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ANALYTICAL SOLUTION OF THE SCALAR WAVE EQUATION FOR SLIGHTLY DEFORMED OPTICAL BRAGG WAVEGUIDE

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Abstract

Using a simple analytical method, we study the electromagnetic-wave propagation in an optical Bragg waveguide which is slightly deformed at one side. We show the results of computations in the form of dispersion curves and provide a comparison with standard circular Bragg waveguides and step-index circular waveguides. We show that the correspondence between the cutoff V values for the standard step-index fiber and the new Bragg waveguide, which is slightly flattened on one side, is quite close for the lowest mode under the weak guidance condition.

Keywords: Bragg waveguide, deformed Bragg waveguide, weak guidance condition, characteristic dispersion relation.

1. Introduction

Conventional optical waveguides confine optical waves to the core due to the total reflection between the core and cladding interface but these optical waves also penetrate into the cladding in the form of evanescent waves which are of an exponentially decaying nature. On the other hand, Bragg waveguides confine their waves in a central region due to the Bragg reflection originating from the periodicity in the claddings where the refractive index, number of layers, profiles, materials, etc., of cladding regions play an essential role as a guiding mechanism in Bragg waveguides [1–7]. Recently, circular Bragg waveguides have attracted much attention because of their extraordinary properties and significant practical applications [5,6]. In contrast to the conventional step-index guiding fibers, the Bragg waveguides have an air core surrounded by some dielectric multilayer claddings that confines light by Bragg reflection, as shown

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in Fig. 1 a. This kind of waveguide is of particular interest in guiding light through the air core in the far infrared region, where conventional fibers are limited by absorption losses due to the fiber materials. These Bragg waveguides have been successfully used for CO_2 laser operating at 10.6 μm [8]. On the other hand, the invention of a perfect dielectric layer by applying high and low refractive-index materials with a large contrast enables great reduction in leakage losses, leading to potential applications in long distance communications.

To date, several numerical approaches have been proposed to analyze the modal properties of Bragg fibers [9–12]. Vivek Singh et al. [13] presented a very simple analytical approach to analyzing the propagation characteristic of a circular Bragg fiber. Based on this study, we also gave the description of unconventional Bragg waveguides [14–17]. However, these descriptions were mainly focused on the formation of modes in such waveguides where some design parameters are considered and compared. Many of these considered geometries are the modifications or distortions of circular and rectangular cross-sections [18, 19]. The basic idea of these works is to see how distortions in the waveguide cross-section change the modal characteristics, such as the number of guided modes sustained by the waveguide.

In this paper, a slightly distorted optical Bragg waveguide having a small number of alternating layers is considered (see Fig. 1 b). This distortion is deliberately introduced in the Bragg waveguide to study its effect on the propagation characteristics. Using a simple matrix method and replacing the boundary condition by a matrix equation, we obtain the modal characteristic equation for the proposed Bragg waveguide. We show the results of computations in the form of dispersion curves and compare them with dispersion curves of a standard Bragg fiber having a circular core [13] and also with a weakly guiding step-index fiber [20].

This paper is organized as follows.

In Sec. 2, we outline the derivation of the characteristic equations from the Helmholtz wave equation and then calculate the characteristic dispersion relation, cutoff frequencies, etc., with the help of Bessel functions using the weak guidance condition. In Sec. 3, we consider numerical results and discuss these results. Finally, the paper ends with some remarks in the conclusion given in Sec. 4.

2. Theoretical Background

The cross-sectional view of the standard Bragg waveguide and proposed unconventional Bragg waveguide are shown in Fig. 1 a and b, respectively. The modal characteristic equation of the standard Bragg waveguide is given in [13] and its diagram is given here only for comparison. We analyze the proposed unconventional Bragg waveguide in the following sections.

2.1. The Characteristic Eigenvalue Equation

A simple matrix method [13] is used to compute the modal characteristics of the proposed waveguide. The basic idea is to replace the boundary condition by a matrix equation. The cross-sectional view of a six layered Bragg waveguide is shown in Fig. 1 b. It has low refractive index n_a in the central region and higher refractive indices n_1 and n_2 ($n_1 > n_2$) in the cladding regions around it. Thereby, we suitably design alternating claddings of high and low refractive indices:

$$n(r) = \left(\begin{array}{l} n_a, \quad 0 < r < b; \quad n_1, \quad b < r < a; \quad n_2, \quad a < r < a+b; \quad n_1, \quad a+b < r < a+2b; \\ n_2, \quad a+2b < r < a+3b; \quad n_1, \quad a+3b < r < a+4b; \quad n_a, \quad r > a+4b. \end{array} \right. \tag{1}$$

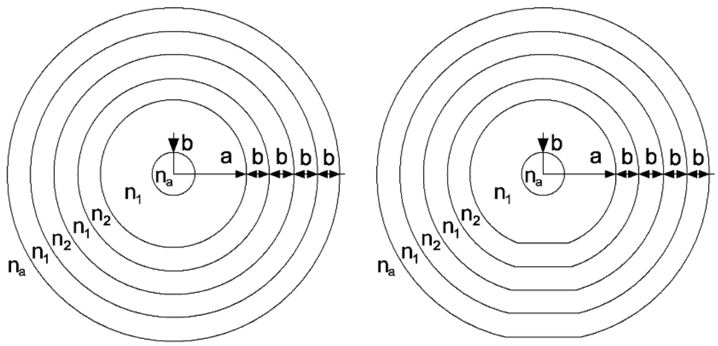


Fig. 1. The cross-sectional view of the standard Bragg waveguide (a) and the cross-sectional view of a slightly flattened core Bragg waveguide (b).

The index profile is then given in Fig. 1.

The polar equation of the curve in polar coordinates is written as

$$r = \rho \exp(1 + \sin\theta), \tag{2}$$

where ξ is a size parameter. At the core–cladding boundary, we put its value equal to a, where a is a fixed constant.

The equation of the normal curve to the curve represented by Eq. (2) in polar coordinates reads

$$r_\perp = \eta \frac{\cos\theta}{1 + \sin\theta}\,. \tag{3}$$

For our proposed waveguide structure, a new coordinate system (ρ, η, z) is suitable. Using the new coordinates and Maxwell equations, we can obtain the expressions for the fields E and H in terms of the new coordinates. The details of this procedure are given in our previous paper [17]. As a first approximation, we write the longitudinal components of the fields for the modes as follows:

$$E(\rho) = \left[A_i J_m(U_i\, d\rho) + B_i Y_m(U_i\, d\rho)\right], \tag{4}$$

$$H(\rho) = \left[C_i J_m(U_i\, d\rho) + D_i Y_m(U_i\, d\rho)\right], \tag{5}$$

where $U_i^2 = k^2 n_i^2 - \beta^2$, $i = 1$, 2 correspond to refractive indices n_1 and n_2. We present the solution for the central region and the outermost region as

$$E(\rho) = \left[E I_m(W\, d\rho) + F K_m(W\, d\rho)\right], \tag{6}$$

$$H(\rho) = \left[G I_m(W\, d\rho) + H K_m(W\, d\rho)\right], \tag{7}$$

where $W = \sqrt{\beta^2 - k^2 n_a^2}$, with n_a being the common refractive index of these regions. In the above equations, J_ν and Y_ν are Bessel functions of the first and second kinds, while I_ν and K_ν are the modified Bessel functions, respectively, and A_i, B_i, C_i, D_i, E, F, G, and H are unknown constants. Here, β is the axial component of the propagation vector, ω is the wave frequency, and μ is the permeability of the non-magnetic medium. Also, d is a number $\sqrt{e/2}$ and $e = 2.718$ which emerges in the analysis because of the peculiarity of the geometrical shape, and A, B, C, D, E, F, G, and H are unknown constants to be determined.

We have the boundary conditions

$$E(\rho)|_r = E(\rho)|_{ri}\,, \tag{8}$$

$$\left.\frac{\partial E(\rho)}{\partial \rho}\right|_r = \left.\frac{\partial E(\rho)}{\partial \rho}\right|_{ri}. \tag{9}$$

Thus, we obtain a set of equations having twenty-two unknown constants. The nontrivial solution will exist only when the determinant formed by the coefficients of the unknown constants is equal to zero. Calling this 12×12 determinant Δ, we have

$$\Delta = 0. \tag{10}$$

The elements in the rows and columns of this determinant can be identified readily.

We also define (see Fig. 1) that

$$\Delta n = n_1 - n_2, \quad \Delta n' = n_1 - n_a, \quad V = k_0(a-b)(n_1^2 - n_a^2)^{1/2} = k_0(a-b)\left[2n(\Delta n + \Delta n')\right]^{1/2}, \tag{11}$$

where k_0 is the vacuum wave number.

We define the usual normalized propagation parameter for the weakly guidance case as follows:

$$B = \frac{\beta^2 - k_0^2 n_a^2}{k_0^2(n_1^2 - n_a^2)} \approx \frac{\beta - k_0 n_a}{k_0(\Delta n + \Delta n')}. \tag{12}$$

We introduce the dimensionless parameter V to incorporate the parameters n_a, n_1, a, b, and k_0 which possibly have an effect on the propagation. One can choose other alternative ways to define the quantities V and B, but as an illustrative case, the present definitions are adequate.

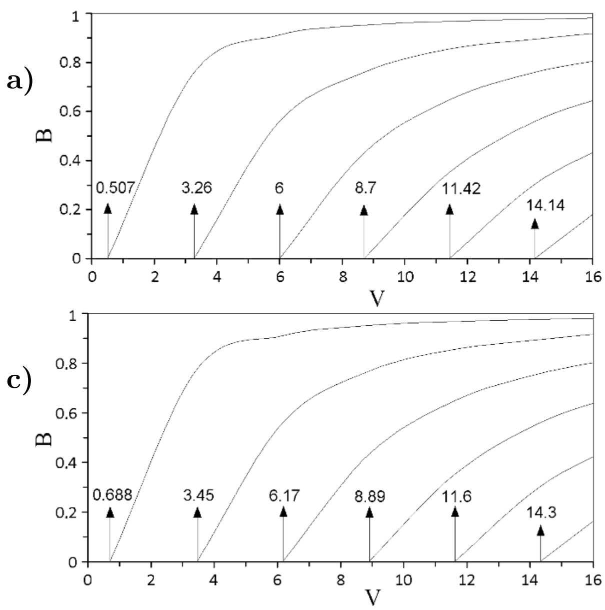
3. Numerical Results and Discussion

Characteristic equation (10) provides all information regarding the modal dispersion characteristics of the proposed Bragg waveguide. Here, we introduced two dimensionless parameters: normalized frequency V and normalized propagation constant B defined in Eqs. (11) and (12), respectively.

Table 1. Cutoff Frequencies (V Values) for Some Modes in the Bragg Waveguide Slightly Flattened on One Side for Three Different Thicknesses of the Cladding Strips.

Mode No.	Cut off frequencies of various modes in flat Bragg fiber, $0 < V < 16$, with thickness of cladding strip								
	$b = 0.01$ μm			$b = 0.10$ μm			$b = 1.00$ μm		
LP_{1m}	Six layered	Four layered	Two layered	Six layered	Four layered	Two layered	Six layered	Four layered	Two layered
LP_{11}	3.26	3.36	3.45	1.40	2.32	3.199	-	-	2.90
LP_{12}	6.00	6.08	6.17	4.12	4.99	5.90	-	-	5.57
LP_{13}	8.70	8.80	8.89	6.84	7.71	8.61	-	-	8.33
LP_{14}	11.42	11.52	11.60	9.51	10.46	11.33	-	2.66	11.01
LP_{15}	14.14	14.23	14.30	12.28	13.14	14.00	-	4.98	13.77
LP_{16}	-	-	-	15.01	15.86	-	-	7.65	-
LP_{17}	-	-	-	-	-	-	1.72	10.42	-
LP_{18}	-	-	-	-	-	-	4.35	13.18	-
LP_{19}	-	-	-	-	-	-	7.07	15.90	-
LP_{110}	-	-	-	-	-	-	9.77	-	-
LP_{111}	-	-	-	-	-	-	12.49	-	-
LP_{112}	-	-	-	-	-	-	15.20	-	-

We analyze Eq. (10) numerically taking a particular case where $n_a = 1.0002$, $n_1 = 1.50$, $n_2 = 1.45$, $b = 0.01$, 0.1, and 1.0 μm, $\lambda_0 = 1.55$ μm, and various values of the dimensional parameter a in a regular increasing order. First, we take a particular value of a and compute the left-hand side (L.H.S.) of Eq. (10) for many equi-spaced values of β lying between $k_0 n_1$ and $k_0 n_2$. Plotting the computed values of the L.H.S. of Eq. (10) versus β, we obtain a curve, whose intersections with the $\beta = 0$ axis provide the β values for the guided modes. We repeat this method for different values of a, using β values of the guided modes [B values can be obtained from Eq. (12)]. Similarly, in view of Eq. (11), the parameter V can also be calculated for each value of a. Now we are able to plot B versus V. These curves are known as dispersion curves and are shown in Figs. 2–4 for different numbers of cladding layers of different thicknesses.



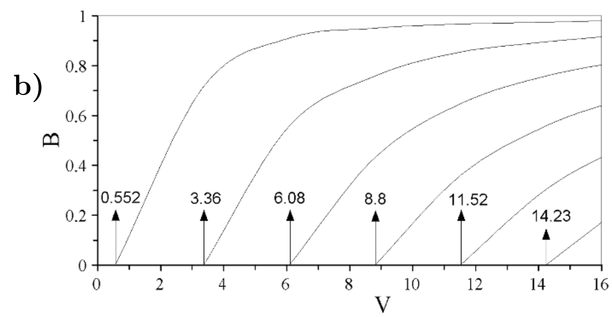


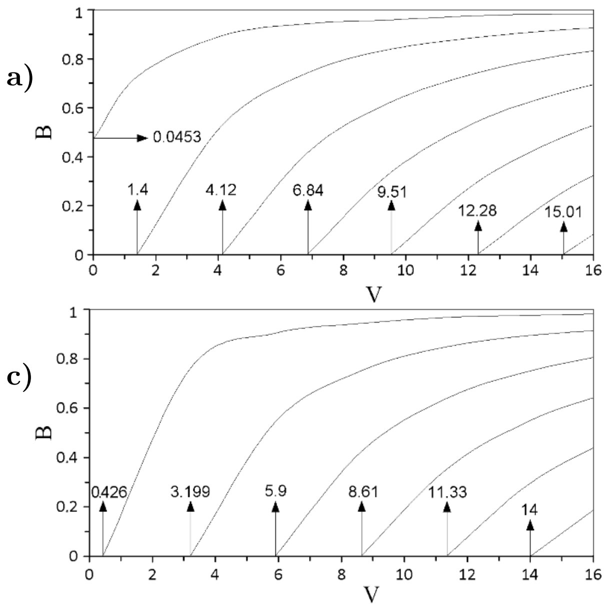
Fig. 2. Dispersion curves of normalized frequency V versus normalized propagation constant B for slightly flattened-core Bragg waveguide with the cladding-region thickness $b = 0.01$ μm for six-layered (a), four-layered (b), and two-layered (c) waveguides.

The cutoff values V and their dependence on the thickness b of the cladding strips for various cladding layers observed for the proposed Bragg waveguide are listed in Table 1.

Figure 2 shows the lowest-order cutoff V values $V = 3.26$, 3.36, and 3.45 for six-, four-, and two-layer waveguides, respectively. We see that these values are greater than the cutoff V values of the standard circular Bragg fiber [13] for the LP_{11} mode. Here we found that an increase in the number of layers from two to six leads to a decrease in the cutoff V value. One can observe this decrease in the cutoff V value, because the field is tightly bound for the six-layered Bragg waveguide in comparison to the two-layered Bragg waveguide. In Bragg waveguides, the wave propagates through the Bragg reflection. Since six layers have a larger reflecting surface as compared to two layers of the proposed waveguide, and two-layered waveguides have only two reflecting surfaces, the field is tightly bound for six layers in comparison with two layers.

Figures 3 and 4 show that, as the thickness of the layers increase from $b = 0.01$ to 1.0 μm, the cutoff V values decrease for all modes corresponding to different cladding layers for both Bragg waveguides. Such a reduction is larger for the circular Bragg waveguide in comparison with the proposed Bragg waveguide.

Again, this reduction in cutoff V values may occur due to a tight bound of the field or, in other words, when the width of the cladding layers decreases, we obtain higher cutoff V values even when the number of cladding regions is only two. We know that the greater the cutoff values, the fewer the number



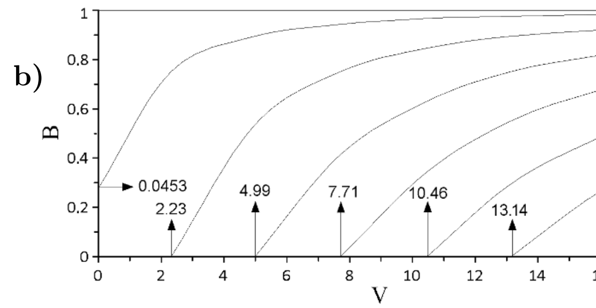
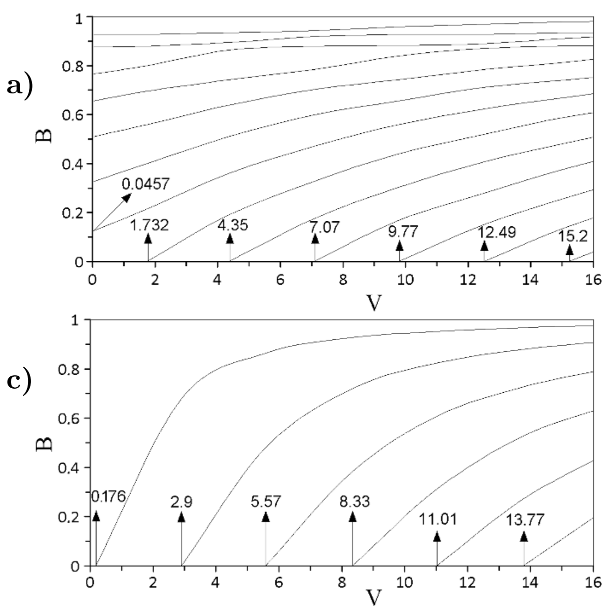


Fig. 3. Dispersion curves of normalized frequency V versus normalized propagation constant B for a slightly flattened core Bragg waveguide with cladding-region thickness $b = 0.10$ μm for six-layered (a), four-layered (b), and two-layered (c) waveguides.



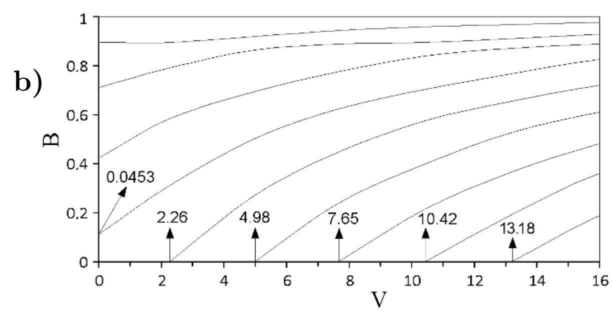


Fig. 4. Dispersion curves of normalized frequency V versus normalized propagation constant B for a slightly flattened core Bragg waveguide with cladding-region thickness $b = 1.0$ μm for six-layered (a), four-layered (b), and two-layered (c) waveguides.

of sustained modes. Hence, Bragg waveguides show good performance regarding the limiting modes of small number.

4. Conclusions

We analyzed numerically the dispersion curves of slightly deformed optical Bragg waveguides with two, four, and six layers. We observed that the cutoff V values increase with decrease in the number of cladding layers. Also these cutoff V values decrease with increase in the cladding-layer thickness. Therefore, these cutoffs V values can tailor up to a certain level by changing these two parameters.

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