DOI 10.1007/s10946-019-09819-y
Journal of Russian Laser Research, Volume 40, Number 5, September, 2019

MOTION OF A CHAIN OF VORTICES UNDER DIFFERENT CONDITIONS OF PASSING A CURRENT THROUGH A JOSEPHSON JUNCTION COUPLED TO A WAVEGUIDE

A. S. Malishevskii,[1*] V. P. Silin,[1,2] and S. A. Uryupin[1,2]

[1] *Lebedev Physical Institute, Russian Academy of Sciences*
Leninskii Prospect 53, Moscow 119991, Russia

[2] *National Research Nuclear University MEPhI*
Kashirskoe Shosse 31, Moscow 115409, Russia

*Corresponding author e-mails: malish@lebedev.ru, vabanque@mail.ru

Abstract

We obtain the current–voltage characteristics for the following three modes of current passing through a system consisting of a Josephson junction coupled to a waveguide: first, only through the waveguide; second, both through the junction and through the waveguide; and third, only through the junction. In the first and second modes, sustaining of the induced motion of fast vortex chains is possible at a lower current density than that in the third mode.

Keywords: Josephson junction, waveguide, fast vortex, current–voltage characteristics (CVC).

1. Introduction

An attractive property of long Josephson junctions is the possibility of motion of Josephson vortices, both solitary and their chains [1]. Josephson vortices have a large number of applications. In particular, they are used to record and transmit information [2]. They find applications in generators of radiation in the flux flow regime [3]. On their basis, qubits are created [4]. Experimental investigation of the dynamics of vortices is possible with the help of registration of the current–voltage characteristics (CVC) of Josephson junctions [5, 6]. The study of the induced motion of vortices under the action of the bias current has long attracted the attention of researchers. In the development of this topic, we have previously predicted the possibility of fast vortex motion [7] and chains of such vortices [8,9]. To advance towards a possible experimental check of the traveling fast vortices, in the present paper we describe the CVC of Josephson junction coupled to a waveguide. Such a system is shown in Fig. 1. We can assume that the waveguide is a limiting case of a Josephson junction with zero critical current. In this sense, the system depicted in Fig. 1 is similar to multilayered systems, which are stacks of Josephson junctions. Such systems have been intensely studied (see, for example, [10–13]) when building models for describing the motion of vortices in layered high T_c superconductors and also to demonstrate the manifestation of the Vavilov–Cherenkov effect in the physics of Josephson junctions.

Below we present an unusual CVC of Josephson junction–waveguide system. We believe that such a CVC can be implemented experimentally. This would give a new impetus to the studies of dynamics of fast vortices, which we predicted earlier [7]. The properties of fast vortices are studied in a number

Manuscript submitted by the authors in English on August 21, 2019.

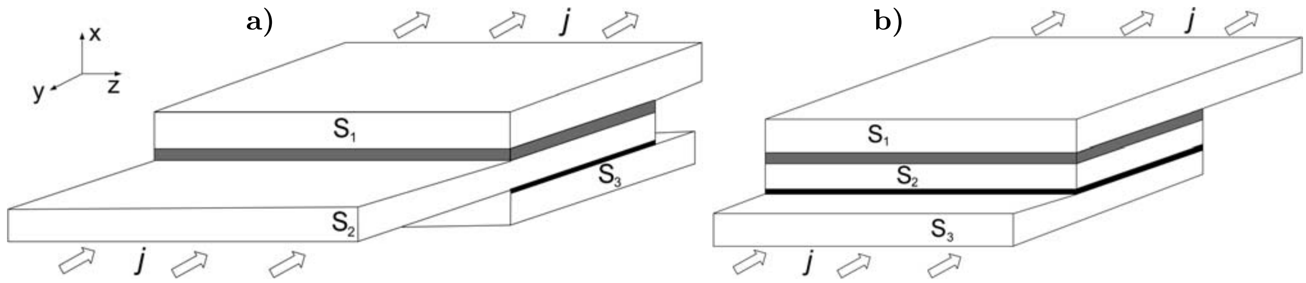

Fig. 1. A schematic (not to scale) image of a system consisting of a coupled Josephson junction (the tunneling area is shown in black) and a waveguide (shown in gray), with S_1, S_2, and S_3 being superconducting electrodes. The arrows indicate the direction of current flow through only the waveguide (a) and through the entire system (b).

of papers; at the same time, in the absence of accompanying experiments, an interesting theoretical prediction has not yet received due recognition. It should be noted that experimental detection of fast vortices by registering new CVC would allow one to take a fresh look at the possibility of using Josephson vortices in mastering the terahertz frequency range and, in the long term, in creating compact terahertz generators. Stable areas of the found CVC can be detected in the mode of a given current. It is shown that in order to maintain the induced motion of fast vortices, it is preferable to pass current only through the waveguide or through the entire system than to pass current only through the junction.

2. Chains of Slow and Fast Vortices

For a description of electrodynamics of a layered system consisting of a long Josephson junction and a magnetically coupled plane waveguide in the dissipation-free limit, the following equations for phase differences of the superconductive order parameter on the junction φ and the waveguide φ_w are usually used:

$$\omega_J^2 \sin\varphi + \varphi_{tt} = V_S^2 \varphi_{zz} + S V_S^2 \varphi_{w,zz}, \tag{1}$$

$$\varphi_{w,tt} = V_{Sw}^2 \varphi_{w,zz} + S V_{Sw}^2 \varphi_{zz}, \tag{2}$$

where $\omega_J^2 \equiv 16\pi^2 c d j_c/\varepsilon\phi_0$ is the Josephson plasma frequency, c is the speed of light, $2d$ is the width of the Josephson junction in the x direction, j_c is the critical Josephson current density, ε is the dielectric constant of a tunneling layer, ϕ_0 is the magnetic flux quanta, V_S and V_{Sw} are renormalized Swihart velocities of the Josephson junction and the waveguide [7], and S and $S_w \le 1$ are coupling constants of the Josephson junction and the waveguide, respectively.

The main difference between Eqs. (1) and (2) is the absence of a nonlinear term on the left-hand side of the Eq. (2). This is due to the fact that Cooper pairs do not tunnel through the waveguide.

For vortex structures traveling with constant velocity v, from Eq. (2) we obtain

$$\psi_w'' = \left[S_w V_{Sw}^2/(v^2 - V_{Sw}^2)\right]\psi'', \tag{3}$$

where $\psi_w(\zeta) \equiv \varphi_w(z,t)$, $\psi_w(\zeta) \equiv \varphi_w(z,t)$, and $\zeta \equiv z - vt$. Relations (1) and (3) lead to the following equation for the phase difference ψ on the Josephson junction:

$$\sin\psi = k_J^{-2}\psi'', \tag{4}$$

where

$$k_J(v) \equiv \sqrt{\frac{V_{Sw}^2 - v^2}{\left(v_1^2 - v^2\right)\left(v_2^2 - v^2\right)}}\,\omega_J, \tag{5}$$

and the velocities v_1 and v_2 are known functions of V_S, V_{Sw}, S, and S_w [7]; they read

$$v_m \equiv \sqrt{\frac{V_S^2 + V_{Sw}^2}{2} + (-1)^m \sqrt{\frac{\left(V_S^2 - V_{Sw}^2\right)^2}{4} + S S_w V_S^2 V_{Sw}^2}}, \quad m = 1, 2. \tag{6}$$

As in the case of an isolated long junction [1], Eq. (4) has solutions describing the vortex chains. Depending on the velocity, these chains have different shapes. In the case where the velocity lies in the intervals $(0, v_1)$ and (V_{Sw}, v_2), k_J is real and $\psi = \pi + 2\mathrm{am}(k_J\zeta/k, k)$, $0 < k < 1$. In other case where the velocity lies in the intervals (v_1, V_{Sw}) and (v_2, ∞), k_J is imaginary, and we have $\psi = 2\mathrm{am}(|k_J|\zeta/k, k)$. In the both cases, the parameter k characterizes the period $2k\mathrm{K}(k)|k_J(v)|^{-1}$ of the chain of traveling vortices, and $\mathrm{K}(k)$ is the complete elliptic integral of the first kind.

We emphasize that usually in Josephson junctions a chain of vortices, whose velocity is small compared to the Swihart velocity, are considered. Taking into account the influence of the waveguide provides the extension of the permissible limits of existence of vortices. If the Swihart velocity of the waveguide V_{Sw} is large compared to the Swihart velocity of the junction V_S, there is a possibility of movement of the so-called fast vortices with a velocity close to V_{Sw}.

In [14], it is shown that there is a peak, associated with the chain of fast vortices, on the CVC of a system consisting of a Josephson junction magnetically coupled to a waveguide. This result is obtained when the bias current passes only through the Josephson junction, when the bias current is applied to the electrode S_3 and removed from the electrode S_2. Next, we consider the CVC of such a system in the case of vortex chain motion under other modes of passage of the bias current.

3. Forced Motion of Vortex Chains

Analysis of forced motion of vortex chains is convenient using the balance of forces acting on the vortex. At small losses and bias current densities significantly less than j_c, it is possible to follow the approach previously proposed for the solitary Josephson junction [15].

According to [14], the friction force per chain period (per unit length along the Oy axis) is

$$\mathbf{F}_f = -\frac{\phi_0^2 \varepsilon \mathrm{E}(k)}{4\pi^3 c^2 k d} v \left|k_J(v)\right| \alpha(v) \mathbf{e}_z, \tag{7}$$

where $\mathrm{E}(k)$ is the complete elliptic integral of the second kind,

$$\alpha(v) \equiv \alpha + S S_w \frac{V_S^2 V_{Sw}^2}{(V_{Sw}^2 - v^2)^2} \alpha_w,$$

and α and α_w characterize ohmic losses in nonsuperconducting layers. At the same time, the Lorenz force is $\mathbf{F}_L = (\phi_0/c)\int_{\text{period}} d\zeta \left(j_J\psi' + j_w\psi_w'\right)\mathbf{e}_z$, where j_J and j_w are values of the bias-current density in the Josephson junction and the waveguide. They are introduced for three possible situations, i.e.,

(1) when the current passes only through the junction, then $j_J = j$ and $j_w = 0$; (2) when the current passes only through the waveguide, then $j_J = 0$ and $j_w = j$; (3) when the current passes through the entire system, then $j_J = j_w = j$. In the same way as the Lorentz force acting on the Abrikosov vortex is determined by the product of the bias current density and the magnetic flux carried by the vortex [17], in the system under consideration, the Lorentz force is determined by the product of the current density in the Josephson junction and (or) the waveguide and the change in the corresponding phase differences at one period of the vortex chain. For small losses and small values of the current density, the derivatives of phase differences on the waveguide and the Josephson junction are linearly related: $\psi'_w = \left[S_w V_{Sw}^2/(v^2 - V_{Sw}^2)\right]\psi'$ (it is assumed that there is no external magnetic field), and for the Lorentz force we have

$$\mathbf{F}_L = \frac{\phi_0}{c}\left(j_J + \frac{S_w V_{Sw}^2}{v^2 - V_{Sw}^2} j_w\right)\mathbf{e}_z. \tag{8}$$

The appearance of the multiplier $S_w V_{Sw}^2/(v^2 - V_{Sw}^2)$ in expression (8) corresponds to a linear relationship for the derivatives of phase differences. The balance of friction and Lorentz forces corresponding to the motion of vortices with a constant velocity results in the current–velocity characteristic.

First, consider the situation where the bias current flows only through the waveguide, when the bias current is applied to the electrode S_2 and removed from the electrode S_1, as depicted in Fig. 1 a. Then, in accordance with Eqs. (7) and (8), we have the following relation between the bias current density j and the vortex chain velocity v:

$$\frac{j}{j_c} = -\frac{4\mathrm{E}(k)}{\pi k}\frac{V_{Sw}^2 - v^2}{S_w V_{Sw}^2}\frac{v|k_J(v)|}{\omega_J^2}\alpha(v). \tag{9}$$

Relation (9) is also applicable to the case of ring geometry, where the Josephson junction and the waveguide have relatively large radii R, which allows us to ignore the curvature and use the equations for the rectilinear system. In this case, the parameter k and the velocity v are related by

$$2\pi R = 2nk\mathrm{K}(k)/|k_J(v)|, \tag{10}$$

where $n = 1, 2\ldots$ is the number of vortices in the ring.

The voltage across the Josephson junction averaged with respect to the time period $2\pi R/nv$ is

$$\langle U\rangle = -\phi_0 nv/2\pi cR. \tag{11}$$

Equations (9), (10), and (11) implicitly describe the CVC of a ring system consisting of the Josephson junction and the waveguide in the case of current passing only through the waveguide. Eliminating the parameter k by numerically solving Eq. (10), in view of the relation between the velocity and voltage (11) and the transcendental equation (9), one can numerically get the CVC (the dependence of j on $\langle U\rangle$).

The other possibility of sustaining the induced motion of a chain of vortices is realized when current is passing through the entire system, as shown in Fig. 1 b. Then we have the following relation between the bias current density j and the vortex chain velocity v [9]:

$$\frac{j}{j_c} = \frac{4\mathrm{E}(k)}{\pi k}\frac{V_{Sw}^2 - v^2}{v_w^2 - v^2}\frac{v\,|k_J(v)|}{\omega_J^2}\alpha(v), \tag{12}$$

where $v_w \equiv \sqrt{1 - S_w}\,V_{Sw}$. The procedure for obtaining the CVC from this current–velocity characteristic is similar to that described above.

4. Calculations and Results

The CVC in the case of passing bias current only through the waveguide are shown in Fig. 2 by dashed curves. The following notation was used: $U_p \equiv -\phi_0 n v_p/2\pi cR$ $(p = 1, 2)$, $U_w \equiv -\phi_0 n v_w/2\pi cR$, $U_S \equiv -\phi_0 n V_S/2\pi cR$, and $U_{Sw} \equiv -\phi_0 n V_{Sw}/2\pi cR$. Calculations were performed for a system with the following parameters: $V_{Sw} = 2V_S$, $S = S_w = 0.9$, $\alpha = 5 \cdot 10^{-4}\omega_J$, $R\omega_J/V_S = 2/\pi$, and $n = 1$. In Fig. 2, only stable parts of the CVC are given, where the absolute value of the current density increases with increasing voltage $\langle U\rangle$. Here, stable sections of the CVC in the cases of current passing through the entire system (solid curves) and only through the Josephson junction (dash-dotted curves) are also shown.

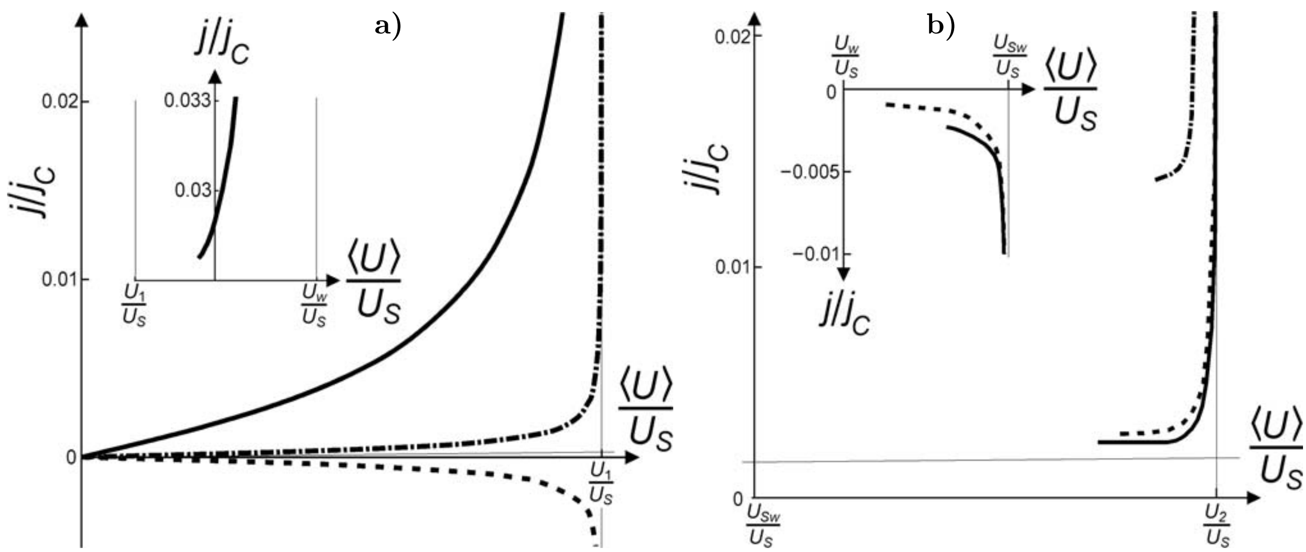

Fig. 2. The CVC of the ring system consisting of the Josephson junction and the waveguide for voltages from 0 to U_w (a) and from $\lesssim U_{Sw}$ to U_2 (b). The CVC for the case of passing current through only the waveguide are shown by the dashed curve, through the entire system by the solid curve, and only through the Josephson junction by the dash-dotted curve, respectively. The inset in (a) shows the stable section of the CVC in the voltage range between U_1 and U_w and the inset in (b) shows stable sections near the voltage U_{Sw}. Only stable parts of CVC are shown. Asymptotes are shown by thin solid lines.

From Fig. 2 a one can see that in the region of voltages smaller than U_1, which correspond to slow vortices moving at velocities of $0 < v < v_1$, all three CVC are stable. At high voltages, at all CVC, instability regions appear.

For a given voltage value, the friction force (7) does not depend on the mode of passing current. At the same time, the multiplier in parentheses in (8) is j in the case of current passing only through the Josephson junction, $S_w V_{Sw}^2 j/(v^2 - V_{Sw}^2)$ in the case of current passing only through the waveguide, and $(v^2 - v_w^2)j/(v^2 - v_{Sw}^2)$ in the case of current passing through the whole system. In the region of existence of a fast vortex, $V_{Sw} < v < v_2$, inequalities are met $1 < S_w V_{Sw}^2/(v^2 - v_{Sw}^2) < (v^2 - v_w^2)/(v^2 - v_{Sw}^2)$. Then it follows from the condition of the balance of forces that, in order to maintain uniform motion of the vortex chain, the least current is required if it is passing through the entire system. When the current passes only through the waveguide, its value is more, $(v^2 - V_{Sw}^2)/S_w V_{Sw}^2 > 1$. A greater value of the current is required when it passes only through the Josephson junction. According to Fig. 2 b, passing current only through the waveguide or through the entire system requires significantly less current than passing current only through the Josephson junction. The closeness of the current supporting the movement of fast vortices in the case of current passing only through the waveguide or through the entire system arises from the fact that the magnetic field of the fast vortex is mainly localized on the waveguide and near it (see, i.e., [18]).

Now we briefly discuss the sections of the CVC away from the region of motion of the fast vortex. Speaking of the motion of a slow vortex at voltages less than U_1, from Fig. 2 a it can be seen that maintaining the uniform motion of vortices when passing current through the waveguide requires a different direction of current than with other passing modes. This is due to the fact that in the case of $j_J = 0$ and at voltages smaller than U_{Sw}, the sign of the Lorentz force is opposite to the sign of j_w. Note that, in this area, the highest absolute value of the current density corresponds to the case of passing current through the entire system.

In the intermediate voltage range, from U_1 to U_{Sw}, for the same reason, changing the sign of the Lorentz force, the current is negative in the case of passing only through the waveguide. Also note a new feature of the voltage U_w. It is characteristic only for passing current through the whole system. This feature is related to the presence of the denominator $v_w^2 - v^2$ in the expression for current density (12). When passing the point $v = v_w$ (or $\langle U \rangle = U_w$) from left to the right, the Lorentz force ($\propto v^2 - v_w^2$) changes sign, which corresponds to the CVC gap by the law $\propto 1/(v^2 - v_w^2)$. Note that Fig. 2 a is plotted in the case $V_1 < v_w$. In the opposite case, a similar gap arises at the point $\langle U \rangle = U_w$, which is located to the left of the point $\langle U \rangle = U_1$. In this case, on a stable part of the CVC near this voltage, the current (12) is negative.

Finally, speaking of the CVC in the voltage region greater than U_2, not depicted in the figures, the continuous and dashed curves yield a linear asymptotic dependence $j/j_c = -2\pi c\alpha \langle U \rangle / \phi_0 \omega_J^2$, corresponding to the vortex-free state according to [14], which is shown in Fig. 2 by a thin solid line. At the same time, in the case of current passing only through the waveguide, the asymptotic dependence is quadratic, due to the fact that the Lorentz force at sufficiently high velocities decreases by the law v^{-2}.

5. Conclusions

From the above consideration of the dynamics of vortex chains in the Josephson junction coupled to the waveguide, two conclusions follow. First, the induced motion of slow vortices is easier to realize by current passing only through this junction. Second, it is advisable to study the dynamics of fast vortex chains by passing the current only through the waveguide or through the entire structure.

Acknowledgments

This work was partially supported by the Ministry of Higher Education and Science of the Russian Federation within the framework of the National Research Nuclear University MEPhI Academic Excellence Project under Contract No. 02.a03.21.0005.

References

1. P. Lebwohl and M. J. Stephen, *Phys. Rev.*, **163**, 376 (1967).
2. P. Guret, *IEEE Trans. Magnetics*, **11**, 751 (1975).

3. V. M. Krasnov, *Phys. Rev. B*, **82**, 134524 (2010).
4. M. H. Devoret and R. J. Schoelkopf, *Science*, **339**, 1169 (2013).
5. T. Doderer, S. G. Lachenmann, A. V. Ustinov, et al., *Phys. Scr.*, **T49A**, 172 (1993).
6. A. V. Ustinov, *Physica D: Nonlin. Phenomena*, **123**, 315 (1998).
7. A. S. Malishevskii, V. P. Silin, and S. A. Uryupin, *Phys. Lett. A*, **306**, 153 (2002).
8. A. S. Malishevskii, V. P. Silin, S. A. Uryupin, and S. G. Uspenskii, *Kratkie Soobscheniya po Fizike FIAN*, **5**, 21 (2005) [*Bull. Lebedev Phys. Inst.*, **5**, 16 (2005)].
9. A. S. Malishevskii, V. P. Silin, and S. A. Uryupin, *Fizika Tverdogo Tela*, **49**, 1000 (2007) [*Phys. Solid State*, **49**, 1051 (2007)].
10. S. Sakai, P. Bodin, and N. F. Pedersen, *J. Appl. Phys.*, **73**, 2411 (1993).
11. S. Sakai, A. V. Ustinov, H. Kohlstedt, et al., *Phys. Rev. B*, **50**, 12905 (1994).
12. S. Sakai, A. V. Ustinov, N. Thyssen, and H. Kohlstedt, *Phys. Rev. B*, **58**, 5777 (1998).
13. J. Zitzmann, A. V. Ustinov, M. Levitchev, and S. Sakai, *Phys. Rev. B*, **66**, 064527 (2002).
14. A. S. Malishevskii, V. P. Silin, and S. A. Uryupin, *Doklady Akademii Nauk*, **478**, 284 (2018) [*Dokl. Phys.*, **63**, 5 (2018)].
15. P. M. Marcus and Y. Imry, *Solid State Commun.*, **33**, 345 (1980).
16. A. S. Malishevskii, V. P. Silin, S. A. Uryupin, and S. G. Uspenskii, *Phys. Lett. A*, **348**, 361 (2006).
17. V. V. Shmidt and G. S. Mkrtchyan, *Phys. Uspekhi*, **17**, 170 (1974).
18. A. S. Malishevskii, V. P. Silin, S. A. Uryupin, and S. G. Uspenskii, *Fiz. Tv. Tela*, **46**, 1544 (2004) [*Phys. Solid State*, **46**, 1591 (2004)].