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SQUEEZING AND ENTANGLEMENT PROPERTIES OF THE CAVITY LIGHT WITH DECOHERENCE IN A CASCADE THREE-LEVEL LASER

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Abstract

We consider a three-level laser coupled to a two-mode thermal reservoir via a single port mirror. Applying the master equation, we study the effect of decoherence on the squeezing, entanglement, and statistical properties of a two-mode cavity light at a steady state. It turns out that the two-mode thermal reservoir substantially degrades the degree of squeezing and entanglement. However, it significantly enhances the mean number of the photon pairs and intensity difference.

Keywords: decoherence, quadrature squeezing, entanglement, mean number of photon pairs.

1. Introduction

During the past few years, the quantum analysis of light generated by three-level cascade lasers has been studied theoretically by several authors in connection with its potential as a source of squeezed and entangled light [1–11]. In three-level lasers, a cascade configuration of three-level atoms are continuously injected into the cavity with a constant rate and removed after some time. When an atom decays to the bottom level from the top level via the intermediate level, two photons are produced. These photons are highly correlated, and the nonclassical features of the cavity light generated by this quantum optical system are due to this correlation. The correlation arises because of the injected atomic coherence, which can be induced either by initially preparing the three-level atoms in a coherent superposition of the top and bottom levels [1–6] or coupling these levels by external coherent light [7, 8, 10, 11]. For instance, Ansari [12] has considered a cascade three-level atomic system with the atomic coherence introduced by the aforementioned two mechanisms simultaneously. He has predicted that the system shows a nearly perfect squeezing outside of the cavity.

Entanglement, the unique characteristics of quantum mechanics, plays a key role in quantum information processing [13–15]. Recently, the generation of macroscopic entangled states has been demonstrated in two-mode three-level cascade lasers. For instance, using the sufficient entanglement measure set by Duan et al. [16], macroscopic entanglement has been realized in a driven two-mode three-level laser [17], when the atoms are ejected from the lower level. Besides, Alebachew [18] has considered a nondegenerate three-level cascade laser with a parametric oscillator coupled to a two-mode vacuum reservoir with the atomic coherence introduced by the superposition of the ground and upper excited states. He has found that the presence of the parametric oscillator enhances the degree of two-mode squeezing and entanglement.

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In the realistic quantum system, the cascade three-level laser is coupled to the unwanted fluctuations in the surrounding environment via the walls of the cavity. The destruction of nonclassical properties due to an interaction with the environment is called the decoherence. Thus, the interaction between the system and the environment degrades the two-mode squeezing and entanglement, where decoherence is usually inevitable. The effect of decoherence can be incorporated by entangling the quantum system with a thermal reservoir. For example, Hiroshima [19] has investigated the decoherence of two-mode squeezed vacuum states by examining the relative entropy of entanglement for phase and amplitude damping when it is coupled to a thermal environment.

In this work, we introduce a model that produces two-mode squeezed and entangled as well as bright light by a three-level cascade laser coupled to a two-mode thermal reservoir, as shown in Fig. 1. The cascade configuration of three-level atoms initially prepared in a coherent superposition of their ground and upper excited states are injected into the laser cavity. The quantum properties of the two-cavity modes generated by quantum optical systems coupled to a two-mode squeezed vacuum reservoir have been studied by applying various methods [20, 21]. These studies show that the squeezed vacuum reservoir enhances the degree of squeezing and entanglement of the two-mode cavity light.

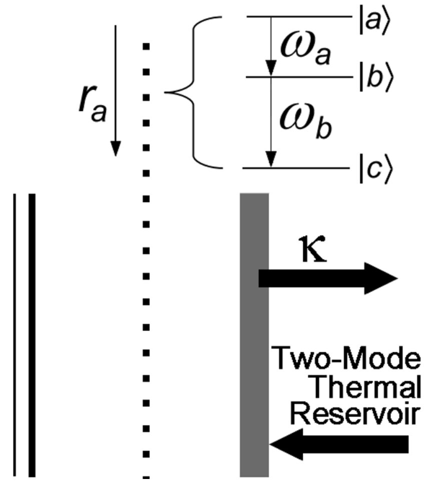

Fig. 1. Cascade configuration of a three-level laser coupled to a two-mode thermal reservoir. The three-level atoms prepared in a coherent superposition of their ground and upper excited states are injected at a constant rate r_a into a laser cavity.

Moreover, the nonclassical properties of the cavity mode with a parametric oscillator coupled to a vacuum reservoir have been investigated employing stochastic differential equations [9]. It has been shown that the parametric oscillator significantly increases the amount of squeezing of the cavity light. Opposed to previous studies, here we investigate the quantum-statistical features of the two-mode cavity light produced by a three-level cascade laser coupled to a two-mode thermal reservoir following a different approach. Applying the master equation, we obtain the evolution equations of the expectation values for the cavity mode variables. The steady-state solutions of these equations are then used to study the squeezing, entanglement, and statistical properties of the two-mode cavity light.

2. Master Equation

Here, we consider a nondegenerate three-level laser coupled to a two-mode thermal reservoir. A cascade configuration of three-level atoms initially prepared in a coherent superposition of their ground and upper excited levels are injected at a constant rate r_a and removed from the laser cavity after some time τ. We denote the top, intermediate, and bottom levels of a three-level atom by $|a\rangle$, $|b\rangle$, and $|c\rangle$, respectively. We assume that the cavity modes are at resonance with the transitions $|a\rangle \to |b\rangle$ and $|b\rangle \to |c\rangle$, and with a direct transition between $|a\rangle$ and $|c\rangle$ to be dipole forbidden.

The interaction of the cavity modes with a single three-level atom can be described in the rotating wave approximation and in the interaction picture by the Hamiltonian of the form

$$\hat{H} = ig[\hat{a}|a\rangle\langle b| - |b\rangle\langle a|\hat{a}^{\dagger} + \hat{b}|b\rangle\langle c| - |c\rangle\langle b|\hat{b}^{\dagger}], \tag{1}$$

where g is the atom–cavity coupling constant assumed to be the same for both transitions, and $\hat{a}$ and

$\hat{b}$ are the annihilation operators for the two cavity modes. The initial state of a single three-level atom assumed to be

$$|\Psi_A(0)\rangle = C_a(0)|a\rangle + C_c(0)|c\rangle, \tag{2}$$

and the corresponding initial density operator of a single atom has the form

$$\hat{\rho}_A(0) = \rho_{aa}^{(0)}|a\rangle\langle a| + \rho_{ac}^{(0)}|a\rangle\langle c| + \rho_{ca}^{(0)}|c\rangle\langle a| + \rho_{cc}^{(0)}|c\rangle\langle c|, \tag{3}$$

where

$$\rho_{aa}^{(0)} = C_a(0)C_a^*(0), \qquad \rho_{cc}^{(0)} = C_c(0)C_c^*(0) \tag{4}$$

are the initial populations of the atoms at the top and bottom levels, respectively, and

$$\rho_{ac}^{(0)} = C_a(0)C_c(0) = \rho_{ca}^{(0)*} \tag{5}$$

represents the initial atomic coherence.

It can be readily established that the evolution equation of the density operator with the damping of the cavity modes by a two-mode thermal reservoir in the linear and adiabatic approximation scheme, following the procedure presented in [22], has the form

$$\begin{aligned} \frac{d\hat{\rho}(t)}{dt} = {} & \frac{1}{2}[A\rho_{aa}^{(0)} + \kappa\bar{n}_a](2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\hat{\rho}) + \frac{1}{2}[\kappa\bar{n}_a + \kappa](2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) \\ & + \frac{1}{2}[A\rho_{cc}^{(0)} + \kappa\bar{n}_b + \kappa](2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger\hat{b}) + \frac{1}{2}\kappa\bar{n}_b[2\hat{b}^\dagger\hat{\rho}\hat{b} - \hat{b}\hat{b}^\dagger\hat{\rho} - \hat{\rho}\hat{b}\hat{b}^\dagger] \\ & + \frac{1}{2}A\hat{\rho}_{ac}^{(0)}[\hat{a}\hat{b}\hat{\rho} - 2\hat{b}\hat{\rho}\hat{a} + \hat{\rho}\hat{a}\hat{b}] + \frac{1}{2}A\hat{\rho}_{ac}^{(0)}[\hat{\rho}\hat{b}^\dagger\hat{a}^\dagger - 2\hat{a}^\dagger\hat{\rho}\hat{b}^\dagger + \hat{b}^\dagger\hat{a}^\dagger\hat{\rho}], \end{aligned} \tag{6}$$

where

$$A = \frac{2g^2 r_a}{\gamma^2} \tag{7}$$

is the linear gain coefficient, κ is the cavity damping constant, γ is the atomic decay rate assumed to be the same for levels $|a\rangle$ and $|b\rangle$, and $\bar{n}_a$ and $\bar{n}_b$ are the mean photon number associated with the thermal reservoir.

3. The Solutions of the Cavity Mode Operators

Now we proceed to determine the evolution equations for the expectation values of the cavity mode operators. Applying the relation

$$\frac{d}{dt}\langle\hat{A}\rangle = \mathrm{Tr}\left(\frac{d\hat{\rho}}{dt}\hat{A}\right), \tag{8}$$

along with Eq. (6), we find

$$\frac{d}{dt}\langle\hat{a}\rangle = -\frac{\beta_a}{2}\langle\hat{a}\rangle - \frac{A}{2}\hat{\rho}_{ac}^{(0)}\langle\hat{b}^\dagger\rangle, \qquad \frac{d}{dt}\langle\hat{b}\rangle = -\frac{\beta_b}{2}\langle\hat{b}\rangle + \frac{A}{2}\hat{\rho}_{ac}^{(0)}\langle\hat{a}^\dagger\rangle, \tag{9}$$

$$\frac{d}{dt}\langle\hat{a}^2\rangle = -\beta_a\langle\hat{a}^2\rangle - A\hat{\rho}_{ac}^{(0)}\langle\hat{b}^\dagger\hat{a}\rangle, \qquad \frac{d}{dt}\langle\hat{b}^2\rangle = -\beta_b\langle\hat{b}^2\rangle + A\hat{\rho}_{ac}^{(0)}\langle\hat{b}\hat{a}^\dagger\rangle, \tag{10}$$

$$\frac{d}{dt}\langle\hat{a}^\dagger\hat{a}\rangle = -\beta_a\langle\hat{a}^\dagger\hat{a}\rangle + [\kappa\bar{n}_a + A\hat{\rho}_{aa}^{(0)}] - \frac{A}{2}\hat{\rho}_{ac}^{(0)}(\langle\hat{a}\hat{b}\rangle + \langle\hat{b}^\dagger\hat{a}^\dagger\rangle), \tag{11}$$

$$\frac{d}{dt}\langle\hat{b}^\dagger\hat{b}\rangle = -\beta_b\langle\hat{b}^\dagger\hat{b}\rangle + \kappa\bar{n}_b + \frac{A}{2}\hat{\rho}_{ac}^{(0)}(\langle\hat{a}\hat{b}\rangle + \langle\hat{b}^\dagger\hat{a}^\dagger\rangle), \tag{12}$$

$$\frac{d}{dt}\langle\hat{a}\hat{b}^\dagger\rangle = -\beta\langle\hat{a}\hat{b}^\dagger\rangle + \frac{\kappa}{2}(\bar{n}_a - \bar{n}_b)\langle\hat{a}\hat{b}^\dagger\rangle + \frac{A}{2}\hat{\rho}_{ac}^{(0)}(\langle\hat{a}^2\rangle - \langle\hat{b}^{\dagger 2}\rangle), \tag{13}$$

$$\frac{d}{dt}\langle\hat{a}\hat{b}\rangle = -\beta\langle\hat{a}\hat{b}\rangle + \frac{A}{2}\hat{\rho}_{ac}^{(0)}(\langle\hat{a}^\dagger\hat{a}\rangle - \langle\hat{b}^\dagger\hat{b}\rangle) + \frac{A}{2}\hat{\rho}_{ac}^{(0)}, \tag{14}$$

where

$$\beta_a = \kappa - A\hat{\rho}_{aa}^{(0)}, \qquad \beta_b = \kappa + A\hat{\rho}_{cc}^{(0)}, \qquad \beta = \kappa + \frac{A}{2}(\hat{\rho}_{cc}^{(0)} - \hat{\rho}_{aa}^{(0)}). \tag{15}$$

The steady-state solution of each of the above equations has the form

$$\langle\hat{a}\rangle = \langle\hat{b}\rangle = 0, \qquad \langle\hat{a}^2\rangle = \langle\hat{b}^2\rangle = 0, \qquad \langle\hat{a}\hat{b}^\dagger\rangle = 0, \tag{16}$$

$$\langle\hat{a}\hat{b}\rangle = \frac{A\hat{\rho}_{ac}^{(0)}[\kappa(\bar{n}_a\beta_b - \bar{n}_b\beta_a) + A\hat{\rho}_{aa}^{(0)}\beta_b + \beta_a\beta_b]}{2\beta\beta_a\beta_b + A^2\hat{\rho}_{ac}^{(0)2}(\beta_a + \beta_b)}, \tag{17}$$

$$\langle\hat{b}^\dagger\hat{b}\rangle = \frac{A^2\hat{\rho}_{ac}^{(0)2}(\kappa\bar{n}_a + \kappa\bar{n}_b + A\hat{\rho}_{aa}^{(0)} + \beta_a)}{2\beta\beta_a\beta_b + A^2\hat{\rho}_{ac}^{(0)2}(\beta_a + \beta_b)} + \frac{2\kappa\bar{n}_b\beta\beta_a}{2\beta\beta_a\beta_b + A^2\hat{\rho}_{ac}^{(0)2}(\beta_a + \beta_b)}, \tag{18}$$

$$\langle\hat{a}^\dagger\hat{a}\rangle = \frac{A^2\hat{\rho}_{ac}^{(0)2}(\kappa\bar{n}_a + \kappa\bar{n}_b - \beta_b + A\hat{\rho}_{aa}^{(0)})}{2\beta\beta_a\beta_b + A^2\hat{\rho}_{ac}^{(0)2}(\beta_a + \beta_b)} + \frac{2\beta_b(\kappa\bar{n}_a\beta + A\hat{\rho}_{aa}^{(0)}\beta)}{2\beta\beta_a\beta_b + A^2\hat{\rho}_{ac}^{(0)2}(\beta_a + \beta_b)}. \tag{19}$$

4. Quadrature Variance

Here, we calculate the quadrature variances for the two-mode cavity light. To study the squeezing properties of a two-mode light, we introduce the quadrature operators defined by

$$\hat{c}_+ = \hat{c} + \hat{c}^\dagger, \qquad \hat{c}_- = i(\hat{c}^\dagger - \hat{c}), \qquad \hat{c} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{b}). \tag{20}$$

The quadrature operators satisfy the commutation relation $[\hat{c}_+, \hat{c}_-] = 2\,i$, and the corresponding uncertainty relation reads $\Delta c_-\Delta c_+ \geq 1$. We note from this result a two-mode light to be in a squeezed state if either $\Delta c_+ < 1$ or $\Delta c_- < 1$. The variances of the quadrature operators are defined by

$$(\triangle c_\pm)^2 = \langle\hat{c}_\pm^2\rangle - \langle\hat{c}_\pm\rangle^2. \tag{21}$$

Because of Eq. (20), this equation can be rewritten as follows:

$$(\triangle c_\pm)^2 = 1 + \langle\hat{a}^\dagger\hat{a}\rangle + \langle\hat{b}^\dagger\hat{b}\rangle + \langle\hat{a}^\dagger\hat{b}\rangle + \langle\hat{a}\hat{b}^\dagger\rangle \pm \frac{1}{2}\Big(\langle\hat{a}^2\rangle + \langle\hat{b}^2\rangle + \langle\hat{a}^{\dagger 2}\rangle + \langle\hat{b}^{\dagger 2}\rangle + 2\langle\hat{a}\hat{b}\rangle + 2\langle\hat{a}^\dagger\hat{b}^\dagger\rangle\Big). \tag{22}$$

With the aid of Eqs. (16) and (17), the variances take the form

$$\Delta c_{\pm}^2 = 1 + \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{b}^\dagger \hat{b} \rangle \pm 2\langle \hat{a}\hat{b} \rangle. \tag{23}$$

Now we introduce a parameter η such that

$$\hat{\rho}_{aa}^{(0)} = \frac{1-\eta}{2}. \tag{24}$$

We consider the case where the atoms are initially in a superposition of the top and bottom level states. Thus, one can write

$$\hat{\rho}_{aa}^{(0)} + \hat{\rho}_{cc}^{(0)} = 1. \tag{25}$$

It is easy to see that

$$\hat{\rho}_{cc}^{(0)} = \frac{1+\eta}{2}, \qquad \hat{\rho}_{ac}^{(0)} = \hat{\rho}_{ca}^{(0)} = \frac{1}{2}\sqrt{1-\eta^2}. \tag{26}$$

Since the value of $\rho_{aa}^{(0)}$ lies between 0 and 1, we see that the values of η are in the interval $-1 \le \eta \le 1$. We also note that for $\eta = 0$, $\rho_{aa}^{(0)} = \rho_{cc}^{(0)} = \rho_{ac}^{(0)} = 1/2$, which corresponds to a maximum initial atomic coherence. However, for $\eta = 1$, $\rho_{aa}^{(0)} = \rho_{ac}^{(0)} = 0$ and $\rho_{cc}^{(0)} = 1$ corresponds to no initial atomic coherence. Now in view of Eqs. (17)–(19) and taking into account Eqs. (15), we arrive at

$$\begin{aligned} \Delta c_{\pm}^2 &= \frac{(A^2/2)(1-\eta^2)(1+\bar{n}_a+\bar{n}_b) \pm (A^2/2)\sqrt{1-\eta^2}(\bar{n}_a+\bar{n}_b)}{(2\kappa + A\eta)(\kappa + A\eta)} \\ &\quad + \frac{2(\kappa + A\eta/2)^2(\bar{n}_a+\bar{n}_b)}{(2\kappa + A\eta)(\kappa + A\eta)} + \frac{(\kappa + A/2)(1+\eta)(2\kappa + A\eta \pm A\sqrt{1-\eta^2})}{(2\kappa + A\eta)(\kappa + A\eta)} \\ &\quad + \frac{(\kappa + A\eta/2)(A \pm A\sqrt{1-\eta^2})(\bar{n}_a - \bar{n}_b)}{(2\kappa + A\eta)(\kappa + A\eta)}. \end{aligned} \tag{27}$$

To investigate the dependence of the maximum quadrature squeezing on the parameter η and the linear gain coefficient A, we collect data from Eq. (27) in Table 1.

In Table 1, we see that the maximum quadrature squeezing occurs for different values of the parameter η corresponding to different values of the linear gain coefficient A. Moreover, it is straightforward to see in Table 1 that the variation of the maximum quadrature squeezing due to the linear gain coefficient increases with decrease of the parameter η.

Table 1. Variation of Maximum Quadrature Squeezing with η and A^a.

A	Maximum squeezing	Occurs at
50	64.2 %	$\eta = 0.19$
100	67.3 %	$\eta = 0.14$
500	71.5 %	$\eta = 0.07$
1000	72.5 %	$\eta = 0.05$

[a]Here, $\kappa = 0.5$ and $\bar{n}_a = \bar{n}_b = 0$.

In Fig. 2 a, we note that the two-mode light exhibits two-mode squeezing for all values of η between zero (maximum coherence) and unity (minimum coherence), and the degree of two-mode squeezing increases with the rate at which the atoms are injected into the cavity. It is also seen that the squeezing vanishes for maximum and minimum values of injected atomic coherence. Moreover, as the linear gain coefficient A increases, the value of η, at which the maximum two-mode squeezing occurs, shifts towards the maximum initial

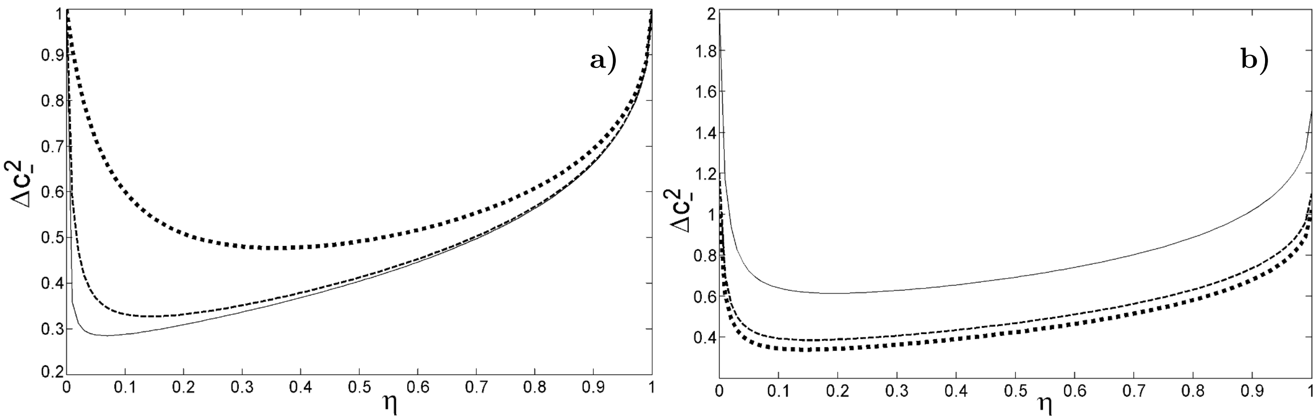

Fig. 2. Plots of the two-mode quadrature variance Δc_-^2 of Eq. (27) for the two-mode cavity light at the steady state vs η for $\kappa = 0.5$. Here, $\bar{n}_a = \bar{n}_b = 0$ (a) with $A = 10$ (the dotted curve), $A = 100$ (the dashed curve), and $A = 500$ (the solid curve) and $A = 100$ (b) with $\bar{n}_a = \bar{n}_b = 0.02$ (the dotted curve), $\bar{n}_a = \bar{n}_b = 0.1$ (the dashed curve), and $\bar{n}_a = \bar{n}_b = 0.5$ (the solid curve).

atomic coherence ($\eta = 0$). We then realize that a significant two-mode squeezing can be achieved for smaller values of η and larger values of A.

To study the effect of entanglement of two-mode cavity light beams with the two-mode thermal reservoir (decoherence) on the degree of two-mode squeezing of the two-mode light beams, in Fig. 2 b, we present Δc_-^2 against η for different values of the mean photon number of the two-mode thermal reservoir. We easily see from this plot that the two-mode thermal reservoir leads to a decrease in the degree of two-mode squeezing. For example, the minimum value of the quadrature variance described by Eq. (27) for $A = 100$, $\kappa = 0.5$, and $\bar{n}_a = \bar{n}_b = 0$ is found to be $\Delta c_-^2 = 0.3270$ and occurs at $\eta = 0.1430$. This result implies that the maximum squeezing for the above value is 67.3% below the vacuum state level. However, in the presence of the thermal noise with $\bar{n}_a = \bar{n}_b = 0.1$ and for the same parameters used above, the minimum value of the quadrature variance is 0.3852 and occurs at $\eta = 0.1560$. We then note that the maximum quadrature squeezing, in this case, is 61.48% below the shot-noise limit. Hence, with this choice of the thermal noise, the amount of squeezing is lowered by over 5%. In Fig. 2 b, we see that, when the mean photon number of the two-mode thermal reservoir decreases, the value of η, at which the maximum two-mode squeezing occurs, shifts towards the value of maximum atomic coherence injected. Moreover, one observes that, as the mean photon number of the two-mode thermal noise increases, the two-mode squeezing disappears for values of η very close to 0 and 1.

5. Entanglement Properties of the Two-Mode Light

In this section, we seek to study the entanglement properties of the two-cavity modes. The composite system of the two modes a and b are said to be entangled, if its state cannot be expressed as a product of the state of the subsystems. Various entanglement measures for continuous variables have been introduced by some authors [16, 23]. According to the inseparability criteria proposed by Duan et al. [16], the quantum state of the system is entangled if the sum of the variances of the EPR-like operators

$$\hat{u} = \hat{x}_a - \hat{x}_b, \qquad \hat{v} = \hat{p}_a + \hat{p}_b \tag{28}$$

satisfy

$$\Delta u^2 + \Delta v^2 < 2, \tag{29}$$

in which

$$\hat{x}_k = \hat{k} + \hat{k}^\dagger, \qquad \hat{p}_k = \frac{\hat{k} - \hat{k}^\dagger}{i}, \tag{30}$$

with $k = a, b$ being the quadrature operators for the two cavity modes. The sum of the variances of the EPR-like operators can then be written as

$$\Delta u^2 + \Delta v^2 = 2 + 2\langle \hat{a}^\dagger \hat{a} \rangle + 2\langle \hat{b}^\dagger \hat{b} \rangle - 4\langle \hat{a}\hat{b} \rangle = 2\Delta c_-^2, \tag{31}$$

with Δc_-^2 given by Eq. (23). This relation shows that the degree of two-mode squeezing and entanglement has a direct relationship, which is in agreement with previous studies [2, 5].

Now making use of Eqs. (17)–(19) along with (15), we arrive at

$$\begin{aligned} \Delta u^2 + \Delta v^2 \;&=\; \frac{2\Big[(A^2/2)(1-\eta^2)(1+\bar{n}_a+\bar{n}_b) \pm (A^2/2)\sqrt{1-\eta^2}(\bar{n}_a+\bar{n}_b)\Big]}{(2\kappa + A\eta)(\kappa + A\eta)} \\ &+ \frac{4(\kappa + A/\eta)^2(\bar{n}_a+\bar{n}_b)}{(2\kappa + A\eta)(\kappa + A\eta)} + \frac{2(\kappa + (A/2)(1+\eta))(2\kappa + A\eta \pm A\sqrt{1-\eta^2})}{(2\kappa + A\eta)(\kappa + A\eta)} \\ &+ \frac{2(\kappa + A\eta/2)(A \pm A\sqrt{1-\eta^2})(\bar{n}_a - \bar{n}_b)}{(2\kappa + A\eta)(\kappa + A\eta)}. \end{aligned} \tag{32}$$

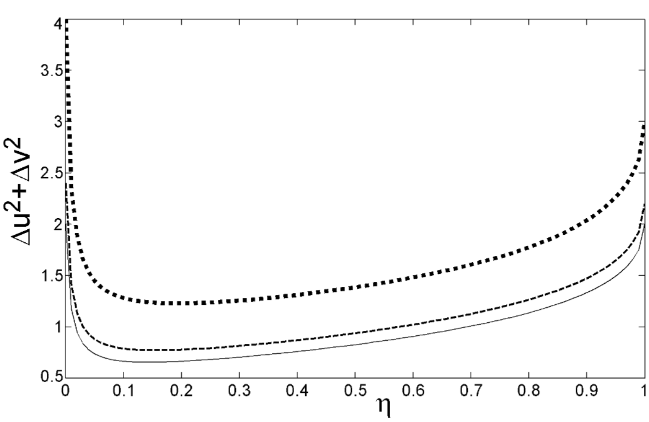

Fig. 3. Plots of $\Delta u^2 + \Delta v^2$ of the two-mode light in the cavity at steady state vs η for $\kappa = 0.5$ and $A = 100$; here, $\bar{n}_a = \bar{n}_b = 0.5$ (the dotted curve), $\bar{n}_a = \bar{n}_b = 0.1$ (the dashed curve), and $\bar{n}_a = \bar{n}_b = 0$ (the solid curve).

Table 2. Variation of the Maximum Degree of Entanglement with η and A^b.

A	Maximum Entanglement	Occurs at
50	64.2 %	$\eta = 0.19$
100	67.3 %	$\eta = 0.14$
500	71.5 %	$\eta = 0.07$
1000	72.5 %	$\eta = 0.05$

[b] Here, $\kappa = 0.5$ and $\bar{n}_a = \bar{n}_b = 0$.

It is indicated in Fig. 3 that the entanglement criterion given by Eq. (29) is satisfied for certain values of η. As can be seen in Fig. 3, the degree of entanglement decreases with the mean photon number of the two-mode thermal reservoir. We then realize that decoherence has an adverse effect on the degree of entanglement. Moreover, as the mean photon number of the two-mode thermal reservoir increases, the entanglement disappears when the atoms are initially prepared with maximum or minimum atomic coherence. However, as shown in Fig. 3, the maximum entanglement occurs when the atoms are prepared with the injected atomic coherence close to the maximum possible value. The maximum degree of entanglement and the corresponding values of η, for which the maximum entanglement occurs, are generated from Eq. (32) and collected in Table 2.

In Table 2, we show that a higher maximum degree of entanglement occurs for smaller values of η and larger values of A. Moreover, a comparison of Table 1 and Table 2 reveals that the maximum degree of squeezing and entanglement are the same for the same choice of the parameters. We also note from these two Tables that the maximum amount of entanglement and squeezing occurs close to a region with an equal number of atoms initially prepared in the bottom and top level states.

6. Mean Number of Photon Pairs

We now seek to determine the mean number of photon pairs of two-mode cavity radiation. To this end, based on Eq. (20), first we write that

$$\langle \hat{c}^\dagger \hat{c} \rangle = \frac{1}{2}[\langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{b}^\dagger \hat{b} \rangle]. \tag{33}$$

Then it follows that

$$\begin{aligned} \bar{n} \;=\; & \frac{A^2(1-\eta^2)(2\kappa(\bar{n}_a+\bar{n}_b)-A\eta)+2\kappa(2\kappa+A\eta)^2(\bar{n}_a+\bar{n}_b)}{4\kappa(2\kappa+A\eta)(2\kappa+2A\eta)} \\ & + \frac{2\kappa(2\kappa+A\eta)A(\bar{n}_a-\bar{n}_b)}{4\kappa(2\kappa+A\eta)(2\kappa+2A\eta)} + \frac{A(1-\eta)(2\kappa+A+A\eta)(2\kappa+A\eta)}{4\kappa(2\kappa+A\eta)(2\kappa+2A\eta)}. \end{aligned} \tag{34}$$

In Fig. 4 a, we represent the plots of the mean number of the photon pairs against η for $\kappa = 0.5$, $\bar{n}_a = \bar{n}_b = 0$, and different values of A. In Fig. 4 a, one can see that the mean number of the photon pairs decreases with the parameter η. Then we infer that the mean number of the photon pairs is maximum

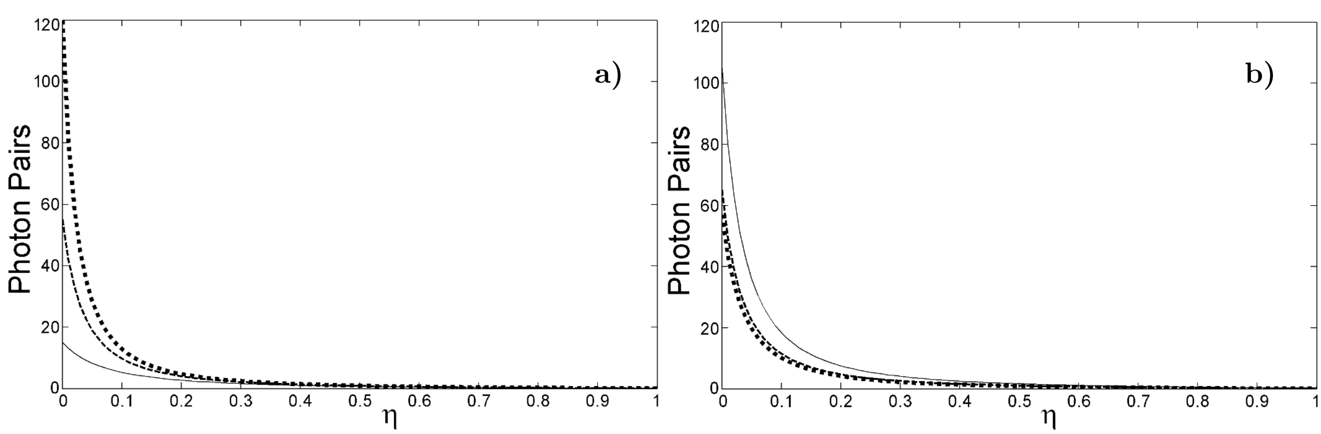

Fig. 4. Plots of the mean number of the photon pairs $\bar{n}$ of Eq. (34) for the two-mode cavity light at the steady state vs η for $\kappa = 0.5$. Here, $\bar{n}_a = \bar{n}_b = 0$ (a) with $A = 5$ (the solid curve), $A = 10$ (the dashed curve), and $A = 15$ (the dotted curve) and $A = 10$ (b) with $\bar{n}_a = \bar{n}_b = 0.5$ (the solid curve), $\bar{n}_a = \bar{n}_b = 0.1$ (the dashed curve), and $\bar{n}_a = \bar{n}_b = 0.02$ (the dotted curve).

at maximum injected atomic coherence and minimum without injected atomic coherence. Also one can see that the mean number of the photon pairs increases with the rate A, at which the atoms are injected into the cavity.

As clearly shown in Fig. 4 b, the mean number of photon pairs increases with the mean photon number of the two-mode thermal reservoir. Hence one can infer that the thermal reservoir has a substantial effect on the brightness of the cavity light. We also notice that the mean number of photon pairs is relatively larger in a region, where the degree of squeezing is higher.

7. Mean of the Intensity Difference

In this section, our goal is to calculate the mean of the intensity difference. To this end, the intensity difference can be defined as

$$\hat{I}_D = \hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}, \tag{35}$$

in which its mean can be expressed as

$$I_D = \langle \hat{a}^\dagger \hat{a} \rangle - \langle \hat{b}^\dagger \hat{b} \rangle. \tag{36}$$

Thus, making use of Eqs. (18) and (19), we obtain

$$\begin{aligned} I_D \;=\; & \frac{-A^2(1-\eta^2)(2\kappa + A\eta)}{4\kappa(2\kappa + A\eta)(\kappa + A\eta)} + \frac{2\kappa[2\kappa + A\eta][2\kappa(\bar{n}_a - \bar{n}_b) + A(\bar{n}_a + \bar{n}_b) + A\eta(\bar{n}_a - \bar{n}_b)]}{4\kappa(2\kappa + A\eta)(\kappa + A\eta)} \\ & + \frac{(2\kappa + A\eta)(A(1-\eta))(2\kappa + A + A\eta)}{4\kappa(2\kappa + A\eta)(\kappa + A\eta)}. \end{aligned} \tag{37}$$

In Fig. 5, we see that the mean of the intensity difference is positive. This shows that the intensity of light mode a is greater than that of light mode b. Moreover, we observe, in general, that the mean of the intensity difference decreases as the parameter η increases. We also realize that the mean of the intensity difference is maximum when the atoms are initially prepared with an equal number of atoms at the top and bottom levels.

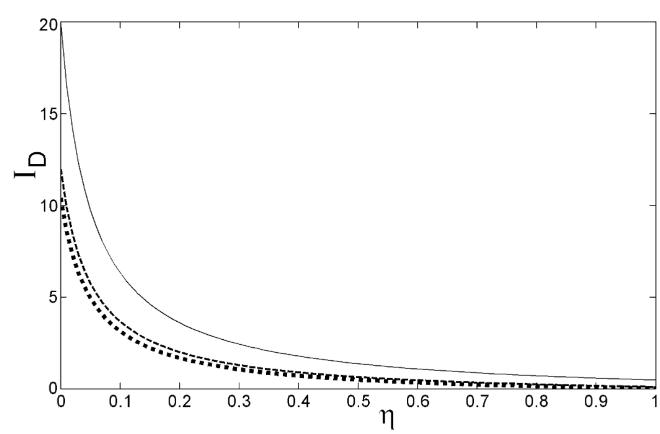

Fig. 5. The mean of the intensity difference I_D of Eq. (37) for the two-mode cavity light at the steady state vs η for $\kappa = 0.5$ and $A = 10$; here, $\bar{n}_a = \bar{n}_b = 0.5$ (the solid curve), $\bar{n}_a = \bar{n}_b = 0.1$ (the dashed curve), and $\bar{n}_a = \bar{n}_b = 0.02$ (the dotted curve).

8. Conclusions

In this paper, we studied the light generated by a nondegenerate three-level laser coupled to a two-mode thermal reservoir with the atomic coherence induced by the initial superposition of top and bottom level states. We found that the two-mode cavity light is in a squeezed state, when the atoms are initially prepared with more atoms at the bottom level than at the top level. We also saw that the degree of squeezing increases with the rate, at which the atoms are injected into the cavity, and the two-mode thermal reservoir has the effect of decreasing the degree of squeezing. Moreover, it is found the maximum quadrature squeezing of the two-mode cavity light to be 71.5% below the vacuum state level for $A = 500$, $\kappa = 0.5$, and $\bar{n}_a = \bar{n}_b = 0$. In addition, we demonstrated that the two cavity modes are entangled in a region, where the squeezing exists. We also found that the degree of squeezing and entanglement are larger particularly, when there is a nearly equal number of atoms initially in the top and bottom level states. We also showed that though decoherence has an adverse effect on the degree of squeezing and entanglement, it increases the mean number of the photon pairs and intensity difference.

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