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SCINTILLATION INDEX OF A SPHERICAL WAVE PROPAGATING THROUGH KOLMOGOROV AND NON-KOLMOGOROV TURBULENCE ALONG LASER-SATELLITE COMMUNICATION UPLINK AT LARGE ZENITH ANGLES

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Abstract

Under the background that the Earth's aerosphere contains Kolmogorov and non-Kolmogorov turbulence, it is significant to research the combined influence of these two kinds of atmospherical turbulence on laser-satellite communication. In this paper, based on the power spectra of refractive-index fluctuations for non-Kolmogorov turbulence in the free troposphere and stratosphere, using the extended Rytov theory, the scintillation indices of the spherical wave in the free troposphere and the stratosphere are derived, respectively, which are valid in all regimes of turbulent fluctuations. On this basis, using a three-layer altitude-dependent turbulent spectrum model for vertical/slant path describing the variations of turbulent statistical characteristics with altitude in the aerosphere, which is more accurate than the two-layer model, we present the scintillation index of a spherical wave propagating through Kolmogorov and non-Kolmogorov turbulence along laser-satellite communication uplink at large zenith angles and estimate the combined influence of Kolmogorov and non-Kolmogorov turbulence on the scintillation index. It is noteworthy that this expression is also valid in all regimes of turbulent fluctuations.

Keywords: laser satellite communication, atmospheric optics, Kolmogorov turbulence, non-Kolmogorov turbulence, scintillation index.

1. Introduction

As one of the solutions of high-capacity communication, laser-satellite communication technology has attracted much attention from the scientific community in recent decades, which has potential advantages of higher data rates, smaller antennas, lower mass, lower volume, low probability of intercept, no restriction for frequency use, and less power consumption comparing to conventional radio-frequency communication technology [1,2]. Nowadays, some spacial experiments have been successfully performed, which are preparing to promote this technology to real applications [3–11]. It may be expected that a new generation of satellite communication network will appear in the near future. However, the aerosphere is one part of the communication channel for satellite-to-ground and ground-to-satellite links, therefore the atmospheric turbulence deteriorates the performance level of laser-satellite communication system [1].

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At the same time, it has been extensively accepted that there exists two kinds of atmospheric turbulence in the aerosphere, Kolmogorov and non-Kolmogorov turbulence, which has been confirmed by increasing experimental evidences and theoretical researches [12–14]. The experts and scholars at home and abroad have begun to study the optical-wave propagation in non-Kolmogorov atmospheric turbulence [15–43], in which the waveform covers the plane wave, the spherical wave, Gaussian beam, Gaussian–Schell beam, Laguerre–Gaussian beam, and so on, the atmospheric turbulent effect includes the scintillation index, the beam wander, the beam spreading, log-amplitude variance, the coherent length, the structure function, the wavefront variance, the temporal frequency spectrum, the cross-correlation function, the Strehl ratio, the AOA variance, and so forth; the fluctuation conditions also extend from the weak one to the strong one.

The theory of optical wave propagation in the atmosphere has been greatly developed; on its basis, the combined influence of Kolmogorov and non-Kolmogorov turbulence on optical wave has been investigated along slant or vertical path [44–52], and the related works also extend the two-layer atmospheric turbulence model to the three-layer one that is more accurate with further measurements in the aerosphere [53–56]. But these works are limited in the weak-fluctuation regime or for small zenith angles (45°–60°) or less along satellite-to-ground and ground-to-satellite links. For greater zenith angles, moderate-to-strong fluctuation theory must usually be applied to predict optical scintillation, but so far no tractable models have been developed for this case.

In this paper, we use the power spectra of refractive-index fluctuations with exponent values of 10/3 and 5 for non-Kolmogorov turbulence in the free troposphere and stratosphere. Employing the extended Rytov theory, we derive the scintillation indices of the spherical wave in the free troposphere and the stratosphere, respectively. Then based on a three-layer altitude-dependent power spectrum of refractive-index fluctuations for satellite-to-ground and ground-to-satellite links, we present an expression governing scintillation of a spherical wave along an uplink slant path, which is valid in all regimes of turbulence. We hope that this work lays a scientific foundation for the establishment of atmospheric turbulence compensation model and has certain scientific significance for improving the performance of satellite-ground laser communication system, on-orbit experiment, and practical applications.

2. Three-Layer Altitude Spectrum Model of Refractive-Index Fluctuations

To date, further additional measurements have shown that the aerosphere may be subdivided into several main layers: the boundary layer (up to 2–3 km), in which the turbulence is characterized by the conventional Kolmogorov model, the free troposphere (up to 8–10 km), and the stratosphere above them, where the turbulence exhibits different non-Kolmogorov characteristics. To study the combined influence of Kolmogorov and non-Kolmogorov turbulence on the laser propagation for an uplink communication channel with large zenith angles or under moderate-to-strong fluctuation conditions, we used a modified three-layer altitude spectrum model of refractive-index fluctuations [43], which has the following form:

$$\Phi_n(\varkappa,z) = \Phi_{nB}(\varkappa,z)G_{nB}(\varkappa,z)+\Phi_{nF}(\varkappa,z)G_{nF}(\varkappa,z)+\Phi_{nS}(\varkappa,z)G_{nS}(\varkappa,z), \tag{1}$$

where $\Phi_{nB}(\varkappa,z)$ is the conventional Kolmogorov refractive-index fluctuation power spectrum in the boundary layer, $\Phi_{nF}(\varkappa,z)$ is non-Kolmogorov refractive-index fluctuation power spectrum in the free troposphere, and $\Phi_{nS}(\varkappa, z)$ is non-Kolmogorov refractive-index fluctuation power spectrum in the stratosphere, which are

$$\Phi_{nB}(\varkappa, z) = 0.033\, C_n^2(z)\, \varkappa^{-11/3}, \tag{2}$$
$$\Phi_{nF}(\varkappa, z) = 0.015\, \widetilde{C}_{nF}^2(z)\, \varkappa^{-10/3}, \tag{3}$$
$$\Phi_{nS}(\varkappa, z) = 0.0024\, \widetilde{C}_{nS}^2(z)\, \varkappa^{-5}, \tag{4}$$

where $\varkappa$ is the magnitude of the spatial frequency vector [in units of rad/m], z is a propagation distance that varies between $z = 0$ and $z = L$, and $C_n^2(z)$ is the conventional Kolmogorov turbulent refractive-index structure parameter in the boundary layer that depends on the altitude and has units of $m^{-2/3}$, $\widetilde{C}_{nF}^2(z)$ is non-Kolmogorov refractive-index structure parameter in the free troposphere that is dependent on the altitude and has units of $\mathrm{m}^{-1/3}$, and $\widetilde{C}_{nS}^2(z)$ is the non-Kolmogorov refractive-index structure parameter in the stratosphere that depends on the altitude and has units of m^{-2}. Also, $G_{nB}(\varkappa, z)$ in Eq. (1) represents the spatial-frequency filter function for conventional Kolmogorov turbulence in the boundary layer, and $G_{nF}(\varkappa, z)$ and $G_{nS}(\varkappa, z)$ in Eq. (1) denote the spatial-frequency filter functions for non-Kolmogorov turbulence in the free troposphere and the stratosphere, respectively, which are adopted in a modified form

$$\begin{aligned} G_{nB}(\varkappa, z) &= G_{nxB}(\varkappa, z) + G_{nyB}(\varkappa, z) \\ &= A_{nB}(H_1, h_0) \exp\left\{-\int_0^1 D_B\left[\frac{\varkappa \rho_{0B}}{\varkappa_{xB}} w(\tau, z)\right] d\tau\right\} + \frac{B_{nB}(H_1, h_0)\varkappa^{11/3}}{\left(\varkappa^2 + \varkappa_{yB}^2\right)^{11/6}}, \end{aligned} \tag{5}$$

$$\begin{aligned} G_{nF}(\varkappa, z) &= G_{nxF}(\varkappa, z) + G_{nyF}(\varkappa, z) \\ &= A_{nF}(H_2, H_1) \exp\left\{-\int_0^1 D_F\left[\frac{\varkappa \rho_{0F}}{\varkappa_{xF}} w(\tau, z)\right] d\tau\right\} + \frac{B_{nF}(H_2, H_1)\varkappa^{10/3}}{\left(\varkappa^2 + \varkappa_{yF}^2\right)^{5/3}}, \end{aligned} \tag{6}$$

$$\begin{aligned} G_{nS}(\varkappa, z) &= G_{nxS}(\varkappa, z) + G_{nyS}(\varkappa, z) \\ &= A_{nS}(H, H_2) \exp\left\{-\int_0^1 D_S\left[\frac{\varkappa \rho_{0S}}{\varkappa_{xS}} w(\tau, z)\right] d\tau\right\} + \frac{B_{nS}(H, H_2)\varkappa^{5}}{\left(\varkappa^2 + \varkappa_{yS}^2\right)^{5/2}}, \end{aligned} \tag{7}$$

where $A_{nB}(H_1, h_0)$ and $B_{nB}(H_1, h_0)$ are weighting constants that allow for altitude variations of the structure parameters $C_n^2(h)$ on the large- and small-scale scintillations in the boundary layer, respectively, $A_{nF}(H_2, H_1)$ and $B_{nF}(H_2, H_1)$ are weighting constants that allow for altitude variations of the structure parameters $\widetilde{C}_{nF}^2(h)$ on the large- and small-scale scintillations n the free troposphere, respectively, and $A_{nS}(H, H_2)$ and $B_{nS}(H, H_2)$ are weighting constants that allow for altitude variations of the structure parameters $\widetilde{C}_{nS}^2(h)$ on the large- and small-scale scintillations in the stratosphere, respectively. Also, $\varkappa_{xB}$ and $\varkappa_{yB}$ in Eq. (5) are the large- and small-scale spatial-frequency cutoffs for the boundary layer, $\varkappa_{xF}$ and $\varkappa_{yF}$ in Eq. (6) are the large- and small-scale spatial-frequency cutoffs for the free troposphere, and $\varkappa_{xS}$ and $\varkappa_{yS}$ in Eq. (7) are the large- and small-scale spatial-frequency cutoffs for the stratosphere, which are directly related to the correlation length and scattering disk, respectively, according to

$$\frac{L}{kl_x}=\frac{1}{\varkappa_{xB}}\sim\begin{cases}\sqrt{\frac{L}{k}}, & \frac{L}{k\rho_{0B}^2}\ll 1,\\ \frac{L}{k\rho_{0B}}, & \frac{L}{k\rho_{0B}^2}\gg 1,\end{cases}\qquad l_y=\frac{1}{\varkappa_{yB}}\sim\begin{cases}\sqrt{\frac{L}{k}}, & \frac{L}{k\rho_{0B}^2}\ll 1,\\ \rho_{0B}, & \frac{L}{k\rho_{0B}^2}\gg 1,\end{cases}\tag{8}$$

$$\frac{L}{kl_x}=\frac{1}{\varkappa_{xF}}\sim\begin{cases}\sqrt{\frac{L}{k}}, & \frac{L}{k\rho_{0F}^2}\ll 1,\\ \frac{L}{k\rho_{0F}}, & \frac{L}{k\rho_{0F}^2}\gg 1,\end{cases}\qquad l_y=\frac{1}{\varkappa_{yF}}\sim\begin{cases}\sqrt{\frac{L}{k}}, & \frac{L}{k\rho_{0F}^2}\ll 1,\\ \rho_{0F}, & \frac{L}{k\rho_{0F}^2}\gg 1,\end{cases}\tag{9}$$

$$\frac{L}{kl_x}=\frac{1}{\varkappa_{xS}}\sim\begin{cases}\sqrt{\frac{L}{k}}, & \frac{L}{k\rho_{0S}^2}\ll 1,\\ \frac{L}{k\rho_{0S}}, & \frac{L}{k\rho_{0S}^2}\gg 1,\end{cases}\qquad l_y=\frac{1}{\varkappa_{yS}}\sim\begin{cases}\sqrt{\frac{L}{k}}, & \frac{L}{k\rho_{0S}^2}\ll 1,\\ \rho_{0S}, & \frac{L}{k\rho_{0S}^2}\gg 1.\end{cases}\tag{10}$$

Here, it should be noted that $\frac{L}{k\rho_{0B}^2}\ll 1$, $\frac{L}{k\rho_{0F}^2}\ll 1$, and $\frac{L}{k\rho_{0S}^2}\ll 1$ represent weak fluctuation conditions, and $\frac{L}{k\rho_{0B}^2}\gg 1$, $\frac{L}{k\rho_{0F}^2}\gg 1$, and $\frac{L}{k\rho_{0S}^2}\gg 1$ denote strong fluctuation conditions in the boundary layer, the free troposphere, and the stratosphere, respectively, while $C_n^2(z)$, $\widetilde{C}_{nF}^2(z)$, and $\widetilde{C}_{nS}^2(z)$ are constant [2], where ρ_{0B}, ρ_{0F}, and ρ_{0S} are the transverse spatial coherence radii in the boundary layer, the free troposphere, and the stratosphere, respectively, which are defined by $\rho_{0B}=\left(1.46\,k^2\mu_{0B}\right)^{-3/5}\cos^{3/5}(\zeta)$, $\rho_{0F}=\left(0.52\,\mu_{0F}k^2\right)^{-3/4}\cos^{3/4}(\zeta)$, and $\rho_{0S}=\left(0.016\,\mu_{0S}k^2\right)^{-2}\cos^2(\zeta)$, with $\mu_{0B}=\int_{h_0}^{H_1}C_n^2(h)\,dh$, $\mu_{0F}=\int_{H_1}^{H_2}\widetilde{C}_{nF}^2(h)\,dh$, and $\mu_{0S}=\int_{H_2}^{H}\widetilde{C}_{nS}^2(h)\,dh$. In addition, here, ζ is the zenith angle, h_0 is the height above ground of an optical transmitter/receiver, H_1 is the altitude of the boundary layer, H_2 is the altitude of the free troposphere, and H is the altitude of the satellite. Terms $D_B(\rho)$, $D_F(\rho)$, and $D_S(\rho)$ appearing in Eqs. (5)–(7) are the phase structure functions of optical wave propagating through Kolmogorov turbulent atmosphere in the boundary layer, non-Kolmogorov turbulent atmosphere in the free troposphere and the stratosphere; they read

$$D_B(\rho)=2.914\,\mu_{0B}k^2\rho^{5/3}\sec(\zeta)=2\left(\rho/\rho_{0B}\right)^{5/3},\tag{11}$$

$$D_F(\rho)=1.046\,\mu_{0F}k^2\rho^{4/3}\sec(\zeta)=2\left(\rho/\rho_{0F}\right)^{4/3},\tag{12}$$

$$D_S(\rho)=0.032\,\mu_{0S}k^2\rho^2\sec(\zeta)=2\left(\rho/\rho_{0S}\right)^2.\tag{13}$$

The function $w(\tau,z)$ in Eqs. (5)–(7) is related to the propagation distance according to

$$w(\tau,z)=\begin{cases}\tau\left[1-(\varepsilon z/L)\right], & \tau<z/L,\\ (1-\varepsilon\tau)\,(z/L), & \tau>z/L,\end{cases}\tag{14}$$

where τ is a normalized distance variable, $\varepsilon=0$ denotes a plane wave, and $\varepsilon=1$ represents a spherical wave.

3. The Rytov Variance and Variable C_n^2 Model

Usually the scintillation index is used to establish the scintillation fluctuations regime, which is given by the Rytov variance of a plane wave. The Rytov variances for Kolmogorov turbulence in the boundary layer and non-Kolmogorov turbulence in the free troposphere and stratosphere are expressed as

$$\sigma_R^2 = 1.23\, C_n^2 k^{7/6} L^{11/6}, \tag{15}$$
$$\sigma_{RF}^2 = 0.713\, \widetilde{C}_{nF}^2 k^{4/3} L^{5/3}, \tag{16}$$
$$\sigma_{RS}^2 = 0.063\, \widetilde{C}_{nS}^2 k^{1/2} L^{5/2}. \tag{17}$$

The propagation along a vertical or slant path requires a $C_n^2(h)$ profile model to describe properly the varying strength of optical turbulence as a function of altitude h. For the $C_n^2(h)$ profile, the most widely used Hufnagel–Valley model is chosen [2],

$$C_n^2(h) = 0.00594\,(v/27)^2 (10^{-5}h)^{10} \exp(-h/1000) + 2.7\cdot 10^{-16} \exp(-h/1500) + C_n^2(0)\exp(-h/100), \tag{18}$$

with the wind speed $v = 21$ m/s and $C_n^2(0) = 1.7\cdot 10^{-14}\,\mathrm{m}^{-2/3}$. For the $\widetilde{C}_{nF}^2(h)$ profile in the free troposphere, this form is taken as suggested in [55],

$$\widetilde{C}_{nF}^2(h) = 2.2\,(k/L_1)^{-1/6}\, C_n^2(h), \tag{19}$$

where L_1 is the propagation distance in the free troposphere and k denotes the optical wave number. For the $\widetilde{C}_{nS}^2(h)$ profile in the stratosphere, this form is taken as suggested in [55],

$$\widetilde{C}_{nS}^2(h) = 13.75\,(k/L_2)^{2/3}\, C_n^2(h), \tag{20}$$

where L_2 is the propagation distance in the stratosphere.

4. Uplink Channel

It is well known that the atmospheric turbulence in the aerosphere causes irradiance scintillations that induce deep random fades and consequently results in performance degradation of laser-satellite communication systems. In the optical wave case for an uplink path from the ground, usually the diverged beam received on the satellite is accurately modeled by a spherical wave. Following [57], the scintillation index for a spherical wave propagating through Kolmogorov turbulence in the boundary layer reads

$$\sigma_{I(B)}^2 = \exp\left[0.695 B_{nB}\,(H_1,h_0)\,\frac{\mu_{0B}}{\mu_{1B}}\sigma_{RB}^2\eta_{yB}^{-5/6} + 0.462 A_{nB}(H_1,h_0)\frac{\mu_{2B}}{\mu_{1B}}\sigma_{RB}^2\eta_{xB}^{7/6}\right] - 1, \quad 0 \le \sigma_R^2 < \infty, \tag{21}$$

where σ_{RB}^2 is the normalized variance of irradiance for Kolmogorov turbulence in the boundary layer, which is given by $\sigma_{RB}^2 = 2.25\,\mu_{1B} k^{7/6}(H_1-h_0)^{5/6}\sec^{11/6}(\zeta)$. In addition, $A_{nB}\,(H_1,h_0) = 9.31\,\mu_{1B}/\mu_{2B}$, $B_{nB}\,(H_1,h_0) = 4.54\,\mu_{1B}/\mu_{0B}$, $\mu_{1B} = \int_{h_0}^{H_1} C_n^2(h)\,[(h-h_0)/(H_1-h_0)]^{5/6}\,[1-(h-h_0)/(H_1-h_0)]^{5/6}\,dh$, and $\mu_{2B} = \int_{h_0}^{H_1} C_n^2(h)\xi^{-1/3}(1-\xi)^{1/3}\,dh$, while $\eta_{xB} = \dfrac{0.156}{1+0.62\,(\mu_{1B}/\mu_{2B})^{6/7}(\mu_{0B}/\mu_{1B})^{6/5}\sigma_R^{12/5}}$ and $\eta_{yB} = 8.92\,(1+0.69\,\sigma_{RB}^{12/5})$. Also, ξ takes the form $\xi = 1-z/L = 1-(h-h_0)/(H-h_0)$ for a uplink communication channel, and finally the scintillation index for arbitrary zenith angle and ground-level strength of turbulence over uplink optical communication channel portion in the boundary layer is described by

$$\begin{aligned}\sigma_{I(B)}^2 &= \exp\left\{\frac{0.49\sigma_{RB}^2}{\left[1+0.62(\mu_{1B}/\mu_{2B})^{6/7}(\mu_{0B}/\mu_{1B})^{6/5}\sigma_R^{12/5}\right]^{7/6}} + \frac{0.51\sigma_{RB}^2}{\left(1+0.69\sigma_{RB}^{12/5}\right)^{5/6}}\right\} - 1, \\ &0 \le \sigma_{RB}^2 < \infty. \end{aligned} \tag{22}$$

In order to find the large-scale log-amplitude variance of a spherical wave in the free troposphere, we first apply Eqs. (12) and (14) with $\varepsilon = 1$ to simplify the low-pass spatial filter for non-Kolmogorov turbulent power in the free troposphere, i.e.,

$$\begin{aligned} G_{nxF}(\varkappa, z) &= A_{nF}(H_2, H_1)\exp\left\{-\int_0^1 D_F\left[\frac{\varkappa\rho_0}{\varkappa_{xF}} w(\tau, z)\right] d\tau\right\} \\ &= A_{nF}(H_2, H_1)\exp\left[-\frac{6}{7}\left(\frac{\varkappa}{\varkappa_{xF}}\right)^{4/3}\xi^{4/3}(1-\xi)^{4/3}\right]. \end{aligned} \tag{23}$$

Following the approach of [2], using Eqs. (3) and (23) and the geometrical-optics approximation, we obtain the large-scale log-amplitude variance of a spherical wave in the free troposphere; it is

$$\sigma_{\mathrm{lnxF}}^2 = 0.806\,A_{nF}(H_2, H_1)\mu_{2F}k^{4/3}(H_2-H_1)^{2/3}\sec^{5/3}(\zeta)\eta_{xF}^{4/3}, \tag{24}$$

where $\eta_{xF} = L\varkappa_{xF}^2/k$ and

$$\mu_{2F} = \int_{H_1}^{H_2} \widetilde{C}_{nF}^2(h)\xi^{-2/3}(1-\xi)^{-2/3}\,dh. \tag{25}$$

Along uplink for non-Kolmogorov turbulence in the free troposphere, the non-Kolmogorov–Rytov variance in the free troposphere reads

$$\sigma_{RF}^2 = 1.19\,\mu_{1F}k^{4/3}(H_2-H_1)^{2/3}\sec^{5/3}(\zeta), \tag{26}$$

where

$$\mu_{1F} = \int_{H_1}^{H_2} \widetilde{C}_{nF}^2(h)\left(\frac{h-H_1}{H_2-H_1}\right)^{2/3}\left(1-\frac{h-H_1}{H_2-H_1}\right)^{2/3} dh. \tag{27}$$

Then we rewrite σ_{lnxF}^2 as

$$\sigma_{\mathrm{lnxF}}^2 = 0.677 A_{nF}(H_2, H_1)\,(\mu_{2F}/\mu_{1F})\,\sigma_{RF}^2\eta_{xF}^{4/3}, \tag{28}$$

where we set $A_{nF}(H_2, H_1) = 18.07\,\mu_{1F}/\mu_{2F}$, in which $A_{nF}(H_2, H_1) = 1$ when $\widetilde{C}_{nF}^2$ is constant. To determine the cutoff spatial wave number $\varkappa_{xF}$, we adopt the asymptotic theory according to

$$\frac{1}{\varkappa_{xF}^2} = c_{1F}\frac{L}{k} + c_{2F}\left(\frac{L}{k\rho_{0F}}\right)^2. \tag{29}$$

In order to calculate the scaling constant c_{1F}, we use the asymptotic behavior under weak fluctuations. Imposing the condition

$$\sigma^2_{\rm lnxF} = 0.677\, A_{nF}(H_2, H_1) \left(\frac{\mu_{2F}}{\mu_{1F}}\right) \sigma^2_{RF}\eta^{4/3}_{xF} \simeq 0.49\, \sigma^2_{RF}, \qquad \sigma^2_{RF} \ll 1, \tag{30}$$

we carry out $c_{1F} = 11.17$. To calculate the scaling constant c_{2S}, we use the asymptotic behavior under strong fluctuation conditions.

Imposing the asymptotic result $A_{nF}(H_2, H_1)\eta^{4/3}_{xF} = \left[k\rho^2_0/L\right]^{4/3} = \left[3.47\,(\mu_{1F}/\mu_{0F})^{3/2}\sigma^{-3}_{RS}\right]^{4/3}$, in view of the expression $L/k\rho^2_0 = 0.289\,(\mu_{0F}/\mu_{1F})^{3/2}\sigma^3_{RS}$, we arrive at

$$\eta_{xF} = \frac{0.09}{1 + 0.23\,(\mu_{1F}/\mu_{2F})^{3/4}(\mu_{0F}/\mu_{1F})^{3/2}\sigma^3_{RS}}. \tag{31}$$

Then it follows that

$$\sigma^2_{\rm lnxF} = \frac{0.49\,\sigma^2_{RF}}{\left[1 + 0.23\,(\mu_{1F}/\mu_{2F})^{3/4}(\mu_{0F}/\mu_{1F})^{3/2}\sigma^3_{RF}\right]^{4/3}}\,. \tag{32}$$

Similarly, using Eqs. (3) and the second item of Eq. (6), we obtain the small-scale log-amplitude variance of a spherical wave in the free troposphere; it is

$$\sigma^2_{\rm lnyF} = 1.776\, B_{nF}(H_2, H_1)\mu_{0F}k^{4/3}\,(H_2 - H_1)^{2/3} \sec^{5/3}(\zeta)\eta^{-2/3}_{yF}, \tag{33}$$

where $\eta_{yF} = L\varkappa^2_{yF}/k$. Using Eq. (26), we rewrite the above expression as

$$\sigma^2_{\rm lnyF} = 1.494\, B_{nF}(H_2, H_1)\,(\mu_{0F}/\mu_{1F})\,\sigma^2_{RF}\eta^{-2/3}_{yF}. \tag{34}$$

Choosing $B_{nF}(H_2, H_1) = 0.29\,\mu_{1F}/\mu_{0F}$, we reduce Eq. (34) to $\sigma^2_{\rm lnyF} = 0.44\,\sigma^2_{RF}\eta^{-2/3}_{yF}$, which is the small-scale scintillation index for a spherical wave propagating along the horizontal non-Kolmogorov turbulence path in the free troposphere.

Similar to the Kolmogorov case in the boundary layer, we determine the scaling constants for the small-scale scintillation index, $c_{3F} = 0.80$ and $c_{4F} = 0.19$; this means that

$$\eta_{yF} = \frac{L\varkappa^2_{yF}}{k} = 0.80\left(1 + 0.13\,\sigma^3_{RF}\right). \tag{35}$$

As a result, Eq. (34) becomes

$$\sigma^2_{\rm lnyF} = \frac{0.51\sigma^2_{RF}}{\left(1 + 0.13\,\sigma^3_{RF}\right)^{2/3}}\,. \tag{36}$$

Finally, the scintillation index for arbitrary zenith angle and ground-level strength of turbulence over a uplink optical communication channel portion in the free troposphere is described by

$$\begin{aligned} \sigma^2_{I(F)} &= \exp\left\{\frac{0.49\,\sigma^2_{RF}}{\left[1 + 0.23\,(\mu_{1F}/\mu_{2F})^{3/4}(\mu_{0F}/\mu_{1F})^{3/2}\sigma^3_{RF}\right]^{4/3}} + \frac{0.51\,\sigma^2_{RF}}{\left(1 + 0.13\sigma^3_{RF}\right)^{2/3}}\right\} - 1, \\ & \quad 0 \le \sigma^2_{RF} < \infty. \end{aligned} \tag{37}$$

Similar to the case of the free troposphere, in order to obtain the large-scale log-amplitude variance of a spherical wave in the stratosphere, first we apply Eqs. (13) and (14) with $\varepsilon = 1$ to simplify the low-pass spatial filter for non-Kolmogorov turbulent power in the stratosphere, i.e.,

$$\begin{aligned} G_{nxS}(\varkappa, z) &= A_{nS}(H, H_2) \exp\left\{-\int_0^1 D_S\left[\frac{\varkappa\rho_0}{\varkappa_{xS}} w(\tau, z)\right] d\tau\right\} \\ &= A_{nS}(H_2, H_1) \exp\left[-\frac{2}{3}\left(\frac{\varkappa}{\varkappa_{xS}}\right)^2 \xi^2 (1-\xi)^2\right]. \end{aligned} \tag{38}$$

Using the geometrical-optics approximation, in view of Eqs. (4) and (38), we arrive at the large-scale log-amplitude variance of a spherical wave in the stratosphere; it is

$$\sigma^2_{\mathrm{lnxS}} = 0.104\, A_{nS}(H, H_2)\mu_{2S} k^{1/2} (H - H_2)^{3/2} \sec^{5/2}(\zeta)\eta_{xS}^{1/2}, \tag{39}$$

where $\eta_{xS} = L\varkappa^2_{xS}/k$ and

$$\mu_{2S} = \int_{H_2}^{H} \widetilde{C}^2_{nS}(h)\xi(1-\xi)\, dh. \tag{40}$$

Along uplink for non-Kolmogorov turbulence in the stratosphere, the non-Kolmogorov–Rytov variance in the stratosphere is redefined by

$$\sigma^2_{RS} = 0.159\, \mu_{1S} k^{1/2} (H - H_2)^{3/2} \sec^{5/2}(\zeta), \tag{41}$$

where

$$\mu_{1S} = \int_{H_2}^{H} \widetilde{C}^2_{nS}(h)\left(\frac{h - H_2}{H - H_2}\right)^{3/2}\left(1 - \frac{h - H_2}{H - H_2}\right)^{3/2} dh. \tag{42}$$

Then we represent σ^2_{lnxS} as follows:

$$\sigma^2_{\mathrm{lnxS}} = 0.655\, A_{nS}(H, H_2)\,(\mu_{2S}/\mu_{1S})\,\sigma^2_{RS}\eta^{1/2}_{xS}, \tag{43}$$

where we set $A_{nS}(H, H_2) = 2.264\,\mu_{1S}/\mu_{2S}$, in which $A_{nS}(H, H_2) = 1$ when $\widetilde{C}^2_{nS}$ is constant. To determine the cutoff spatial wave number $\varkappa_{xS}$, we employ the asymptotic theory according to

$$\frac{1}{\varkappa^2_{xS}} = c_{1S}\frac{L}{k} + c_{2S}\left(\frac{L}{k\rho_{0S}}\right)^2. \tag{44}$$

Also, we use the asymptotic behavior under weak fluctuations to calculate the scaling constant c_{1S}. Imposing the condition

$$\sigma^2_{\mathrm{lnxS}} = 0.655\, A_{nS}(H, H_2)\,(\mu_{2S}/\mu_{1S})\,\sigma^2_{RS}\eta^{1/2}_{xF} \simeq 0.49\,\sigma^2_{RS}, \qquad \sigma^2_{RS} \ll 1, \tag{45}$$

we carry out $c_{1S} = 9.16$. At the same time, we use the asymptotic behavior under strong fluctuations to calculate the scaling constant c_{2S}.

Imposing the asymptotic result $A_{nS}(H, H_2)\eta^{1/2}_{xF} = \left[(L/k)^5(k\rho_0^2/L)\right]^{1/2} = \left[9.94^4(\mu_{1S}/\mu_{0S})^4\sigma^{-8}_{RS}\right]^{1/2}$, where the expression $(L/k)^5(L/k\rho_0^2) = (1/9.94)^4(\mu_{0S}/\mu_{1S})^4\sigma^8_{RS}$ is applied, we arrive at

$$\eta_{xS} = \frac{0.11}{1 + 0.57\cdot 10^{-4}(\mu_{1S}/\mu_{2S})^2(\mu_{0S}/\mu_{1S})^4\sigma^8_{RS}}. \tag{46}$$

Consequently, we rewrite Eq. (45) as follows:

$$\sigma^2_{\rm lnxS} = \frac{0.49\,\sigma^2_{RS}}{\left[1 + 0.57\cdot 10^{-4}(\mu_{1S}/\mu_{2S})^2(\mu_{0S}/\mu_{1S})^4\sigma^8_{RS}\right]^{1/2}}\,. \tag{47}$$

Using Eqs. (4) and the second item of Eq. (7), we obtain the small-scale log-amplitude variance of a spherical wave in the stratosphere; it is

$$\sigma^2_{\rm lnyS} = 0.063\,B_{nS}(H,H_2)\mu_{0S}k^{1/2}\,(H-H_2)^{3/2}\sec^{5/2}(\zeta)\eta_{yS}^{-3/2}, \tag{48}$$

where $\eta_{yS} = L\varkappa^2_{yS}/k$. Then, in view of Eq. (41), we rewrite Eq. (48) as

$$\sigma^2_{\rm lnyS} = 0.396\,B_{nS}(H,H_2)\,(\mu_{0S}/\mu_{1S})\,\sigma^2_{RS}\eta_{yS}^{-3/2}. \tag{49}$$

Choosing $B_{nS}(H,H_2) = 13.58\,\mu_{1S}/\mu_{0S}$, we reduce Eq. (49) to $\sigma^2_{\rm lnyS} = 5.38\,\sigma^2_{RS}\eta_{yS}^{-3/2}$, which is the small-scale scintillation index for a spherical wave propagating along the horizontal non-Kolmogorov turbulence path in the stratosphere.

Similarly, the scaling constants are determined for the small-scale scintillation index, $c_{3S} = 4.81$ and $c_{4S} = 2.11$; it follow that

$$\eta_{yS} = \frac{L\varkappa^2_{yS}}{k} = 4.81\left(1 + 0.82\,\sigma^{4/3}_{RS}\right). \tag{50}$$

As a consequence, Eq. (49) becomes

$$\sigma^2_{\rm lnyS} = \frac{0.51\sigma^2_{RS}}{(1+0.82\sigma^{4/3}_{RS})^{3/2}}\,. \tag{51}$$

Finally, the scintillation index for arbitrary zenith angle and ground-level strength of turbulence over a uplink optical communication channel portion in the stratosphere is described by

$$\begin{aligned}\sigma^2_{I(S)} \quad &= \quad \exp\left\{\frac{0.49\,\sigma^2_{RS}}{\left[1 + 0.57\cdot 10^{-4}(\mu_{1S}/\mu_{2S})^2(\mu_{0S}/\mu_{1S})^4\sigma^8_{RS}\right]^{1/2}} + \frac{0.51\,\sigma^2_{RS}}{\left(1+0.82\,\sigma^{4/3}_{RS}\right)^{3/2}}\right\} - 1,\\ &\qquad 0 \le \sigma^2_{RF} < \infty.\end{aligned} \tag{52}$$

According to Eq. (1), we can conclude that the scintillation index over the whole satellite-to-ground uplink for arbitrary zenith angle and ground-level strength of turbulence is expressed as the sum of scintillation indices caused by the turbulence in the boundary layer, the free troposphere, and the stratosphere,

$$\begin{aligned}\sigma^2_I \;&=\; \sigma^2_{I(B)} + \sigma^2_{I(F)} + \sigma^2_{I(S)}\\ &=\; \exp\left[\frac{0.49\sigma^2_{RB}}{\left[1+0.62(\mu_{1B}/\mu_{2B})^{6/7}(\mu_{0B}/\mu_{1B})^{6/5}\sigma_R^{12/5}\right]^{7/6}}\frac{0.51\sigma^2_{RB}}{(1+0.69\sigma^{12/5}_{RB})^{5/6}}\right] - 1\\ &\quad + \exp\left\{\frac{0.49\sigma^2_{RF}}{[1+0.23(\mu_{1F}/\mu_{2F})^{3/4}(\mu_{0F}/\mu_{1F})^{3/2}\sigma^3_{RF}]^{4/3}} + \frac{0.51\sigma^2_{RF}}{(1+0.13\sigma^3_{RF})^{2/3}}\right\} - 1\\ &\quad + \exp\left\{\frac{0.49\sigma^2_{RS}}{[1+0.57\cdot 10^{-4}(\mu_{1S}/\mu_{2S})^2(\mu_{0S}/\mu_{1S})^4\sigma^8_{RS}]^{1/2}} + \frac{0.51\sigma^2_{RS}}{(1+0.82\sigma^{4/3}_{RS})^{3/2}}\right\} - 1.\end{aligned} \tag{53}$$

5. Simulation and Analysis

For laser-satellite communication uplink with the transmitter position on the ground and the receiver installed on the satellite, using the three-layer scintillation index model, Eq. (53), we present the scintillation index as a function of zenith angles in Fig. 1 a for $A = 1.7 \cdot 10^{-14}$ m$^{-2/3}$ and in Fig. 1 b for $A = 1.7 \cdot 10^{-13}$ m$^{-2/3}$, respectively, taking $h_0 = 0$ and $\lambda = 1.55$ μm. For comparison, the scintillation indices described by the conventional Kolmogorov theoretical model are also shown in Fig. 1. One can see that the scintillation index increases with increase in the zenith angle, and the result predicted by the three-layer theoretical model is smaller than one predicted by the conventional theory, this is due to weaker fluctuations of non-Kolmogorov turbulence in the free troposphere and the stratosphere than for the case of the conventional Kolmogorov theoretical model. The difference between the scintillation index predicted by the three-layer theoretical model and the one predicted by the conventional Kolmogorov model decreases lightly with zenith angles. Furthermore, as shown in Fig. 1 a, the index value is less than unity at a zenith angle of 30° and does not still reach the strong region of the fluctuation conditions, this is due to the fact that the scintillation index for uplink is smaller than for downlink. The three-layer model weak-fluctuation theory is valid for zenith angles larger than 70° or 80°, while the conventional weak-fluctuation theory is restricted to around zenith angles of 70° for different ground turbulent conditions.

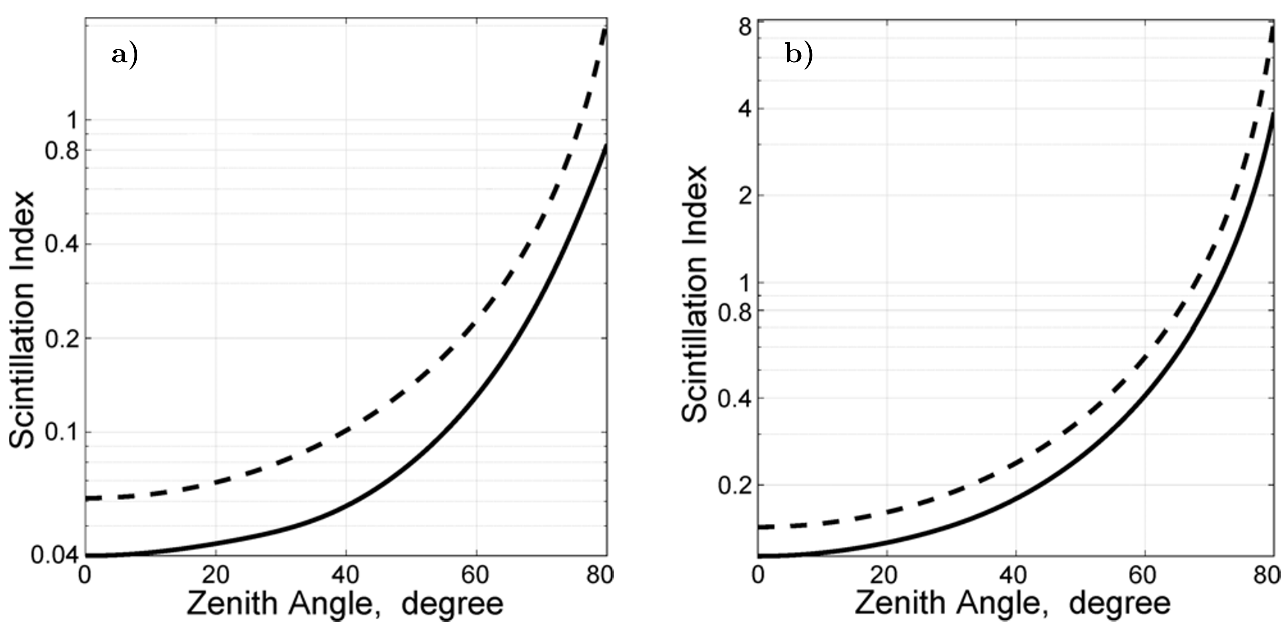

Fig. 1. Scintillation index of an uplink optical wave with the wavelength $\lambda = 1.55$ μm to a receiver on the satellite as a function of zenith angle and the value of ground level $A = 1.7 \cdot 10^{-14}$ m$^{-2/3}$ (a) and $A = 1.7 \cdot 10^{-13}$ m$^{-2/3}$ (b). Here, solid curves correspond to the three-layer model and dashed curves, to the conventional Kolmogorov model.

6. Conclusions

In this paper, based on the power spectrum of refractive-index fluctuations with an exponent value of 10/3 for non-Kolmogorov turbulence in the free troposphere and the power spectrum of refractive-index fluctuations with an exponent value of 5 for non-Kolmogorov turbulence in the stratosphere, using the extended Rytov theory, we derived the scintillation indices of the spherical wave in the free troposphere and the stratosphere, respectively. Utilizing a three-layer altitude-dependent power spectrum of refractive-index fluctuations, the scintillation index of a spherical wave propagating through Kolmogorov and non-Kolmogorov turbulence along laser-satellite communication uplink is obtained, which is applied to characterize the intensity fluctuations for large zenith angles. Finally, we used the expression obtained to analyze the variations of scintillation index with zenith angles. It is noteworthy that we obtained the expression valid in all regimes of turbulent fluctuations.

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