and T_2. When $T_1 = 0$ and $T_2 = 0$, Eq. (14) is the analytical formula of the K parameter of arbitrary beams propagating through the free space.

For the discussion of the coupling property of PCTG beam propagating through the turbulent atmosphere, we introduce the C coefficient defined as [24–27]

$$C = \frac{\langle\theta_x^2\theta_y^2\rangle}{\langle\theta_x^2\rangle\langle\theta_y^2\rangle} - 1, \tag{15}$$

with

$$\langle\theta_\eta^2\rangle = \langle\theta'^2_\eta\rangle_0 + 2T_1 z, \qquad \eta = x, y, \tag{16}$$

$$\langle\theta_x^2\theta_y^2\rangle = \langle\theta'^2_x\theta'^2_y\rangle_0 + 2zT_1\left(\langle\theta'^2_x\rangle_0 + \langle\theta'^2_y\rangle_0\right) + 4z^2T_1^2 + \frac{1}{2k^2}zT_2. \tag{17}$$

Substituting Eqs. (16) and (17) into Eq. (15), we arrive at the C coefficient [24–27]; it is

$$C = \frac{\langle\theta'^2_x\theta'^2_y\rangle_0 - \langle\theta'^2_x\rangle_0\langle\theta'^2_y\rangle_0 + k^{-2}zT_2/2}{\langle\theta'^2_x\rangle_0\langle\theta'^2_y\rangle_0 + 2zT_1\left(\langle\theta'^2_x\rangle_0 + \langle\theta'^2_y\rangle_0\right) + 4z^2T_1^2}. \tag{18}$$

Equation (18) implies the C coefficient of arbitrary beams in non-Kolmogorov atmospheric turbulence. One can see that the C coefficient is only determined by the initial second-order moments and fourth-order moments and turbulence quantities T_1 and T_2. Especially, the C coefficient remains invariant during the free-space propagation ($T_1 = 0$ and $T_2 = 0$).

Now, we will take PCTG beam as an example to study the K parameter and the C coefficient of the PCTG beam in non-Kolmogorov atmospheric turbulence.

The CSDF of PCTG beam at the initial plane ($z = 0$) based on the theoretical model elaborated reads [13–17]

$$W(\mathbf{r}', \mathbf{r}'_d; 0) = \exp\left(-\frac{2\mathbf{r}'^2}{w_0^2} - \frac{\mathbf{r}'^2_d}{2w_0^2}\right)\exp\left(-\frac{\mathbf{r}'^2_d}{2\delta_{xx}^2}\right)\exp[-ik\mu(x'_d y' - x' y'_d)]$$
$$+\exp\left(-\frac{2\mathbf{r}'^2}{w_0^2} - \frac{\mathbf{r}'^2_d}{2w_0^2}\right)\exp\left(-\frac{\mathbf{r}'^2_d}{2\delta_{yy}^2}\right)\exp[-ik\mu(x'_d y' - x' y'_d)], \quad (19)$$

where w_0 denotes the waist width of the fundamental Gaussian mode, μ is the twisted factor, δ_{xx} and δ_{yy} are the initial coherent lengths of the x component and y component of the field, respectively. It should be noted that Eq. (19) will reduce to CSDF of partially coherent Gaussian (PCG) beam as $\mu = 0$.

Letting $z = 0$, $n_1 = 4$, and $n_2 = m_1 = m_2 = 0$ in Eq. (7), in view of Eq. (9), we obtain the initial fourth-order moments of PCTG beam in the x direction as follows:

$$\langle x^4\rangle_0 = 3w_0^4/16; \quad (20)$$

similarly, the other second-order and fourth-order intensity moments are, respectively, given by

$$\langle x^2\rangle_0 = \langle y^2\rangle_0 = w_0^2/4, \quad (21)$$
$$\langle y^4\rangle_0 = \langle x^4\rangle_0 = 3\langle x^2y^2\rangle_0 = 3w_0^4/16, \quad (22)$$

and

$$\langle \theta_x^2\rangle_0 = \langle \theta_y^2\rangle_0 = u^2w_0^2/4 + (2+\varepsilon_x^2+\varepsilon_y^2)/2w_0^2k^2, \quad (23)$$
$$\langle \theta_x^4\rangle_0 = \langle \theta_y^4\rangle_0 = 3\langle \theta_x^2\theta_y^2\rangle_0 = \frac{3u^4w_0^4}{16} + \frac{3u^2}{4k^2}(2+\varepsilon_x^2+\varepsilon_y^2) + \frac{3}{2w_0^4k^4}[2+\varepsilon_x^4+\varepsilon_y^4+2(\varepsilon_x^2+\varepsilon_y^2)], \quad (24)$$
$$\langle x^2\theta_x^2\rangle_0 = \langle y^2\theta_y^2\rangle_0 = u^2w_0^4/16 + (2+\varepsilon_x^2+\varepsilon_y^2)/8k^2, \quad (25)$$
$$\langle x^2\theta_y^2\rangle_0 = \langle y^2\theta_x^2\rangle_0 = 3u^2w_0^4/16 + (2+\varepsilon_x^2+\varepsilon_y^2)/8k^2, \quad (26)$$
$$\langle xy\theta_x\theta_y\rangle_0 = -u^2w_0^4/16, \quad (27)$$
$$\langle x\theta_x\rangle_0 = \langle y\theta_y\rangle_0 = \langle x\theta_x^3\rangle_0 = \langle y\theta_y^3\rangle_0 = \langle x^3\theta_x\rangle_0 = \langle y^3\theta_y\rangle_0$$
$$= \langle x^2y\theta_y\rangle_0 = \langle xy^2\theta_x\rangle_0 = \langle x\theta_x\theta_y^2\rangle_0 = \langle y\theta_x^2\theta_y\rangle_0 = 0, \quad (28)$$

with $\varepsilon_\eta = w_0/\delta_{\eta\eta}$; $\eta = x,\ y$.

Substituting Eqs. (21)–(28) into Eq. (14), one can easily obtain the K parameter of the PCTG beam in non-Kolmogorov atmospheric turbulence.

Substituting Eqs. (23) and (24) into Eq. (18), one can easily obtain the C coefficient of the PCTG beam in non-Kolmogorov atmospheric turbulence.

3. Numerical Calculations

In Fig. 1, we present the K parameter of a PCTG beam versus the propagation distance z for different beam parameters and turbulence parameters. One can see that K first increases up to the maximum value at $z = z_0$ with increase in the transmission distance, then it gradually decreases, and

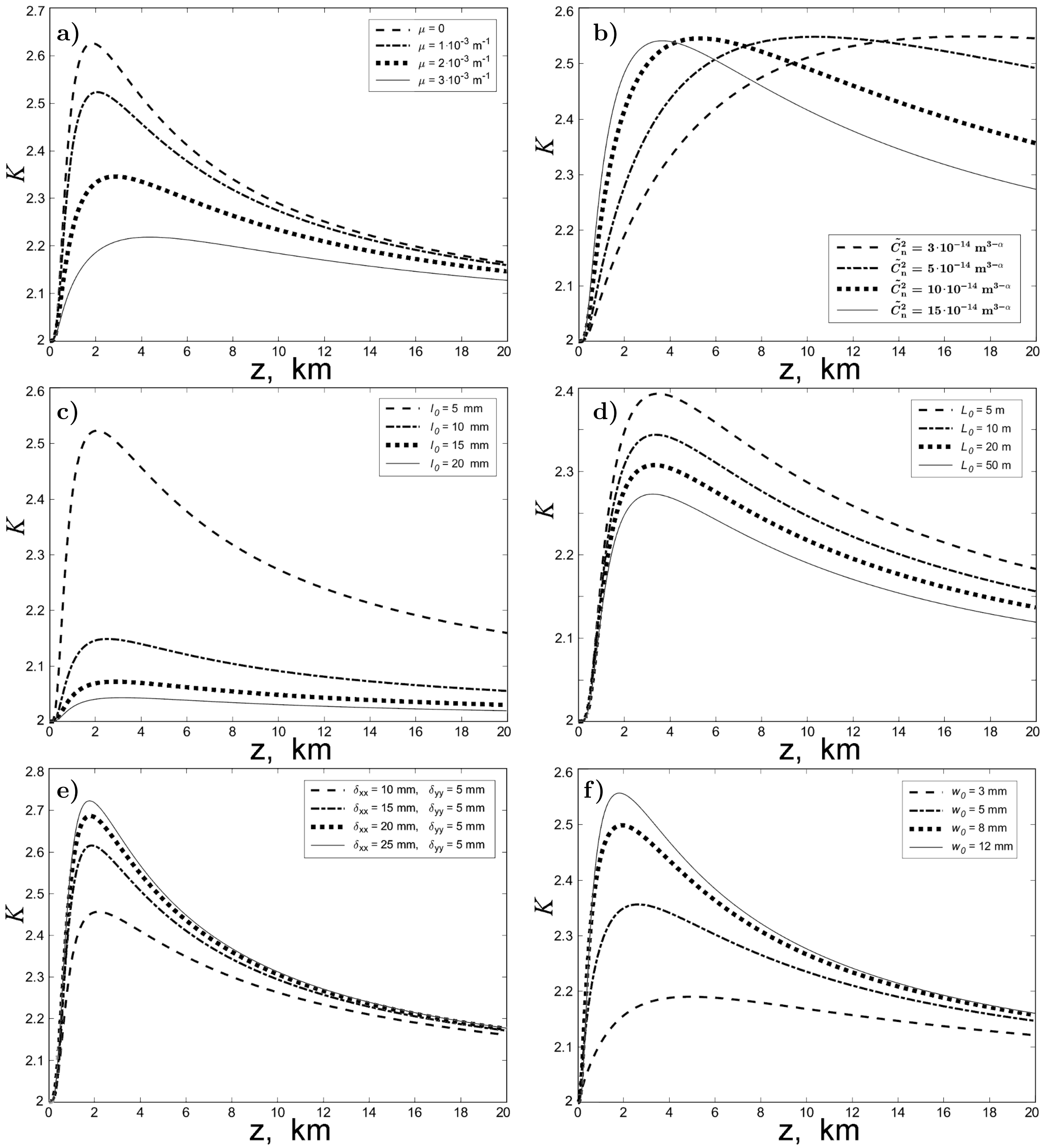

Fig. 1. Kurtosis parameter of a PCTG beam with different beam parameters and turbulence parameters in non-Kolmogorov atmospheric turbulence. Calculation parameters are $\lambda = 632.8$ nm and $\alpha = 11/3$. Here, $\tilde{C}_n^2 = 3 \cdot 10^{-13}$ m$^{3-\alpha}$, $l_0 = 5$ mm, $L_0 = 10$ m, $\delta_{xx} = \delta_{yy} = 5$ mm, and $w_0 = 20$ mm (a), $\mu = 1 \cdot 10^{-3}$ m^{-1}, $l_0 = 5$ mm, $L_0 = 10$ m, $\delta_{xx} = \delta_{yy} = 5$ mm, and $w_0 = 20$ mm (b), $\mu = 1 \cdot 10^{-3}$ m^{-1}, $\tilde{C}_n^2 = 3 \cdot 10^{-13}$ m$^{3-\alpha}$, $L_0 = 10$ m, $\delta_{xx} = \delta_{yy} = 5$ mm, and $w_0 = 20$ mm (c), $\mu = 1 \cdot 10^{-3}$ m^{-1}, $\tilde{C}_n^2 = 1 \cdot 10^{-13}$ m$^{3-\alpha}$, $l_0 = 10$ mm, $\delta_{xx} = \delta_{yy} = 10$ mm, and $w_0 = 20$ mm (d), $\mu = 1 \cdot 10^{-4}$ m^{-1}, $\tilde{C}_n^2 = 1 \cdot 10^{-13}$ m$^{3-\alpha}$, $l_0 = 10$ mm, $L_0 = 10$ m, and $w_0 = 20$ mm (e), and $\mu = 1 \cdot 10^{-3}$ m^{-1}, $\tilde{C}_n^2 = 3 \cdot 10^{-13}$ m$^{3-\alpha}$, $l_0 = 5$ mm, $L_0 = 10$ m, and $\delta_{xx} = \delta_{yy} = 5$ mm (f).

finally converges to constant 2. One can easily see from Eq. (14) that z_0 is defined as the solution of equation $\dfrac{\partial K}{\partial z} = 0$. Equation (14) also shows that $K \to 2$ as $z \to \infty$, indicating that the PCTG beam profile in non-Kolmogorov atmospheric turbulence finally converges to the fundamental Gaussian distribution [23, 24].

In Fig. 1 a, we show the influence of twisted factor μ on the K parameter. When $\mu = 0$ and $\mu = 3{\cdot}10^{-3}$ m^{-1}, the maximum values of K are 2.63 and 2.22, respectively. One can see that the maximum value of K decreases with increase in μ, and K falls down more slowly as μ increases. Therefore, K of the PCTG beam with lager μ is less affected by the turbulence.

Figure 1 b gives the influence of generalized structure parameter $\tilde{C}_n^2$ on K. Obviously, K for the PCTG beam in a stronger turbulence falls down more rapidly when $z > z_0$. It also can be found that z_0 decreases as $\tilde{C}_n^2$ increases, which means that the tendency for K to approach constant 2 is accelerated by stronger turbulence.

Figure 1 c and d show the K of the PCTG beam versus z for the different inner scale l_0 and outer scale L_0, respectively. In Fig. 1 c and d, one can find that, for the lager l_0 and L_0, the maximum value of K is smaller, and K falls down more slowly when $z > z_0$, indicating that the K parameter of the PCTG beam is less affected by the turbulence with larger l_0 and L_0.

Figure 1 e and f represent the influence of initial coherent lengths $\delta_{\eta\eta}$ ($\eta = x,\ y$) and waist width w_0 on K parameter, respectively. It can be seen that K decreases more slowly for smaller $\delta_{\eta\eta}$ and w_0. Therefore, the PCTG beam with smaller $\delta_{\eta\eta}$ and w_0 can resist the effect of turbulence on K parameter more effectively.

In Fig. 2, we show variations of the K parameter with twisted factor μ, when a PCTG beam propagates in non-Kolmogorov atmospheric turbulence under different beam parameters and turbulence parameters. One can see that the variation of K with μ is symmetrical, with the axis of symmetry being $\mu = 0$. The K parameter gets the maximum value at $\mu = 0$, then gradually decreases as $|\mu|$ increases, and finally converges to a constant 2.

Figure 2 a represents the effect of initial coherent lengths $\delta_{\eta\eta}$; $\eta = x$ or y on K. One can find that K decreases more slowly with decrease of $\delta_{\eta\eta}$, indicating that the PCTG beam with smaller $\delta_{\eta\eta}$ can resist the effect of turbulence on the K parameter more effectively.

Figure 2 b–d show variations of the K parameter of the PCTG beam with μ for various generalized structure parameter $\tilde{C}_n^2$, generalized exponent parameter α, and inner scale l_0, respectively. One can see that for the lager $\tilde{C}_n^2$ and smaller α and l_0, K falls down more rapidly. Therefore, the K parameter of the PCTG beam is more affected by the turbulence with the lager $\tilde{C}_n^2$ and smaller α and l_0.

Figure 3 gives the C coefficient of a PCTG beam in non-Kolmogorov atmospheric turbulence for different beam parameters and turbulence parameters versus the propagation distance z. One can see that C first increases up to the maximum value at $z = z_1$ as z increases, then decreases, and finally tends to stabilize. One can easily find from Eq. (18) that z_1 is defined as the solution of equation $\dfrac{\partial C}{\partial} z = 0$. From Eq. (18), we can also obtain that $C(0) = 0$ when $\delta_{xx} = \delta_{yy}$; however, $C(z) > 0$ for the PCTG beam propagating in the turbulence, meaning that there exists a coupling effect, which is only caused by turbulence.

Figure 3 a–c represent effects of the twisted factor μ, waist width w_0, and initial coherent lengths $\delta_{\eta\eta}$; $\eta = x$ or y on the C coefficient, respectively. Obviously, the maximum value of C decreases with increase in μ and decrease of w_0 and $\delta_{\eta\eta}$; for the larger μ and smaller w_0 and $\delta_{\eta\eta}$, C falls down more slowly when $z > z_1$, indicating the PCTG beam with larger μ and smaller w_0 and $\delta_{\eta\eta}$ can effectively resist the

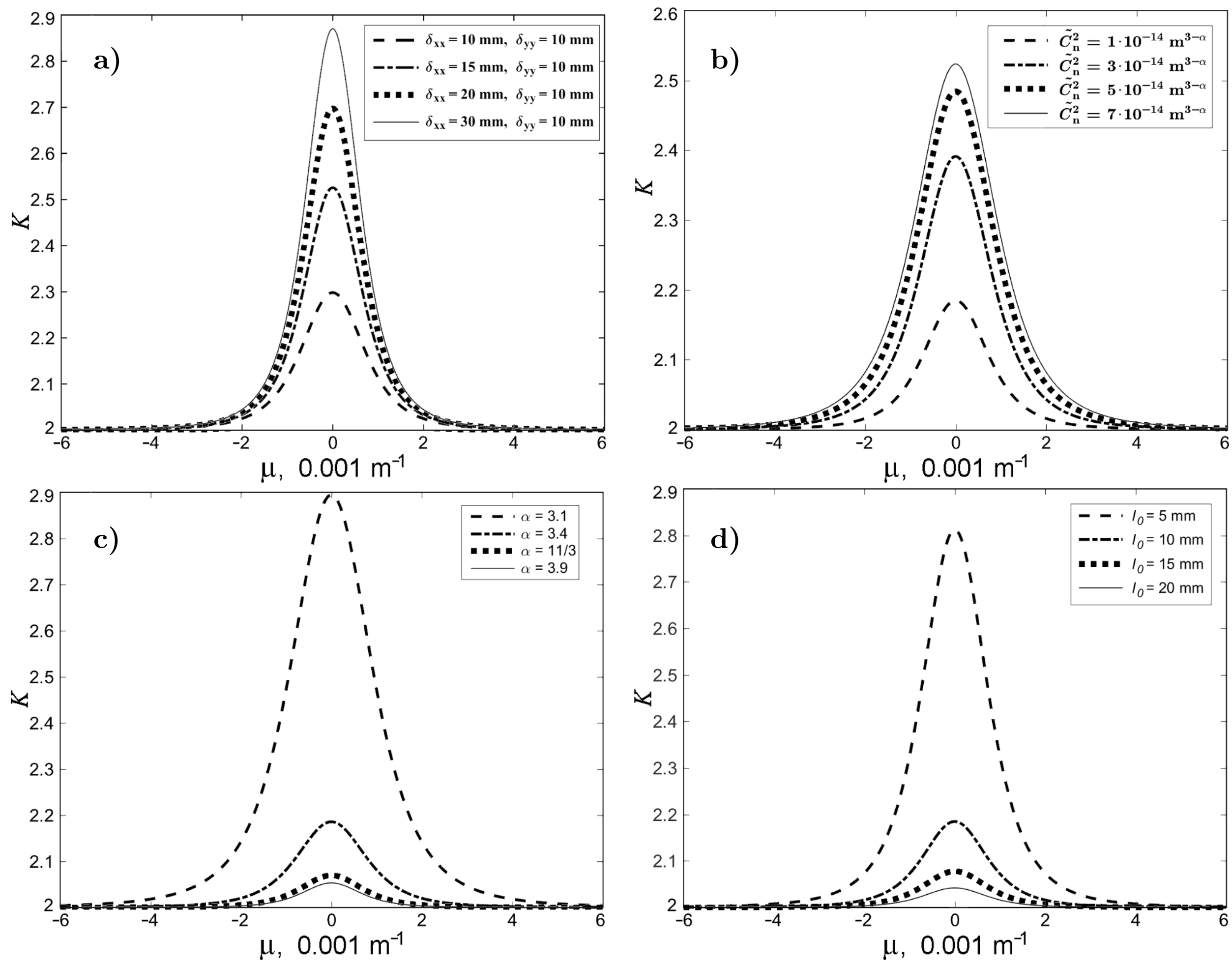

Fig. 2. Kurtosis parameter of a PCTG beam with different beam parameters and turbulence parameters in atmospheric turbulence. Calculation parameters are $\lambda = 632.8$ nm, $w_0 = 20$ mm, and $L_0 = 10$ m. Here, $\tilde{C}_n^2 = 1 \cdot 10^{-14}$ m$^{3-\alpha}$, $\alpha = 11/3$, $l_0 = 10$ mm, and $z = 3$ km (a), $\delta_{xx} = \delta_{yy} = 10$ mm, $\alpha = 11/3$, $l_0 = 10$ mm, and $z = 2$ km (b), $\delta_{xx} = \delta_{yy} = 10$ mm, $\tilde{C}_n^2 = 1 \cdot 10^{-14}$ m$^{3-\alpha}$, $l_0 = 20$ mm, and $z = 3$ km (c), and $\delta_{xx} = \delta_{yy} = 10$ mm, $\tilde{C}_n^2 = 1 \cdot 10^{-14}$ m$^{3-\alpha}$, $\alpha = 11/3$, and $z = 2$ km (d).

coupling effect caused by the turbulence.

Figure 3 d shows the relation between the C coefficient of the PCTG beam with z for various generalized structure parameter C_2^n. One can see that z_1 decreases as $\tilde{C}_n^2$ increases, and the influence of $\tilde{C}_n^2$ on the maximum value of C can be neglected, which indicates that the coupling effect due to the turbulence is stronger with increase in $\tilde{C}_n^2$.

Figure 3 e and f represent the effect of the inner scale l_0 and outer scale L_0 on the C coefficient. One obtain that for the smaller l_0 and L_0, C decreases more rapidly when $z > z_1$, meaning that the turbulence with smaller l_0 and L_0 has a stronger coupling effect on the PCTG beam.

In addition, comparing Fig. 1 with Fig. 3, one also find that the larger values of K parameter also reveal that the coupling effect due to the turbulence is stronger.

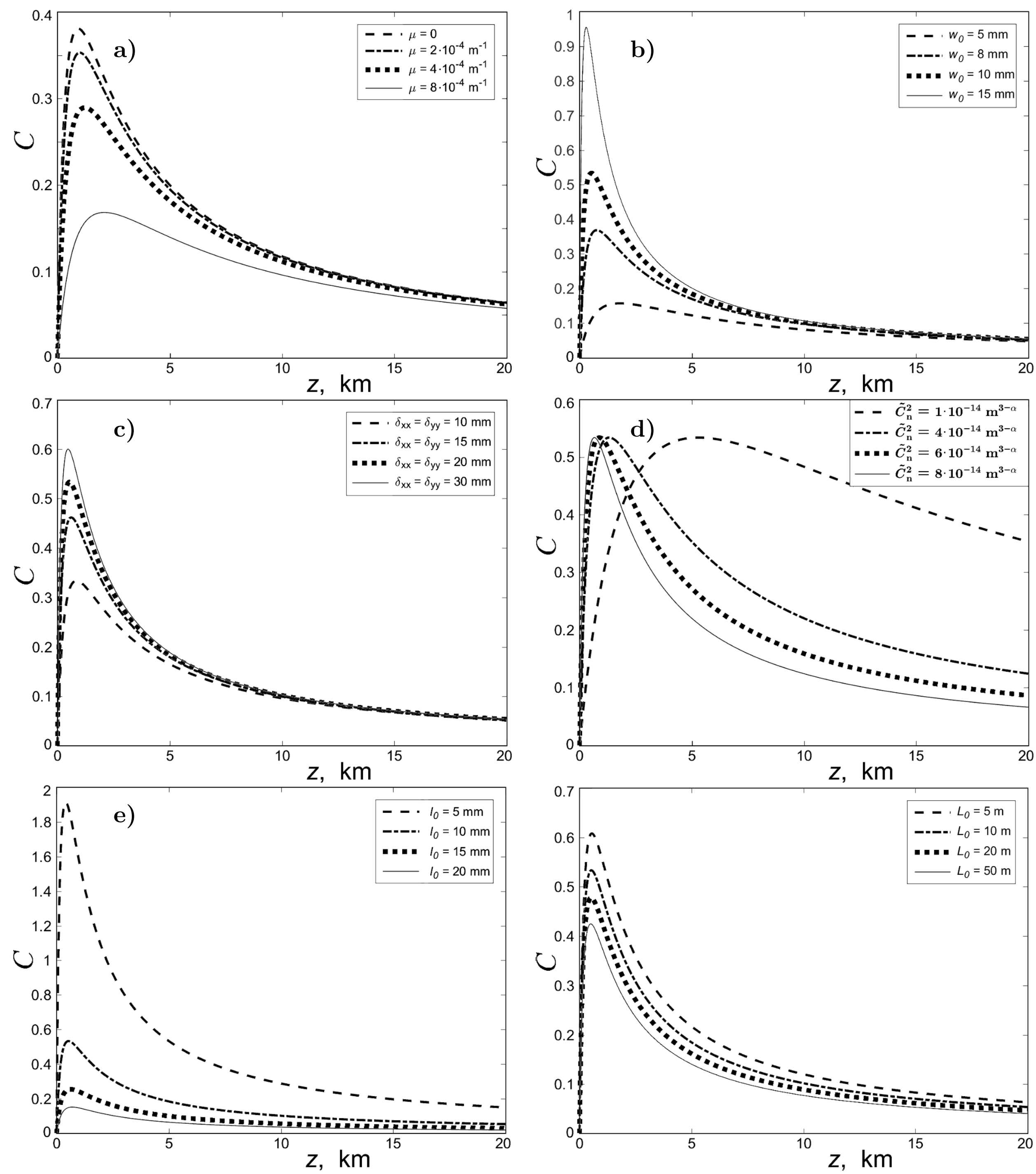

Fig. 3. Coupling coefficient of a PCTG beam with different beam parameters and turbulence parameters in atmospheric turbulence. Calculation parameters are $\lambda = 632.8$ nm and $\alpha = 11/3$. Here, $w_0 = 20$ mm, $\delta_{xx} = \delta_{yy} = 20$ mm, $\tilde{C}_n^2 = 3 \cdot 10^{-14}$ m$^{3-\alpha}$, $l_0 = 20$ mm, and $L_0 = 10$ m (a), $\mu = 1 \cdot 10^{-4}$ m^{-1}, $\delta_{xx} = \delta_{yy} = 20$ mm, $C_n^2 = 1 \cdot 10^{-13}$ m$^{3-\alpha}$, $l_0 = 10$ mm, and $L_0 = 10$ m (b), $\mu = 1 \cdot 10^{-4}$ m^{-1}, $w_0 = 10$ mm, $\tilde{C}_n^2 = 1 \cdot 10^{-13}$ m$^{3-\alpha}$, $l_0 = 10$ mm, and $L_0 = 10$ m (c), $\mu = 1 \cdot 10^{-4}$ m^{-1}, $w_0 = 10$ mm, $\delta_{xx} = \delta_{yy} = 20$ mm, $l_0 = 10$ mm, and $L_0 = 10$ m (d), $\mu = 1 \cdot 10^{-4}$ m^{-1}, $w_0 = 10$ mm, $\delta_{xx} = \delta_{yy} = 20$ mm, $\tilde{C}_n^2 = 1 \cdot 10^{-13}$ m$^{3-\alpha}$, and $L_0 = 10$ m (e), $\mu = 1 \cdot 10^{-4}$ m^{-1}, $w_0 = 10$ mm, $\delta_{xx} = \delta_{yy} = 20$ mm, $\tilde{C}_n^2 = 1 \cdot 10^{-13}$ m$^{3-\alpha}$, and l_0= 10 mm (f).

4. Summary

In this paper, we derived the analytical expressions of the K parameter and the C coefficient for a PCTG beam propagating through non-Kolmogorov atmospheric turbulence based on the extended Huygens–Fresnel principle and second-order and fourth-order moments of the Wigner distribution function. We found that the K parameter and the C coefficient of the PCTG beam in the turbulence depend on the initial second-order moments, fourth-order moments, and turbulence quantities.

The K parameter and C coefficient of the PCTG beam in non-Kolmogorov atmospheric turbulence are discussed in detail by numerical examples. Our results indicate that the K parameter of the PCTG beam has a maximum value for different beam parameters and turbulence parameters as z increases and finally converge to a constant 2 as the propagation distance $z \to \infty$. We also found that the K parameter and the C coefficient of the PCTG beam are less affected by the turbulence with smaller $\tilde{C}_n^2$ and larger α and l_0. In addition, our numerical examples also show that the PCTG beam with larger μ and smaller w_0 and $\delta_{\eta\eta}$ can resist the effect of turbulence more effectively. Our results may be useful in free-space optical communication, lidar detection, and imaging.

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