DOI 10.1007/s10946-022-10035-4
Journal of Russian Laser Research, Volume 43, Number 2, March, 2022

DISPERSION LAW AND DECREMENT OF SUPERLUMINAL SURFACE WAVE IN JOSEPHSON SANDWICH

Andrei S. Malishevskii[1]* and Sergey A. Uryupin[1,2]

[1]*Lebedev Physical Institute, Russian Academy of Sciences*
Leninskii Prospect 53, Moscow 119991, Russia

[2]*National Research Nuclear University MEPhI*
Moscow 115409, Russia

*Corresponding author e-mail: malish @ lebedev.ru

Abstract

For a radiating Josephson sandwich embedded in a dielectric, we find the dispersion law of a surface wave with frequency close to the frequency of Josephson plasmon and the decrement corresponding to this wave. Parameters of the layered structure and the range of wave numbers, for which such an electromagnetic wave exists, are specified. We find the long-wave surface wave energy losses in the radiation propagating along the normal to the sandwich surface.

Keywords: Josephson sandwich, surface wave, Josephson plasmon, radiation, dispersion law, decrement.

1. Introduction

Terahertz waves are usually understood as electromagnetic waves with frequencies from ∼0.1 to ∼10 THz [1]. Since typical values of the Josephson plasma frequency are of the order of hundreds of gigahertz, it is worthy to understand the features of electromagnetic-wave emission of the superconducting sandwich, whose frequency is close to the Josephson frequency, in order to design the terahertz range. The phenomena taking place at these frequencies have been actively studied in recent years; in particular, in the electrodynamics of Josephson junctions; see, e.g., [2–6] and references therein.

It is known that surface electromagnetic waves can propagate in systems with Josephson interactions. In [7], the propagation of surface waves in a Josephson junction of finite thickness, embedded in a dielectric, was considered and the dispersion of superluminal waves, with the phase velocity exceeding the speed of light in dielectric, was qualitatively described. In contrast to [7], in this communication, we present the quantitative study of the dispersion law of surface waves under conditions, where the effect of their emission into the medium surrounding the junction, is significant. The emission of electromagnetic waves from the lateral surfaces of the Josephson junction is due to the Vavilov–Cherenkov effect. The approach proposed in [8] to the consideration of superluminal waves allows one to describe such emission in vacuum under the conditions of resonance interaction of the generalized Swihart wave and the electromagnetic wave in vacuum. The formalism developed in [8] allows one to describe the radiation of the Josephson vortex into decelerating medium surrounding the sandwich [9]. This radiation is due to the Vavilov–Cherenkov effect. It is emitted from the whole length of the superconducting electrodes of the sandwich, and its frequency lies in the terahertz range. The consideration in [9] was carried out

Manuscript submitted by the authors in English on December 9, 2021.

for a relatively narrow region of velocities of the radiating Josephson vortex. This consideration can be generalized to other values of vortex velocities. For this purpose, it is necessary to extend the description of the regions of emission of surface electromagnetic waves. In this communication, we propose such an extension for waves with small wave numbers.

Below we consider the dispersion law and decrement of surface electromagnetic waves in a system consisting of a Josephson sandwich embedded in a dielectric. We describe the waves with frequencies close to the frequency of the Josephson plasmon and find the decrement of such waves. We determine the parameters of the system and the values of the wave numbers, for which the propagation of weakly damped surface waves, with a velocity greater than the speed of light in the dielectric, is possible and show that electromagnetic radiation into the dielectric, corresponding to these waves, is mainly directed along the normal to the sandwich surface.

2. Main Equations

In [9], the equation for the phase difference φ of the order parameter of superconducting electrodes of a long Josephson sandwich

$$\omega_J^2 \sin\varphi(z,t) + \frac{\partial^2 \varphi(z,t)}{\partial t^2} = \frac{\partial}{\partial z} \iint dz' dt' Q(z-z', t-t') \frac{\partial \varphi(z',t')}{\partial z'} \tag{1}$$

was first derived; see formula (1) in [9]. Here, ω_J is the Josephson plasma frequency. Fourier transform of the kernel Q is given by

$$Q(k,\omega) = V_S^2 \frac{\lambda\omega^2 \coth(L/\lambda) - c_m^2 \kappa}{\lambda\omega^2 \tanh(L/\lambda) - c_m^2 \kappa},$$

where V_S is the Swihart velocity of the sandwich, λ is the London magnetic-field penetration depth in superconducting sandwich electrodes, L is the sandwich electrode thickness, c_m is the speed of light in the medium surrounding the sandwich, $\kappa \equiv \sqrt{|k^2 - \omega^2/c_m^2|}\left[\Theta(c_m^2 k^2 - \omega^2) - i\Theta(\omega^2 - c_m^2 k^2)\operatorname{sgn}\omega\right]$, and Θ is the Heaviside step function.

Equation (1) allows one to describe electromagnetic fields in the sandwich, taking into account the effect of their Cherenkov radiation into the external medium. This equation provides the possibility to write down the following dispersion equation for the frequency ω and the wave number k:

$$\omega^2 = \omega_J^2 + k^2 Q(k,\omega), \tag{2}$$

where

$$Q(k,\omega) = V_S^2 \frac{\lambda\omega^2 \coth(L/\lambda) + i c_m \sqrt{\omega^2 - c_m^2 k^2}}{\lambda\omega^2 \tanh(L/\lambda) + i c_m \sqrt{\omega^2 - c_m^2 k^2}}. \tag{3}$$

This equation is valid for long superluminal surface waves traveling along the Josephson junction, for which

$$\lambda k \ll 1, \tag{4}$$

$$\omega > c_m k, \tag{5}$$

without loss of generality; also we assumed that $\operatorname{Re}\omega > 0$ and $k > 0$.

In [7], formula (3.6) was written to describe surface waves. We believe that the consequences that can be obtained from this formula differ from Eqs. (2) and (3). Indeed, in the limit $d/\lambda \ll \tanh(L/\lambda)$, where d is a half-width of the tunnel layer, the above mentioned formula of [7] looks like (2). But instead of k^2 the aforesaid formula contains the value $(k^2 - \epsilon\omega^2/c^2)$, and instead of V_S^2 it contains ϵV_S^2, which does not allow the transition to the case of bulk superconductors ($L \to \infty$). Also in the previously mentioned formula, instead of $ic_m\sqrt{\omega^2 - c_m^2 k^2}$ there is $-c\sqrt{c^2k^2 - \epsilon\omega^2}$, where ϵ is the dielectric constant of the tunnel layer. The formula under discussion of [7] contains obvious misprints. In addition to the impossibility of going to the limit of large L, this is indicated by the fact that on page 565 of [7] it talks about the wave propagation into the medium, in which the junction is embedded, if the inequality $\omega > ck$ is fulfilled. While the obvious condition for the Cherenkov radiation into the medium is $\omega > c_m k$. Note that the qualitative analysis of the dispersion curve of the radiating Josephson sandwich performed in [7] is consistent with our analysis performed below. The fact that the correct qualitative behavior of the dispersion curve and the analytical calculations [7] diverge makes it difficult to compare the analytical results of [7] with those given below.

To analyze the solutions of Eq. (2), we rewrite the value of Q in the form

$$Q(k,\omega) = Q'(k,\omega) + iQ''(k,\omega), \tag{6}$$

where

$$Q'(k,\omega) \equiv V_S^2 \frac{\lambda^2\omega^4 + c_m^2(\omega^2 - c_m^2 k^2)}{\lambda^2\omega^4 \tanh^2(L/\lambda) + c_m^2(\omega^2 - c_m^2 k^2)}, \tag{7}$$

$$Q''(k,\omega) \equiv -2\frac{V_S^2}{\sinh(2L/\lambda)} \cdot \frac{\lambda\omega^2 c_m\sqrt{\omega^2 - c_m^2 k^2}}{\lambda^2\omega^4 \tanh^2(L/\lambda) + c_m^2(\omega^2 - c_m^2 k^2)}. \tag{8}$$

3. Dispersion Law of Surface Electromagnetic Waves

One can see that formulas (2) and (3) already allowed us to draw some conclusions concerning the dispersion properties of the radiating Swihart waves [10]. However, the region on the $k-\omega$ plane, in which the wave frequency $\omega' \equiv \mathrm{Re}\,\omega$ is close to the frequency of the long-wave surface electromagnetic wave, sometimes called the Josephson plasmon,

$$\omega_{Jp}^2 \simeq \omega_J^2 + k^2 V_S^2, \tag{9}$$

was not earlier considered. The dispersion law of the radiating surface waves is described by (2), (6), and (7). One can state that these formulas describe the interaction of two waves — a Josephson plasmon (9) and an electromagnetic wave in the sandwich surrounding medium. Such electromagnetic wave has a simple dispersion law $\omega = c_m k$.

For a wave with frequency close to ω_{Jp}, in the region of relatively small wave numbers, from formulas (2), (6), and (7), we obtain

$$\omega'^2 \simeq \omega_{Jp}^2 + \frac{V_S^2 k^2}{\cosh^2(L/\lambda)} \cdot \frac{\lambda^2\omega_{Jp}^4}{\lambda^2\omega_{Jp}^4 \tanh^2(L/\lambda) + c_m^2(\omega_{Jp}^2 - c_m^2 k^2)}. \tag{10}$$

The second term in (10) is due to the effect of the electromagnetic wave in the medium surrounding the Josephson sandwich on the Josephson plasmon. Since, for the large wavelengths, in view of (5), the frequency of the Josephson plasmon is greater than the frequency of the electromagnetic wave in the medium, their interaction leads to "repulsion" upwards of the dispersion curve of the Josephson plasmon, which corresponds to the plus sign in the last expression. This effect is qualitatively described in [7].

In Fig. 1, we plot formula (10) for $\lambda/\lambda_J = 0.01$ and $v_S = c_m$, where v_S is the Swihart velocity of the sandwich with bulk electrodes. In the cases $L = \lambda$ and $L = 2\lambda$, in the wave number range shown in Fig. 1, the values ω_{Jp} and ω' are indistinguishable at the selected scale. At the same time, for sufficiently thin electrodes ($L = \lambda/2$), dependences $\omega'(k)$ and $\omega_{Jp}(k)$ (the solid and the dotted curves, respectively) diverge with increase in k (see the inset in Fig. 1), which can be interpreted as increasing interaction of Josephson plasmon and electromagnetic wave in the medium with increase in the value of k.

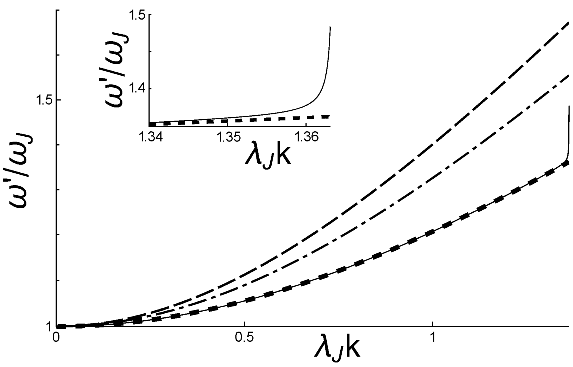

Fig. 1. Dependence of the dimensionless frequency ω'/ω_J on the dimensionless wave number $\lambda_J k$ for $L/\lambda = 2$ (the dashed curve), 1 (the dash-dotted curve), and 1/2 (the solid curve). Here, the dependence of $\omega_{Jp}(k)/\omega_J$ for $L/\lambda = 1/2$ is shown by the dotted curve. The inset shows the wave number region, in which the solid curve visibly "repulses" from the dotted curve.

Note that, if the thicknesses L of the superconducting electrodes of the sandwich are small compared to λ, and the following conditions are met

$$\frac{c_m}{\lambda} \ll \omega_J \ll \frac{c_m}{L}, \tag{11}$$

then, for the wave numbers

$$k \ll c_m/\lambda V_S, \tag{12}$$

formula (10) reads

$$\omega'^2 \simeq \omega_J^2 \left(1 + \frac{V_S^2 \lambda^2 k^2}{c_m^2}\right). \tag{13}$$

In this case, the correction to the Josephson plasmon frequency described by the second term in formula (13), $\simeq \omega_J V_S^2 \lambda^2 k^2 / 2c_m^2$, is greater than the contribution to the value ω_{Jp} from the second term in formula (9). We emphasize that the dispersion law (13) corresponds to $Q' \simeq V_S^2 \lambda^2 \omega^2 / c_m^2$. The corresponding kernel of nonlocal interaction was not considered before.

4. Decrement and Direction of Radiation

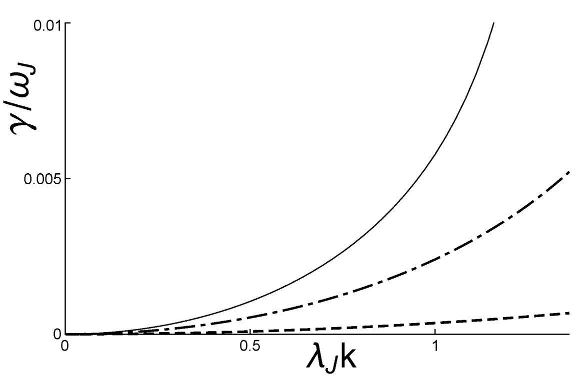

Fig. 2. Dimensionless decrement γ/ω_J versus dimensionless wave number $\lambda_J k$ for $L/\lambda = 2$ (the dashed curve), 1 (the dash-dotted curve), and 1/2 (the solid curve).

From (2), (6), and (7), we determine the surface wave decrement $\gamma \equiv -\operatorname{Im}\omega$; it reads

$$\gamma \simeq -\frac{k^2 Q''(k,\omega')}{2\omega'} > 0, \tag{14}$$

where ω' is given by expression (10).

In Fig. 2, we show the dependence (14) for $\lambda/\lambda_J = 0.01$ and $v_S = c_m$. One can see that increase in the thickness of superconducting electrodes leads to decrease in γ. This is due to a decrease of the interaction of the considered electromagnetic waves, when L increases. That is, in our consideration, the smaller the interaction of the waves, the smaller γ. Note that a wave with spectrum (13) has a decrement $\gamma \simeq v_S^2 \lambda k^2/2c_m$. The inequality $\lambda k \ll (c_m\lambda/v_s\lambda_J)^{1/2}$ should be met for the decrement to be small compared to the wave frequency. Due to restrictions (11), this inequality is stronger than inequalities (4) and (12).

Now we estimate the angle, at which the radiation propagates in the dielectric surrounding the sandwich. The wave vector component of the electromagnetic wave in the external medium, being perpendicular to the side surfaces of the sandwich, is equal to $\sqrt{(\omega/c_m)^2 - k^2}$, and the wave vector component of the surface wave along the sandwich is k. Then, the angle counted from the surface of the sandwich electrodes, at which the radiation propagates, is

$$\arctan\left(\sqrt{\frac{\omega^2}{c_m^2 k^2} - 1}\right). \tag{15}$$

Since, for the considered surface waves, the wave number k is small compared to the wave vector component of the electromagnetic wave in the medium, one can argue that the radiation propagates almost perpendicularly to the superconducting electrodes. This fact distinguishes our result from that obtained in [8], where it was shown that, near the resonance of interacting electromagnetic waves (at $k \sim \omega_J/c$), the Cherenkov radiation propagates along the sandwich surfaces. Note that for the waves, with frequency close to the frequency of the Josephson plasmon, the angle (15) decreases with increase in k. In other words, if the phase velocity of the wave decreases, the radiation pattern becomes wider. A similar phenomenon was found in [9, 11] for a radiating Josephson vortex, whose Poynting vector "presses" to the side surfaces of the sandwich, if the vortex velocity decreases.

5. Conclusions

From the above analysis follows that, in the radiating Josephson sandwich, weakly-damped surface waves can propagate with frequency close to the frequency of the Josephson plasmon. We obtained explicit dependences of the frequency and decrement of these waves on the wave number. We showed that the emission of these waves, caused by the Vavilov–Cherenkov effect, leads to narrow-directional emission into the dielectric surrounding sandwich.

References

1. X. C. Zhang and J. Xu, "THz spectroscopy and imaging," in: *Introduction to THz Wave Photonics*, Springer, Boston, MA (2010).
2. N. V. Kinev, K. I. Rudakov, L. V. Filippenko, et al., *J. Phys.: Conf. Ser.*, **1559**, 012021 (2020).
3. N. V. Kinev, K. I. Rudakov, L. V. Filippenko, et al., *Phys. Solid State*, **62**, 1543 (2020).
4. N. V. Kinev, K. I. Rudakov, L. V. Filippenko, et al., "A superconducting terahertz flux-flow oscillator: Estimation of the emission power to open space" in: *Radiation and Scattering of Electromagnetic Waves (RSEMW)* (2021), p. 235; DOI: 10.1109/RSEMW52378.2021.9494096
5. P. N. Dmitriev, A. B. Ermakov, N. V. Kinev, et al., *J. Commun. Technol. Electron.*, **66**, 473 (2021).
6. M. A. Galin, I. A. Shereshevsky, N. K. Vdovicheva, and V. V. Kurin, *Supercond. Sci. Technol.*, **34**, 075005 (2021).
7. K. L. Ngai, *Phys. Rev.*, **182**, 555 (1969).
8. K. N. Ovchinnikov, V. P. Silin, and S. A. Uryupin, *Phys. Met. Metallogr.*, **83** (1997).
9. A. S. Malishevskii, V. P. Silin, S. A. Uryupin, and S. G. Uspenskii, *Phys. Lett. A*, **372**, 712 (2008).
10. A. S. Malishevskii, V. P. Silin, S. A. Uryupin, and S. G. Uspenskii, *Low Temp. Phys.*, **36**, 728 (2010).
11. A. S. Malishevskii, V. P. Silin, S. A. Uryupin, and S. G. Uspenskii, *J. Exp. Theor. Phys.*, **107**, 263 (2008).