DOI 10.1007/s10946-016-9599-z
Journal of Russian Laser Research, Volume 37, Number 5, September, 2016

AMPLIFICATION OF ELECTROMAGNETIC PULSES BY PHOTOIONIZED PLASMA

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Abstract

We consider the conditions of aperiodic instability appearance in photoionized plasma produced at the impact of a pulse of circularly polarized radiation on the gas. We show the possibility of electromagnetic radiation amplification under reflection by photoionized plasma. We give a comparative analysis of amplification patterns taking place in hydrodynamic and kinetic description of the photoelectrons dynamics.

Keywords: photoionized plasma, circularly polarized radiation, aperiodic instability, pulse amplification.

1. Introduction

The action of laser radiation on the atoms of a substance is accompanied by the formation of plasma with an anisotropic electron-velocity distribution (see, e.g., [1–7]). At sufficiently intense radiations, the conditions under which the time of atomic ionization is several orders of magnitude shorter than the lifetime of the anisotropic photoelectron distribution are easily implemented. In this case, one can speak of a photoionized plasma, the physical properties of which differ qualitatively from those of nearly equilibrium plasma. In particular, the anisotropy of the photoelectron distribution leads to a change in the optical properties of hot dense plasma, because the alternating magnetic field of laser radiation affects the electron kinetics [8–11].

Plasma with an anisotropic electron distribution is unstable [12–14]. As was shown in [15–17], such aperiodic instability can lead to the amplification of electromagnetic radiation reflected from photoionized plasma, as well as transmitted through a plasma slab [18, 19]. The spectrum of the amplified radiation is rather wide, with characteristic frequencies on the order of the instability growth rate. Note that, for typical parameters of photoionized plasma, the characteristic frequencies lie in the terahertz range [15–19].

Bearing in mind the existing interest in this frequency range and the qualitative novelty of the properties of photoionized plasma, we consider in this paper the interaction of test electromagnetic radiation with plasma formed as a result of tunnel ionization of atoms in the field of circularly polarized high-power radiation pulse. In Sec. 2, we show an explicit form of the photoelectron distribution function presented and the time interval during which the nonequilibrium distribution exists. In Sec. 3, we describe the growth rate of aperiodic instability and determine the boundaries of an instability domain in the wave-vector space and the maximum growth rate. In Sec. 4, we describe the anomalous amplification of the test laser pulse reflected by a nonequilibrium plasma using the hydrodynamic approximation.

Manuscript submitted by the authors in English on August 15, 2016.
1071-2836/16/3705-0473

We present the kinetic theory of the radiation amplification in Sec. 5 and show that the amplification efficiency is determined by the maximum value of the instability growth rate, which depends on the degree of anisotropy of the photoelectron distribution function.

2. Photoelectron-Velocity Distribution Function

Consider the interaction of ionizing ultrashort laser pulse with matter. We assume that the pulse duration is larger than the atom-ionization time but smaller than the time of change in the distribution of nonequilibrium photoelectrons over velocities. Also we assume that the laser field has a circular polarization and approximated by the expression

$$\mathbf{E}_{\text{pump}} = E_{\text{pump}}\big\{-\sin(\omega_{\text{pump}}\, t), \cos(\omega_{\text{pump}}\, t), 0\big\}, \tag{1}$$

where ω_{pump} is the carrier frequency of the pulse and E_{pump} is the electric field strength. We assume that the frequency and field strength (1) satisfy the conditions

$$mv_E^2 \gg 2I \gg \frac{3}{2}\hbar\omega_{\text{pump}}\sqrt{\frac{mv_E^2}{2I}}, \tag{2}$$

where $v_E = \left|\dfrac{eE_{\text{pump}}}{m\omega_{\text{pump}}}\right|$, e and m are the electron charge and mass, and I is the ionization potential of the substance atoms. Under these conditions, the regime of tunnel ionization of atoms in the electric field (1) is realized, and the distribution of photoelectrons over velocities $\mathbf{v}$ corresponds to the probability of ionization $W(\mathbf{v})$ derived in [1],

$$W(\mathbf{v}) \propto \exp\left\{-\frac{2}{3\hbar\omega_{\text{pump}}\sqrt{m}v_E}\left[2I + mv_z^2 + m(v_\perp - v_E)\right]^{3/2}\right\}. \tag{3}$$

From this formula and inequalities (2) it follows that the distribution function of the bulk photoelectrons with velocities

$$v_z^2 \ll 2I/m, \qquad (v_\perp - v_E)^2 \ll 2I/m \tag{4}$$

can be approximated as follows:

$$f_a(\mathbf{v}) \simeq \frac{n}{4\pi^2 v_E v_T^2}\exp\left[-\frac{(v_\perp - v_E)^2}{2v_T^2} - \frac{v_z^2}{2v_T^2}\right], \tag{5}$$

where n is the photoelectron density

$$v_T = \sqrt{\frac{\hbar\omega_{\text{pump}} v_E}{2\sqrt{2mI}}} \ll v_E. \tag{6}$$

From (5) for the average velocities of photoelectrons, we have $\sqrt{\langle v_z^2\rangle} \sim v_T$, $\langle v_\perp\rangle \sim v_E$. Taking into account this evaluation, in view of the degree of anisotropy of the distribution of photoelectrons, we can understand the relationship v_E/v_T.

Collisions of electrons lead to the isotropization of the distribution of photoelectrons. If the multiplicity of the ion ionization $Z_i > 1$, the process of isotropization is mainly determined by the collisions of electrons with ions. Herewith, the evolution of the initial distribution of photoelectrons (5) is described by the equation

$$\frac{\partial f}{\partial t} = \frac{1}{2}\nu(v)\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial f}{\partial \xi}, \qquad -1 < \xi < 1, \tag{7}$$

where $f = f(v, \xi, t)$, $\xi = \cos\theta$, θ is the angle between the velocity vector $\mathbf{v}$ and the anisotropy axis of distribution (5),

$$\nu(v) = 4\pi Z_i e^4 n \Lambda m^{-2} v^{-3} \tag{8}$$

is the collision frequency of electrons with ions, and Λ is the Coulomb logarithm. As for the initial distribution $v_E \gg v_T$, in the first time moments the bulk of the photoelectrons are localized in a relatively narrow velocity interval $v_E - v_T < v < v_E + v_T$ and in the region of angles close to $\pi/2$, when $\xi \leqslant v_T/v_E \ll 1$. Thus, for the first time moments, to describe the relaxation of the initial distribution of photoelectrons, we can approximately replace Eq. (7) by a more simple one, namely,

$$\frac{\partial f}{\partial t} \approx \frac{1}{2}\nu\frac{\partial^2 f}{\partial \xi^2}, \tag{9}$$

where $\nu \approx \nu(v_E)$. Considering $\nu t \ll 1$, we can write the approximate solution to Eq. (9) in the form

$$f(v, \xi, t) = \int\limits_{-\infty}^{+\infty} \frac{d\xi'}{\sqrt{2\pi\nu t}} \exp\left[-\frac{(\xi - \xi')^2}{2\nu t}\right] f(v, \xi', t = 0), \tag{10}$$

where $\xi \leqslant \nu t \ll 1$, and the initial distribution $f(v, \xi, t = 0)$ is described by (5), with $v_z = v\xi$ and $v_\perp = v\sqrt{1-\xi^2}$.

At small times, when $\nu t \ll v_T^2/v_E^2$, the distribution (10) is close to the initial distribution of photoelectrons. If

$$v_T^2/v_E^2 \ll \nu t \ll 1,$$

then from (10) we approximately find

$$f(v, \xi, t) \approx \frac{n}{4\pi^2 v_E^2 v_T \sqrt{\nu t}} \exp\left[-\frac{\xi^2}{2\nu t} - \frac{(v_\perp - v_E)^2}{2v_T^2}\right]. \tag{11}$$

According to Eq. (10), electron collisions lead to a broadening of the distribution of photoelectrons over the velocity angles. As can be seen from (11), the domain of the bulk electron localization in the velocity space expands with time according to the law $-\sqrt{\nu t} \leqslant \xi \leqslant \sqrt{\nu t}$. In view of (11), we have for the degree of electron distribution anisotropy $v_E/\sqrt{\langle v_z^2\rangle}$

$$\frac{v_E}{v_T} \gg \frac{v_E}{\sqrt{\langle v_z^2\rangle}} \propto \frac{1}{\sqrt{\nu t}} \gg 1.$$

When $\nu t \approx 1$, the electron–ion collisions lead to the isotropic distribution of photoelectrons. If $Z_i \gg 1$, at the time $\nu t \leqslant 1$, the effect of electron–electron collisions on the relaxation of the initial photoelectron distribution can be neglected. If $Z_i = 1$, the electron–electron collisions speed up the process of isotropization of the photoelectron distribution. In addition, the electron–electron collisions lead to the energy relaxation accompanied by the formation of Maxwell electron distribution at times of the order of ν^{-1}. From this consideration it follows that the characteristic preservation time of the photoelectron distribution anisotropy is $\sim \nu^{-1}$.

3. Growth Rate of Aperiodic Instability

As is well known (see, for example, [12]), the plasma with anisotropic electron velocity distribution is unstable with respect to the development of non-potential aperiodic instabilities. Recall the basic statements of the corresponding theory.

Consider small perturbations of electric and magnetic fields of the form

$$\mathbf{E} \sim \mathbf{B} \sim \exp(-i\omega t + i\mathbf{kr}). \tag{12}$$

Neglecting electron collisions, from the kinetic equation for electrons and the Maxwell equations, we obtain the following system of equations:

$$\left\{ k^2\delta_{ij} - k_i k_j - \frac{\omega^2}{c^2}\varepsilon_{ij} \right\} E_j = 0, \qquad i,j = x,y,z, \tag{13}$$

with the permittivity tensor of the form

$$\varepsilon_{ij} \equiv \varepsilon_{ij}(\omega,\mathbf{k}) = \delta_{ij}\left(1 - \frac{\omega_L^2}{\omega^2}\right) + \frac{\omega_L^2}{n\omega^2}\int d\mathbf{v}\,\frac{v_i v_j}{\omega - \mathbf{kv}}\left(\mathbf{k}\frac{\partial f_a}{\partial \mathbf{v}}\right). \tag{14}$$

Equations (13) have a nontrivial solution if the determinant of this system of equations is equal to zero.

In the low-frequency range $|\omega| \ll \omega_L$, there is a purely imaginary solution $\omega = i\gamma$ corresponding to the possibility of aperiodic instability development with growth rate γ. When the average kinetic energy of the electrons perpendicular to the anisotropy axis exceeds the energy of their motion along it, the perturbations with wave vectors directed along this axis are built up most effectively (for details, see, for example, [15, 17, 20]).

Being interested in optimum conditions of the instability development, we restrict our consideration to perturbations with wave vectors $\mathbf{k} = \{0, 0, k\}$. For such perturbations, the electromagnetic field is transverse and, to be specific, we assume that $\mathbf{E} = \{E, 0, 0\}$ and $\mathbf{B} = \{0, B, 0\}$. Then we obtain the equation for $\omega = i\gamma$ of the form

$$D(k,\omega) \equiv k_E^2 + \frac{\omega^2}{c^2} - \left(k_E^2 + \frac{\omega_L^2}{c^2}\right) Q\left(\frac{\omega}{kv_T}\right) - k^2 = 0, \tag{15}$$

where

$$k_E^2 = \frac{\omega_L^2}{2c^2}\left(\frac{v_E^2}{v_T^2} + 1\right), \tag{16}$$

$$Q(z) = \int_{-\infty}^{+\infty} \frac{d\xi}{\sqrt{2\pi}}\, e^{-\xi^2/2}\,\frac{z}{z - \xi} = -i\sqrt{\frac{\pi}{2}}\, zw\left(\frac{z}{\sqrt{2}}\right), \tag{17}$$

with $w(z)$ being the function related to the error function (see Eq. (7.1.4) in [21]).

In the limit case $kv_T \ll |\omega| \ll \omega_L$, from Eq. (15) we obtain an approximate expression for the growth rate as follows:

$$\gamma \approx \frac{\omega_L k v_E}{\sqrt{2}\sqrt{\omega_L^2 + k^2c^2}}. \tag{18}$$

Expression (18) is valid in the range of relatively small wave numbers corresponding to the hydrodynamic limit. From (18), we see that, in this limit, the growth rate of aperiodic instability is a monotonically increasing function of the wave number. With increasing in k (18), the γ tends to the value $\gamma_E = k_E v_T \approx \omega_L(v_E/\sqrt{2}c) \ll \omega_L$, which is the maximum possible growth rate of aperiodic instability in unbounded plasma with the electron distribution (5) [22]. However, for sufficiently large values of $k \sim |\omega|/v_T$, the approximate expression (18) is not valid, and it is necessary to solve numerically Eq. (18) in order to obtain the growth rate $\gamma(k)$.

In Fig. 1, we show the function $\gamma(k)/\gamma_E$ in the region $k > 0$ for three values of the ratio v_E/v_T; here, curve 1 corresponds to $v_E/v_T = 6$, curve 2 to $v_E/v_T = 10$, and curve 3 to $v_E/v_T = 15$. From Fig. 1, we see that instability can develop only if $|k| < k_E$, which corresponds to not very small spatial scales. The growth rate $\gamma(k)$ has a strongly marked maximum.

The maximum growth rate for a given value of v_E/v_T, $\gamma_m \equiv \gamma_m(v_E/v_T) = \gamma(k_m)$, which takes place at $k_m \equiv k_m(v_E/v_T)$, corresponds to the field perturbations that are most efficiently amplified during the aperiodic instability development. The values of γ_m and k_m are indicated on curve 1 in Fig. 1.

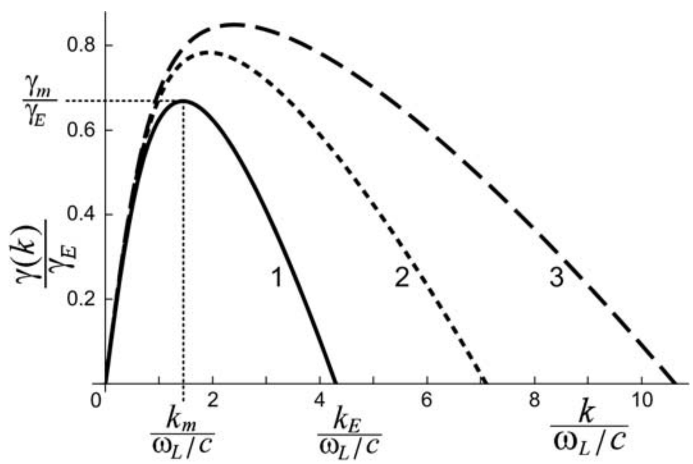

Fig. 1. The growth rate of aperiodic instability $\gamma(k)/\gamma_E$ versus the wave number $k\,c/\omega_L$ for different values of the ratio $v_E/v_T = 6$ (curve 1), 10 (curve 2), and 15 (curve 3).

In Figs. 2 and 3, we present the dimensionless quantities γ_m/γ_E and $k_m c/\omega_L$ as functions of the ratio $v_E/v_T \gg 1$, which characterizes the degree of anisotropy of the initial distribution (5).

From Fig. 2, we see that the maximum growth rate satisfies the inequality $\gamma_m < \gamma_E$, and the quantity γ_m monotonically increases with increase in v_E/v_T, asymptotically approaching γ_E.

According to Fig. 3, the wave number k_m corresponding to the most efficiently amplified perturbations also increases with increase in v_E/v_T.

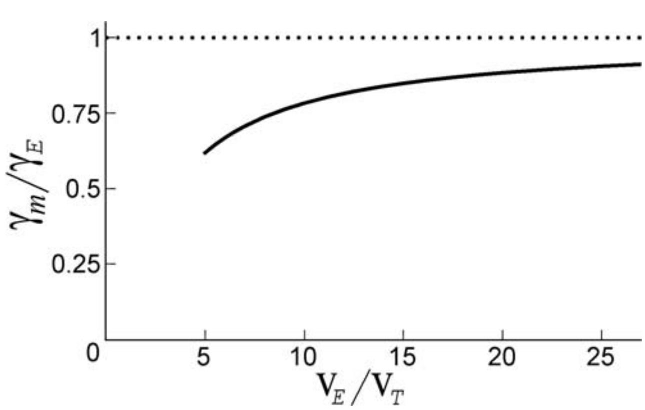

Fig. 2. The growth rate of aperiodic instability γ_m/γ_E versus the ratio v_E/v_T.

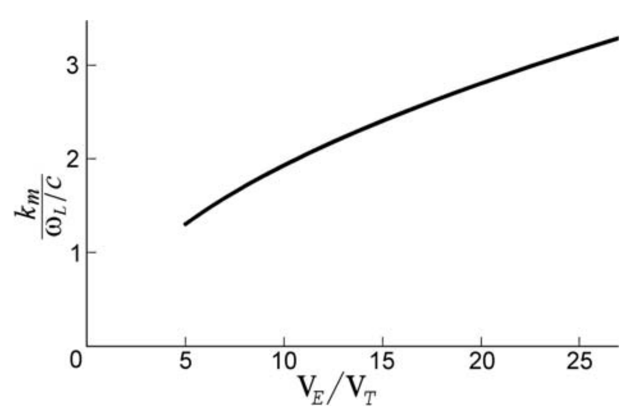

Fig. 3. The wave number $k_m c/\omega_L$, at which the growth rate of aperiodic instability γ_m reaches its maximum value, versus the ratio v_E/v_T.

4. Amplification of Reflected Field. Hydrodynamic Approach

Assume that the half-space $z > 0$ is occupied by the plasma formed under the ionization of matter in the strong laser pulse of circularly polarized radiation. At time $t = 0$, the test electromagnetic pulse

$$\mathbf{E}_i(z,t) = \{E_i(t - z/c), 0, 0\}, \qquad \mathbf{B}_i(z,t) = \{0, E_i(t - z/c), 0\} \tag{19}$$

falls on the plasma surface. Here, $E_i(t - z/c) = E_L\,\eta(t - z/c)\sin[\omega_0(t - z/c)]$, $\omega_0 < \omega_L$, and $\eta(\tau)$ is the Heaviside function.

In this section, we describe the response of a nonequilibrium plasma with the electron velocity distribution (5) on the pulse action (19) in the hydrodynamic approximation. We adopt a system of equations for the mean electron velocity $\mathbf{u}(z,t)$, momentum flux density tensor $P_{ij}(z,t)$, vortex electric $\mathbf{E}(z,t)$, and magnetic $\mathbf{B}(z,t)$ fields (see, for example, [15]). For an electron velocity distribution of the form (5), the momentum flux density tensor $P_{ij} = m\int d\mathbf{v}\ v_i v_j f_a(\mathbf{v})$ is diagonal,

$$P_{xx} = P_{yy} \simeq \frac{1}{2} nmv_E^2 \gg P_{zz} \simeq nmv_T^2. \tag{20}$$

Assuming that the field (19) action on plasma is weak, we describe its influence on the field strength in the linear approximation.

The pulse of the form (19) leads to the excitation of the plasma electromagnetic field with components $\mathbf{E}(z,t) = \{E(z,t), 0, 0\}$ and $\mathbf{B}(z,t) = \{0, B(z,t), 0\}$.

At the same time, one velocity component $\delta\mathbf{u}(z,t) = \{\delta u(z,t), 0, 0\}$ and two identical momentum flux density tensor components $\delta P_{xz}(z,t) = \delta P_{zx}(z,t) \equiv \delta P(z,t)$ are nontrivial. For these quantities, we have the linearized system of equations

$$\begin{aligned} &\frac{\partial \delta u}{\partial t} + \frac{1}{mn}\frac{\partial \delta P}{\partial z} = \frac{e}{m}E, \qquad \frac{1}{mn}\frac{\partial \delta P}{\partial t} + v_T^2\frac{\partial \delta u}{\partial z} = \left(\frac{1}{2}v_E^2 - v_T^2\right)\frac{eB}{mc}, \\ &\frac{\partial B}{\partial t} + c\frac{\partial E}{\partial z} = 0, \qquad \frac{\partial E}{\partial t} + c\frac{\partial B}{\partial z} = -4\pi en\,\delta u. \end{aligned} \tag{21}$$

To solve the system of equations (21) for $t > 0$, we use the Laplace transform where the original function $F(t)$ and its image $F(\omega)$ are related as follows:

$$F(\omega) = \int_0^{+\infty} dt\, F(t)\exp(i\omega t), \qquad F(t) = (2\pi)^{-1}\int_{-\infty+i\Delta}^{+\infty+i\Delta} d\omega\, F(\omega)\exp(-i\omega t),$$

where $\Delta > 0$ is larger then the exponential growth rate of the function $F(t)$. We assume that in plasma the perturbations of E, B, δu, and δP due to the action of pulse (19) are much higher than their values due to thermal fluctuations. This fact allows us to assume that, at time $t = 0$, the initial values of perturbations of these quantities are approximately equal to zero.

Under the conditions (20), the system of equations (21) permits significant simplification if the characteristic electron velocity along the anisotropy axis satisfies the inequality $v_T \ll L_z|\omega|$, where L_z is the minimum characteristic-length scale of variation in physical quantities along the z axis. Then, from

(21), we obtain the second-order differential equation for the Laplace image of the electric field $E(z,\omega)$ in plasma

$$\left\{\frac{\partial^2}{\partial z^2} - k^2(\omega)\right\} E(z,\omega) = 0, \tag{22}$$

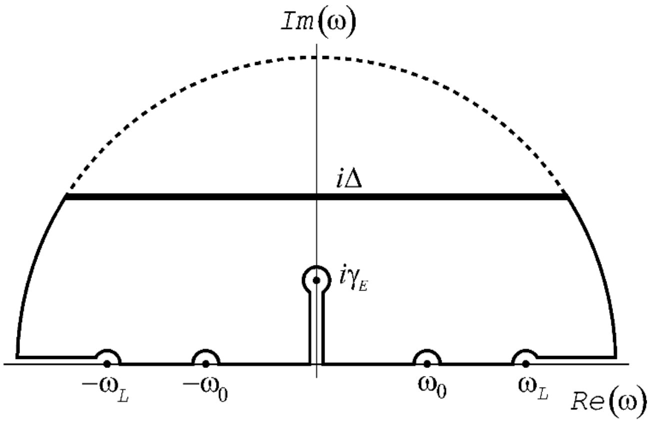

Fig. 4. Path of integration in the complex plane of variable ω.

where $k^2(\omega) = \dfrac{\omega^2}{c^2}\dfrac{\omega_L^2-\omega^2}{\gamma_E^2+\omega^2}$. The solution to Eq. (22) that does not grow deep into the plasma is $E(z=0,\omega)\exp[-k(\omega)z]$, $z>0$, where for the function $k(\omega)$ we chose the branch in the plane of the complex variable ω satisfying the condition $\mathrm{Re}\,[k(\omega)] \geqslant 0$ on the straight line $\mathrm{Im}\,[\omega]=\Delta$ along which the integration is performed in the inverse Laplace transform (see Fig. 4).

In the case of the electron distribution axially symmetric with respect to the z axis, the reflected field satisfying the Maxwell equations at $z<0$ reads

$$\mathbf{E}_r(z,t) = \{E_r(t+z/c),0,0\}\,, \qquad \mathbf{B}_r(z,t) = \{0,-E_r(t+z/c),0\}\,. \tag{23}$$

Using the continuity of tangential electric and magnetic field components at the plasma boundary $z=0$, for the electric field outgoing from the plasma we have

$$E_r\left(t+\frac{z}{c}\right) = -E_i\left(t+\frac{z}{c}\right) + \int\limits_{-\infty+i\Delta}^{+\infty+i\Delta} \frac{d\omega}{2\pi}\,E_i(\omega)\frac{2\omega}{\omega+i\,k(\omega)c}\exp\left[-i\omega\left(t+\frac{z}{c}\right)\right], \tag{24}$$

where $E_i(\omega)=E_L\,\omega_0/(\omega_0^2-\omega^2)$ is the Laplace image of the electric field (19) incident on the plasma.

In order to calculate the integral in (24), we use the integration contour in the plane of the complex variable ω shown in Fig. 4. According to (24), for $t>0$, the front of the reflected field reaches $z_f=-c\,t$. At $z=z_f$, the field (24) is zero. The reflected field is localized in the space $z_f<z<0$ and for time $t>0$ in the point $z<0$ is specified by the field strength on the plasma surface at an earlier time $\tau=t+z/c<t$. Departure from the plasma surface field (24) consists of two parts:

$$E_r = E_r^{(\omega)} + E_r^{(\gamma)}. \tag{25}$$

The first part $E_r^{(\omega)}$ corresponds to the sum of the field $-E_i(t+z/c)$ and the contributions to the integral term in (24) from the contour segments in Fig. 4 passing along the real axis. This part describes the non-increasing contribution to the reflected field, the spatio–temporal structure of which is similar to the radiation field reflected by equilibrium plasma. As plasma is in the nonequilibrium state, there is an additional contribution $E_r^{(\gamma)}$ to the outgoing field due to the aperiodic instability development. This contribution to the integral part of (24) comes from the sides of the cut along the imaginary axis on the interval $(0,i\gamma_E)$ and has the form

$$E_r^{(\gamma)}(\tau) = -\eta[\tau]\,E_S\int\limits_{-\gamma_E}^{\gamma_E}\frac{d\mu}{\pi}\frac{\sqrt{\gamma_E^2-\mu^2}}{\omega_0^2+\mu^2}\exp(\mu\tau)\mathrm{sign}(\mu), \tag{26}$$

where $E_S = (2\omega_0/\omega_L)\,E_L$ is the amplitude of the electric field in plasma near the surface under the conditions of the high-frequency skin-effect for a time interval less than the reverse instability growth rate.

Far from the front of the outgoing radiation, where $\tau = t + z/c \gg (\omega_0^{-1}, \gamma_E^{-1})$, the growing field (26) may be approximated by the expression

$$E_r^{(\gamma)}(\tau) \simeq -\frac{E_S}{\sqrt{2\pi}}\frac{1}{1+\omega_0^2/\gamma_E^2}\frac{\exp(\gamma_E\tau)}{(\gamma_E\tau)^{3/2}}. \tag{27}$$

We see from (27) that the amplitude of the function $E_r^{(\gamma)}(\tau)$ has a maximum when the frequency ω_0 coincides with γ_E.

5. Amplification of Reflected Field. Kinetic Approach

In this section, we consider the features of reflected field amplification, using the kinetic equation to describe the electron motion. Thus, for the fields in plasma and a small perturbation of the electron distribution function $\delta f \equiv \delta f(\mathbf{v}, z, t)$, we have the equations

$$\frac{\partial \delta f}{\partial t} + v_z\frac{\partial \delta f}{\partial z} = -\frac{e}{m}\left\{\mathbf{E} + \frac{1}{c}\,[\mathbf{vB}]\right\}\frac{\partial f_a}{\partial \mathbf{v}}, \tag{28}$$

$$\frac{\partial B}{\partial t} + c\frac{\partial E}{\partial z} = 0, \qquad \frac{\partial E}{\partial t} + c\frac{\partial B}{\partial z} = -4\pi e\int d\mathbf{v}\, v_x\, \delta f. \tag{29}$$

As before, we assume zero initial conditions for perturbations of all the quantities in plasma and consider only the forced solutions to Eqs. (28) and (29) induced by the field (19).

As in the previous section, for solving Eqs. (28) and (29), we use the Laplace transform and the simplest boundary condition corresponding to the mirror reflection of electrons from the plasma boundary. In this case, after reflection the electron velocity component normal to the plasma boundary changes its sign, while the electron velocity components parallel to the plasma boundary remain unchanged. Also we take into account that the perturbed electromagnetic field and function δf vanish deep in the plasma. Then, from Eqs. (28) and (29), we obtain the integro-differential equation for $E(z,\omega)$ in plasma $z > 0$,

$$\left[\frac{\partial^2}{\partial z^2} + \left(k_E^2 + \frac{\omega^2}{c^2}\right)\right] E(z,\omega) = \left(k_E^2 + \frac{\omega_L^2}{c^2}\right)\int\limits_0^{+\infty} dz'\,\left[Q(z+z',\omega) + Q(|z-z'|,\omega)\right]\, E(z',\omega), \tag{30}$$

where the kernel entering into the integral reads

$$Q(|z|,\omega) = -\frac{i\,\omega}{\sqrt{2\pi}\,v_T}\int\limits_0^{+\infty}\frac{d\xi}{\xi}\,\exp\left[-\frac{\xi^2}{2} + i\frac{\omega}{v_T\xi}|z|\right]. \tag{31}$$

Equation (30) should be supplemented with conditions corresponding to the continuity of the tangential components of electric and magnetic fields on the surface $z = 0$.

Following [17] for the electric field reflected from the plasma, we arrive at

$$E_r\left(t + \frac{z}{c}\right) = -E_i\left(t + \frac{z}{c}\right) + \int\limits_{-\infty+i\Delta}^{+\infty+i\Delta}\frac{d\omega}{\pi}\frac{E_i(\omega)\,Z(\omega)}{1+Z(\omega)}\exp\left[-i\omega\left(t + \frac{z}{c}\right)\right], \qquad z < 0, \tag{32}$$

where $Z(\omega) = E(z = 0, \omega)/B(z = 0, \omega)$ is the frequency-dependent surface impedance determined according to

$$Z(\omega) = \lim_{z\to+0} \frac{i\,\omega}{c} \int_{-\infty}^{+\infty} \frac{dk}{\pi} \frac{\mathrm{e}^{ikz}}{D(k,\omega)}. \tag{33}$$

Taking into account the properties of the function $E_i(\omega)$ and solutions to Eq. (15) in the region $\mathrm{Im}[\omega] \geqslant 0$, in order to calculate the integral over ω in (32), we choose the integration contour shown in Fig. 4, in which the maximum possible increment γ_E should be replaced by the greatest one γ_m for a given degree of anisotropy v_E/v_T increment, $\gamma_m < \gamma_E$. This change takes place in a more precise kinetic approach and is essential in describing the exponential increase in the reflected field.

The reflected field (32) is the sum of two parts — the increasing and non-increasing (in time) fields (25). From Eq. (32), we find that the increasing component of the reflected field is

$$E_r^{(\gamma)}(\tau) = \eta[\tau] \int_0^{\gamma_m} \frac{i\,d\eta}{\pi} E_i(i\eta)\, e^{\eta\,\tau} \left[\left(1 + \frac{1}{Z(i\eta - 0)}\right)^{-1} - \left(1 + \frac{1}{Z(i\eta + 0)}\right)^{-1} \right], \tag{34}$$

where $Z(i\eta - 0)$ corresponds to the value of the function $Z(\omega)$ on the left bank of the cut along the imaginary axis, and $Z(i\eta + 0)$ corresponds to that on the right bank (see Fig. 4). For times exceeding the reciprocal of the growth rate of aperiodic instability, it is the contribution (34) exponentially increasing with time that dominates and determines the structure of the reflected field $E_r \simeq E_r^{(\gamma)}$ far from its leading edge, where the inequality $\tau = t + z/c \gg \gamma_m^{-1}$ is satisfied. Under this condition, we obtain the following asymptotic expression for the increasing part of the reflected field, which is valid at long times and sufficiently large distances behind the leading edge of the radiation propagating away from the plasma:

$$E_r^{(\gamma)}(\tau) \approx \frac{E_S}{\sqrt{2\pi}} \frac{1 + 2\,v_T/v_E}{1 + \omega_0^2/\gamma_m^2} \frac{\exp(\gamma_m \tau)}{\sqrt{\gamma_m \tau}}. \tag{35}$$

Since the tangential components of the field are continuous at the plasma boundary, expression (35) calculated at $z = 0$ (which corresponds to $\tau = t$) describes the field amplification in the skin layer.

Qualitatively, the phenomenon of amplification can be understood from the following considerations.

A test pulse, with carrier frequency $\omega_0 < \omega_L$ and electric field amplitude E_L, penetrates over a time $\sim 1/\omega_0$ to the skin depth and produces an electric field with strength of $\sim (2\omega_0/\omega_L)\,E_L$. In the skin layer, this field is amplified due to the onset of instability development and is radiated back into vacuum.

Expression (35), which was obtained within the framework of the kinetic approach, differs from expression (27), which was obtained in the previous section within the framework of the hydrodynamic consideration of the electron dynamics. In contrast to the result (27), where the exponential increase with time was characterized by the growth rate γ_E,

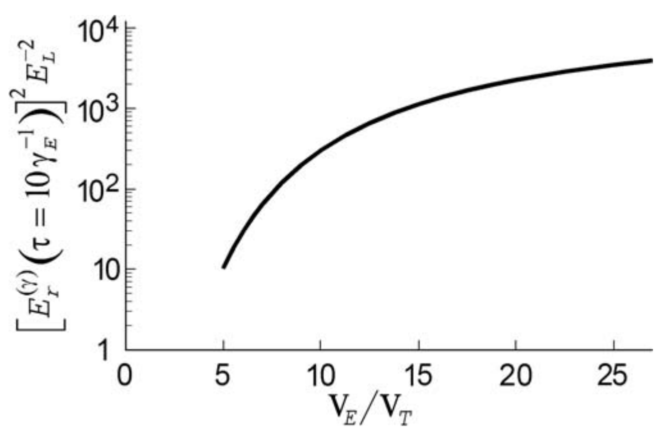

Fig. 5. Dependence of the ratio $[E_r^{(\gamma)}(\tau = 10\,\gamma_E^{-1})]^2/E_L^2$ illustrating the amplification of the reflected field at the instant $\tau = 10\,\gamma_E^{-1}$ on the ratio v_E/v_T.

the amplification of the reflected field described by expression (35) proceeds more slowly because it is determined by a smaller growth rate γ_m, which depends on the degree of anisotropy v_E/v_T of the initial distribution (5); see Fig. 2. Another difference between expression (35) and (27) is the different pre-exponential factor. Instead of $(\gamma_E\tau)^{-3/2}$, expression (35) contains the large quantity $(1+2\,v_T/v_E)/\sqrt{\gamma_m\tau}$, and the field $E_r^{(\gamma)}(\tau)$ has the opposite sign.

To illustrate the amplification of the reflected field, in Fig. 5 we show the ratio between the squared amplitudes of the reflected and test fields $[E_r^{(\gamma)}(\tau = 10\gamma_E^{-1})]^2/E_L^2$ as a function of the parameter v_E/v_T, which characterizes the anisotropy of the electron distribution. The curve was calculated for $\tau = 10\gamma_E^{-1}$ and the following parameters of the plasma and radiation: $n = 10^{18}$ cm^{-3}, $mv_E^2 \simeq 330$ eV, and $\omega_0 \simeq \gamma_E$. We see from Fig. 5 that, as the ratio v_E/v_T increases from 5 to 25, the amplification factor of the reflected field increases by more than two orders of magnitude.

6. Conclusions

We considered the amplification of test electromagnetic radiation by plasma with an anisotropic electron velocity distribution formed as a result of the tunnel ionization of gas atoms by a circularly polarized radiation pulse. We showed that the effect of amplification of the reflected pulse takes place within the framework of the kinetic and hydrodynamic approaches. However, the quantitative properties of the amplification are different. In the kinetic approach, the amplification of the reflected pulse is characterized by a smaller growth rate, which coincides with that obtained in the hydrodynamic approach in the case of a very large degree of electron distribution anisotropy. These quantitative amplification characteristics of the reflected pulse seem to be important because, under actual conditions, the degree of anisotropy is usually moderate.

Let us consider typical conditions under which the above amplification effect can take place. As a result of ionization of hydrogen atoms by a circularly polarized laser pulse with an intensity of $I_{\text{pump}} = 2 \cdot 10^{15}$ W/cm^2, a frequency of $\omega_{\text{pump}} = 2 \cdot 10^{15}$ s^{-1}, and duration of a few tens of femtoseconds, plasma with anisotropic photoelectron distribution (5), in which $mv_E^2 \simeq 330$eV and $v_E/v_T \simeq 12$, is produced. For such plasma with an electron density of $n = 10^{18}$ cm^{-3}, the growth rate of aperiodic instability is $\gamma_m \approx 0.83 \cdot 10^{12}$ s^{-1}.

At times longer than the reciprocal of the growth rate of aperiodic instability but shorter than both the time over which the initial distribution becomes isotropic and the time of nonlinear saturation of instability, the reflected field increases exponentially with time. The reflected field is maximum when the frequency of the test radiation incident on the plasma is close to the growth rate of instability. However, as is seen from formula (35), the dependence of the amplified field on the frequency of the incident test pulse has the form $\sim \omega_0/(\omega_0^2 + \gamma_m^2)$. This allows one to speak of the possibility of amplification of not only monochromatic signals with $\omega_0 \sim \gamma_m$, but also signals with a spectral width of $\sim \gamma_m$.

The function $\omega_0^2/(\omega_0^2 + \gamma_m^2)^2$ characterizes the amplification efficiency in a given frequency band. Since the amplified fraction of the reflected field is concentrated within a wide frequency band near γ_m, the above mechanism of amplification of the reflected field can be used to amplify and generate electromagnetic radiation, in particular, in the terahertz frequency range. In this case, the typical values of the amplification factor can reach several orders of magnitude (see Fig. 5). Thus, upon the incidence of radiation with intensity $I_L = c\,E_L^2/8\pi = 10^3$ W/cm^2 and frequency $\omega_0 \approx \gamma_m$ on plasma with the above parameters, the intensity of radiation reflected from the plasma at the time $t = 10\,\gamma_m^{-1}$ can reach the value $I^{(\gamma)} = c\,(E_r^{(\gamma)})^2/8\pi \approx 1.4 \cdot 10^6$ W/cm^2.

Comparison of the results of Sec. 4, where the effect of amplification was described without regard for details of the electron velocity distribution, and the results of Sec. 5 allow us to conclude that amplification of the test signal by anisotropic plasma is, to some extent, a universal effect. To observe this effect, it is sufficient to create conditions under which aperiodic instability is possible upon atomic ionization.

Acknowledgments

This work was partially supported by the Russian Foundation for Basic Research under Project No. 15-02-07490.

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