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MATHEMATICAL MODELING OF THE HEATING OF A FAST IGNITION TARGET BY AN ION BEAM

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Abstract

We develop a BIN computer code for simulating the interaction of a monochromatic ion beam with a plasma, which takes into account changes in the spatial distribution of the heated-plasma temperature. This enables us to calculate the heating of both homogeneous and inhomogeneous plasmas with parameters corresponding to their real spatial distributions at the time of maximum compression of the inertial confinement fusion (ICF) target. We present the results of a numerical simulation using the BIN code for the heating of a homogeneous deuterium–tritium plasma by a short pulse of monochromatic ions at various ion velocity and plasma–electron thermal velocity ratios. We also present the results of calculations for the heating of an inhomogeneous plasma of a non-cryogenic target formed as a beryllium deuteride–tritide shell by beams of light, medium, and heavy ions. As the initial distributions, we use the results of numerical simulations for such a target, precompressed by a laser pulse (carried out at the M. V. Keldysh Institute of Applied Mathematics using the DIANA code). We demonstrate the possibility of forming the central ignitor with the parameters sufficient for igniting the targets by beams of ions with energies $E \sim 100 - 400$ MeV/u and specific energy densities of the beam $Q \sim 5 - 20$ GJ/cm^2. The required specific energy density drops with increase in the ion energy; however, due to the increased path length, larger-charge ions have to be used.

Keywords: inertial confinement fusion, fast ignition, high-energy ion beam, plasma stopping power.

1. Introduction

Heating by a beam of high-energy ions appears to date the most promising method for fast ignition [1,2] of a precompressed inertial confinement fusion (ICF) target. The advantage of the method in comparison with the use of the electron beam is due to an insignificant scattering of ions both in the residual plasma of the evaporated part of the target and in a fusionable matter. Especially attractive is the possibility to use the Bragg peak to form a hot spot (the ignitor) deep inside the target.

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However, plasma heating in fast ignition is a nonlinear process with a strong feedback — an increase in temperature up to values of ∼10–20 keV results in a significant decrease of its stopping power and an increase of the ion path length, which ultimately can lead to blurring of the Bragg peak and formation of a quasihomogeneous distribution of temperature in the heated plasma region, including a negative gradient in the beam-propagating direction. Theoretical and numerical studies of the heating of initially homogeneous plasma by an ion beam conducted earlier [3–5]* made it possible to reveal various ignition regimes and formulate the requirements on the beam parameters.

In [5], numerical calculations were performed on the model of a spatially-averaged change in the plasma stopping power by increasing the ion path length $\lambda(\langle T\rangle)$ in a plasma with a homogeneous temperature distribution $\langle T\rangle$. The change in the plasma mean temperature was, in turn, in agreement with the ion-beam propagation. This approach can lead to a slight overestimation of the Bragg-peak value.

We have developed a new BIN computer code for simulating the interaction of a monochromatic ion beam with a plasma, which takes into account the change in the spatial distribution of the temperature field in the heating process. This enables us to calculate the heating for both homogeneous and inhomogeneous plasmas with parameters corresponding to their real spatial distributions at the time of maximum compression of the ICF target.

In this work, using the BIN code, we present the results of numerical simulation of the heating of a homogeneous deuterium–tritium (DT) plasma by a short pulse of monochromatic ions at various ratios of the ion velocity and plasma-electron thermal velocity, as well as of the heating by light beams and medium and heavy ions of the inhomogeneous plasma of fast-ignition targets.

As a target, we chose a noncryogenic beryllium deuteride–tritide (BeDT) shell, which conforms to the achievement of high values of target areal density ρR – the product of the density ρ by the size R of the target – at the time of maximum compression. As the initial distributions of parameters of a fast-ignition BeDT target, we use the results of simulating the compression of such a target under the action of a profiled laser pulse; the simulation was done using the DIANA one-dimensional two-temperature hydrodynamic code [6].

2. Model and Algorithm for Computing the Heating of the Plasma by the Ion Beam

We consider a one-dimensional problem with a cylindrical ion beam incident on the plasma surface boundary. The action of the ion beam should provide the formation of the igniter (the region of initiation of thermonuclear burning) with areal density and temperature sufficient to generate a self-sustained wave of thermonuclear burning. For DT fuel, the ignition conditions are as follows: $(\rho R)_{\rm ig} \geq 0.4$ g/cm^2 and $T_{\rm ig} \geq 7$ keV [7]; for noncryogenic BeDT fuel $(\rho R)_{\rm ig} \geq 1 - 1.5$ g/cm^2 and $T_{ig} \geq 15 - 20$ keV [8].

We take into account the most important mechanism of the energy transfer to the plasma from the ion beam determined by the Coulomb collisions of ions with plasma electrons. In the process of heating, the temperature of matter changes significantly and, correspondingly, the stopping power of the plasma also changes substantially. The beam duration t is assumed to be sufficiently short as compared with the characteristic hydrodynamic times τ_h. At the same time, the beam duration is considered to be sufficiently large in comparison with the ion deceleration time at a path length τ_λ, such that $\tau_\lambda \ll t \ll \tau_h$. Thus, the only time scale in our problem is the pulse duration. Deceleration of the ion occurs in the

*Those papers include a detailed bibliography on the feasibility of using the ion ignition driver.

stationary plasma with a slowly changing temperature field $T(x,t)$ formed by the previous part of the beam. In a one-temperature approximation, the dynamics of plasma temperature change is determined by the following equation:

$$\frac{dT(x,t)}{dt} = \left(\frac{1}{E_{j0}}\,\frac{dQ(t)}{dt}\right)\frac{1}{\rho C_V}\,\frac{dE_j}{dx}(x,t), \tag{1}$$

where E_j and E_{j0} are the current and initial values of the ion energy, $\rho(x)$ and $T(x,t)$ are the spatial distribution of the plasma density and temperature, $dQ/dt = E_{jo}(dn/dt)$ is the density of the ion-beam energy flow at the target boundary (dn/dt is the density of the ion flow), C_V is the specific heat capacity of the plasma, and dE_j/dx is the energy losses of one ion in a point x at a given distribution of the plasma temperature, density, and composition.

At a given distribution of the plasma parameters, the energy losses are known functions of the ion energy $E_j : dE_j/dx = f(E_j,T,\rho)$. To find the distribution of the current values of the ion energy in the plasma $E_j(x,t)$, it is necessary to solve a simple differential equation for the deceleration of the ion in the plasma with the given distributions $T(x,t)$, $\rho(x)$, and the boundary value of the energy E_{j0}

$$\left(\frac{dE_j}{dx}\right) = f(E_j, T(x,t), \rho(x)), \qquad E_{j(x=0)} = E_{j0}. \tag{2}$$

Time is contained in Eqs. (1) and (2) only through the flow of the ion energy dQ/dt at the target boundary. This means that, in the given statement of the problem, the dependence of its solution on time can be replaced by a dependence on the specific energy of the ion beam Q (per unit area of its cross section) consumed to heat the medium, $T(x,t) \to T(x,Q)$, and the equations can be rewritten as follows:

$$T(x,Q) = \int_0^Q \frac{1}{E_{j0}\rho C_V}\,\frac{dE_j}{dx}(x,Q')\,dQ', \tag{3}$$

$$\left(\frac{dE_j}{dx}\right) = f(E_j, T(x,Q), \rho(x)), \qquad E_{j(x=0)} = E_{j0}. \tag{4}$$

The system of equations (3) and (4) provides the physical basis for the BIN code. The developed algorithm for the numerical solution of the system of equations (3) and (4) is as follows.

At a fixed spatial distribution of the plasma parameters, the solution of the Cauchy problem (4) was found by the Runge–Kutta method. The general beam of ions splits into N portions equal by their specific energies $\Delta Q = Q/N$ and those N beams were successively passed through the plasma (each succeeding beam in the temperature field formed by the heating of the plasma by the preceding portioned beams). Integration in (3) was replaced by summation for N beams. The size of portions ΔQ was chosen sufficiently small so that the change in the plasma stopping power at the propagation of one beam is insignificant. As calculations show, sufficient smallness is provided by the condition $\Delta Q \ll 0.1$ GJ/cm^2. Our calculations, as a rule, made use of portions with specific energies $\Delta Q \ll 0.01$ GJ/cm^2.

A simplified description of the ion-beam deceleration in [5] consisted in calculating the energy losses of the beam at each point of time in a homogeneous plasma with the average temperature $\langle T\rangle(Q)$ changing at the ion path length λ_T, $d\langle T\rangle = (1/\lambda_T \rho C_V)\,dQ$,

$$dT(x) = \frac{dn}{\rho C_V}\,\frac{dE_j}{dx}(x,\langle T\rangle) = \frac{\lambda_T}{E_{j0}}\,\frac{dE_j}{dx}(x,\langle T\rangle)\,d\langle T\rangle.$$

Herewith, the spatial step in the Runge–Kutta method was constant at all N iterations. This significantly simplified the logic of the code, as there was no need to agree on the spatial networks at various N.

The real state of a precompressed target is to a significant degree inhomogeneous, both with respect to temperature and density of the target, and its chemical composition. The density change is especially strong. As shown by the calculations of the ICF target compression by a profiled laser pulse in the hydrodynamic code DIANA, in the compressed region of the target, changes in the density from the periphery to the center can be up to 2 orders of magnitude. They can reach 9 orders of magnitude if we take into account processes in the regions of the evaporated part of the target ablator (corona). Though the heating process can still be described by the system of equations (3) and (4), the code implementation of their solution becomes much more complicated.

It is obvious that the simplified variant of the calculation described above, with a constant, preset spatial step during all N iterations, is unacceptable under these conditions. The code should choose steps itself automatically, proceeding from the ion deceleration conditions, but these conditions change at each of N iterations. Thereby, the necessity arises to recalculate the spatial networks at various iterations of N, which complicates the logic of the code. Clearly, when choosing a step, the code should proceed from the smallness of changes in the ion energy at that step (the kinetic step) on the one hand, and the plasma parameters (the hydrodynamic step) on the other hand. But the parameters of the kinetic network and hydrodynamic network differ very strongly.

In the target corona, the ion energy does not, in practice, change, and the kinetic step is several orders of magnitude greater than the hydrodynamic step, whereas in the compressed part the correlation of the steps is inverse. In this context, for the BIN code we developed the following algorithm for calculating the ion beam deceleration in an inhomogeneous plasma. At each new iteration, simulating the passage of a portion of the beam with energy ΔQ (or, in other words, the temporal step), the kinetic step is determined from the condition of smallness of ion energy change $\Delta E_j/E_j$ compared with the distance to the nearest node of the spatial network formed in the previous iteration, and the smallest value of them is chosen.

Therewith, the previous network is completely included into the newly forming spatial network. In particular, the initial hydrodynamic network formed by the DIANA code is (at all iterations) a part of the spatial network of the Runge–Kutta method for calculating the distribution of energy losses. This approach leads to some increase in the size of the arrays with increase in the number of iterations, but this increase is not too large. In particular, in the target corona, even at a large number of iterations the final spatial network coincides with the initial network. As our calculations show, with selection of the step from the condition of energy change $\Delta E_j = 0.01 E_{j0}$ at $N = 2000$, the size of the network does not exceed 10,000 nodes.

One more problem is associated with calculating the temperature field in the target corona. With about 500 layers in the initial hydrodynamic distribution, the density on one layer in the corona changes severalfold. This is absolutely insignificant for solving Eq. (4) by the Runge–Kutta method, because the change in the ion energy on one layer in the corona is negligibly small, but a strong change in the value of dE_j/dx at one step yields a large error in calculating the temperature change.

An actual change in the density should not have any significant effect on the temperature change, because the energy losses per particle are almost independent of the density. In the BIN code, in calculating the temperature change, in view of Eq. (3), we initially average the value of $(dE_j/dx)/n_e$ (here n_e is the concentration of plasma electrons), which does solve the problem.

3. Results of Numerical Simulation and Discussion

3.1. Heating of an Initially Homogeneous Plasma

A characteristic feature of the deceleration of fast ions in a plasma is the occurrence of a maximum in the energy losses at the ion velocity u close to the thermal velocity of electrons v_e. The corresponding value of the ion energy E_s ("thermal threshold") is determined by the following expression:

$$E_s\ (\text{MeV}) = \frac{m_j}{m_e} T_e \approx 1.85 \mu_j T_e, \tag{5}$$

where T_e is the temperature of plasma electrons measured in kiloelectronvolts, μ_j is the ion atomic weight, and m_j and m_e are the masses of the ion and electron, respectively. If the beam ion energy E_{j0} is smaller than the thermal threshold, the Bragg peak is not formed and the hot region will be generated at the plasma boundary (the boundary igniter). The energy losses of ions in the process are described by a simple analytical dependence: $dE/dx \propto \sqrt{E}/T^{3/2}$ (see, e.g., [4]), and the heating of plasma, in this case, is described by a self-similar solution in the form of an exponential function with the exponent close to the value of 2/5, $T(x) \cong T_0(1 - x/\lambda)^{2/5}$ [9]. The use of this solution yields a sufficiently exact analytical description of the plasma heating process [4], where it is shown that the optimum choice of values produces an igniter with the required parameters at a specific energy of the beam of light ions $Q = 0.75$ GJ/cm^2.

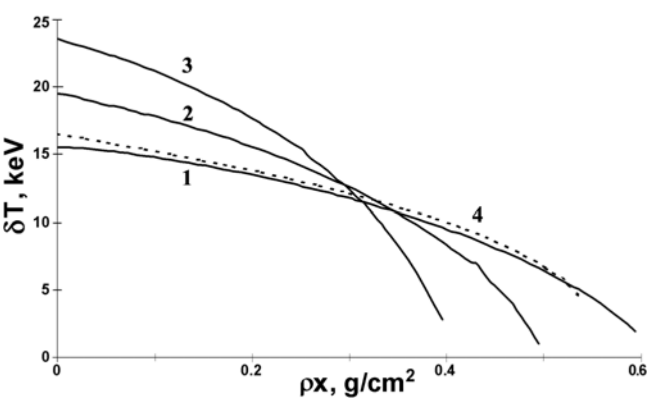

Fig. 1. Spatial distribution of the temperature increment δT in the plasma at initial temperature $T_0 = 1$ keV during its heating by a carbon ion beam of specific energy $Q = 0.75$ GJ/cm^2 at various initial ion energies: 80 (1), 40 (2), and 20 MeV (3). An analytical solution for an optimum edge igniter [4] (dashed curve).

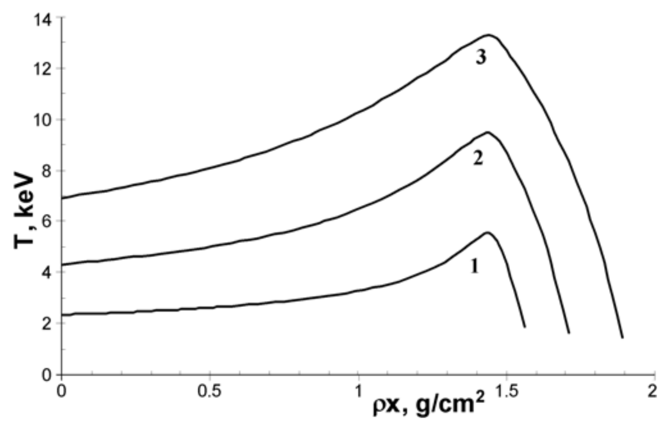

Fig. 2. Spatial distribution of temperature in a DT plasma of density $\rho = 300$ g/cm^3 during its heat-up by a vanadium ion (V^{23+}) beam of energy $E_{j0} = 5.1$ GeV ($E_{j0}/\mu_j = 100$ MeV/u) at various specific-energy values of the beam $Q = 0.4$ (1), 1 (2), and 1.8 GJ/cm^2 (3).

Figure 1 presents the results of numerical simulation, using the BIN code, of the heating of an initially homogeneous plasma by beams of carbon ions at various initial energies near the thermal threshold E_{s0} calculated for the initial temperature $T_0 = 1$ keV. As calculations show, the shapes of all curves are in good agreement with the self-similar solution [9], and the optimum parameters of the igniter are achieved at the initial energy of carbon ions $E_{j0} = 80$ MeV (6.7 MeV/u).

To form a sufficiently pronounced temperature peak deep inside the target, the initial energy of ions should be increased to a significantly higher level than the thermal threshold at the ignition temperature

$E_{\rm sf}$, i.e., $E_{j0}/\mu_j \gg 20$ MeV/u. As shown in [4, 5], for this, the ion beams should have an energy $E_{j0}/\mu_j \geq 100$ MeV/u. This requires the use of beams of medium and heavy ions, because the path lengths of light ions at these energies exceed the characteristic size of the ICF target.

The results of calculations of the heat-up of a DT plasma by a vanadium V^{23+} ion beam of energy $E_{j0} = 5100$ MeV ($E_{j0}/\mu_j = 100$ MeV/u) are shown in Fig. 2.†

As seen in Fig. 2, there is a pronounced temperature peak ($T_{\rm max}/T(0) = 2$) at a depth of $x \sim 47$ μm ($\rho x \sim 1.4$ g/cm^2) in the temperature distribution. Formation of the ignition spot in the inner region of the target requires approximately two times as high values of the beam specific energy than for the boundary igniter: $Q \sim 1.5 - 2$ GJ/cm^2, because, in this case, a part of the beam energy is used to heat the thermonuclear matter in the peripheral parts of the target. It is of interest to compare the results of the calculations by means of the BIN code with the calculations performed in [5] within the framework of a simplified description using the average plasma temperature $\langle T\rangle(Q)$ instead of the real profile $T(x, Q)$ in the calculation of ion deceleration (Fig. 3). The simplified description yields the correct position of the peak and describes very well the temperature distribution in the periphery region, but noticeably overestimates the value of the Bragg peak.

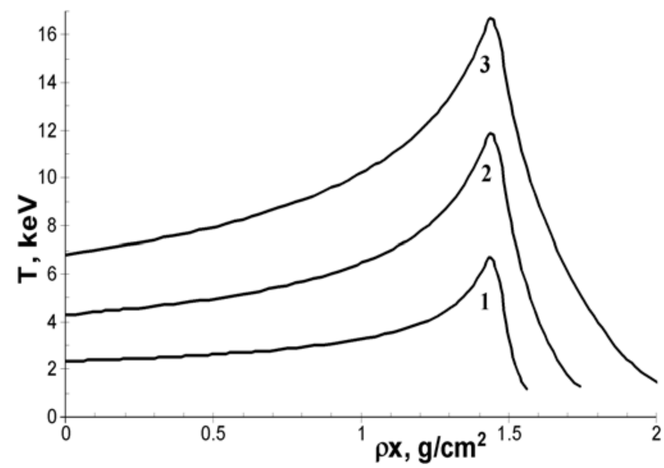

Fig. 3. Results of calculations for the heating of a DT plasma of density $\rho = 300$ g/cm^3 by a vanadium ion (V^{23+}) beam of energy $E_{j0} = 5.1$ GeV [5] at a simplified description of the deceleration of the ion beam with various specific energies $Q = 0.4$ (1), 1 (2), and 1.8 GJ/cm^2 (3).

3.2. Heating of Inhomogeneous Plasma

As one more example of using the BIN code, consider the heating of a noncryogenic target with BeDT fuel by an ion beam [8]. The target compression was calculated using the hydrodynamic code DIANA. In the initial state, the target is a hollow BeDT shell of 8 mg weight, 0.26 cm outer radius, and aspect ratio 20, filled with a background DT gas of density 10^{-5} g/cm^3, which corresponds to a pressure of 0.1 atm at the initial room temperature. The target was irradiated by a profiled laser pulse of wavelength corresponding to the third harmonic of Nd laser radiation (duration $\tau_{\rm laser} = 36$ ns and energy $E_{\rm laser} = 1.27$ MJ). At the time of maximum compression of the target (32.9 ns), the achieved value of areal density ρR was greater than 6 g/cm^2 at the maximum density close to 900 g/cm^3. The corresponding spatial distributions of the plasma parameters in the nonevaporated part of the target are given in Fig. 4.

Numerical simulation using the Monte-Carlo TERA code [11] showed that such a target is ignited by forming an igniter in the target center of size $R_{\rm ig} = 0.003$ cm and temperature $T_{\rm ig} = 25$ keV ($E_{\rm ig} =$ 180 kJ), and the thermonuclear gain of the target – the ratio of the energy evolved in fusion reactions to the total energy of the laser and ignition energy – is close to 40. At such parameters of the igniter, the

†The possibility to use beams of such ions with the average value of Z and the values of parameter E_{j0}/μ_j within the range of 20–120 MeV/m was considered in [10].

value of specific energy in the central part of the target is $Q_0 \cong 3$ GJ/cm^2. The aim of simulating the heating of the target by an ion beam (incident on the target from the left) was to form in it an igniter with the above ignition parameters and determine the overstatement of Q/Q_0 due to the energy losses in the incident beam for the heat-up of the substance from the target boundary to the igniter region.

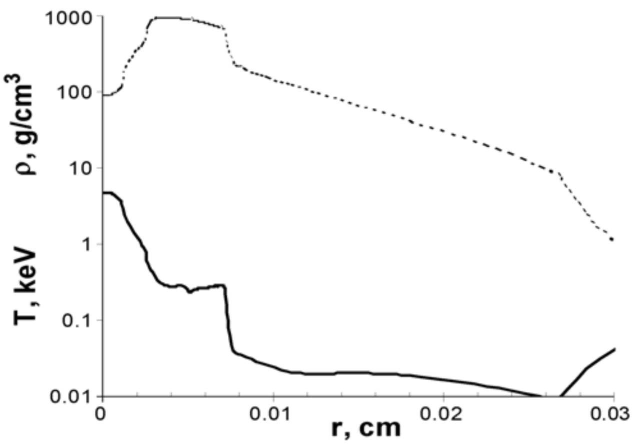

Fig. 4. Spatial distributions of the plasma density and temperature in a fast-ignited precompressed BeDT target obtained by a numerical simulation according to the DIANA code. The density distribution (dotted curve) and the temperature distribution (solid curve).

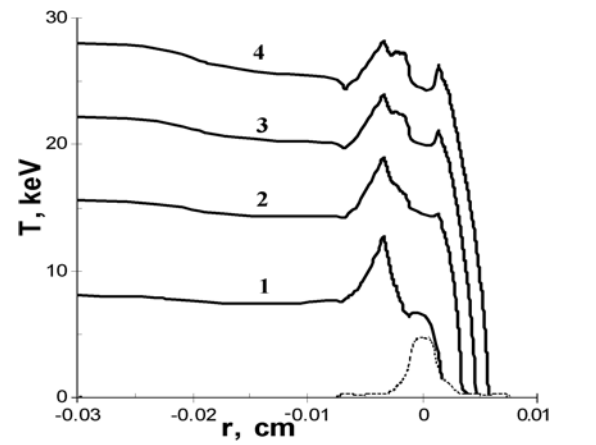

Fig. 5. Spatial distribution of temperature in a fast-ignited BeDT target during heating by a carbon ion beam of energy $E_{j0} = 1.5$ GeV with various specific energies $Q = 5$ (1), 10 (2), 15 (3), and 20 GJ/cm^2 (4). The initial distribution of temperature (dashed curve).

First of all, as the calculations show, it is virtually impossible to form the central igniter in this target using a beam of protons. At an initial proton energy lower than 50 MeV, the temperature distribution is practically homogeneous along the entire channel of the beam passage, and at an increase in the proton energy up to 70 MeV the beam passes through the target. To form the igniter deep inside the target, use should be made of beams of medium and heavy ions with energies $E_{j0}/\mu_j \geq 100$ MeV/u, choosing their energies such that their path lengths are close to the target radius.

Figure 5 presents the results of calculations for the heating of a target by a carbon ion beam of energy $E_{j0} = 1.5$ GeV ($E_{j0}/\mu_j = 125$ MeV/u). Though, in this case, the igniter does form in the central region, it is too wide, and the Bragg peak is insufficiently pronounced. As the result, the energy excess proves sufficiently high, $Q/Q_0 \sim 7$. To reduce the ratio of Q/Q_0, the value of E_{j0}/μ_j and, correspondingly, the ion charge should be increased.

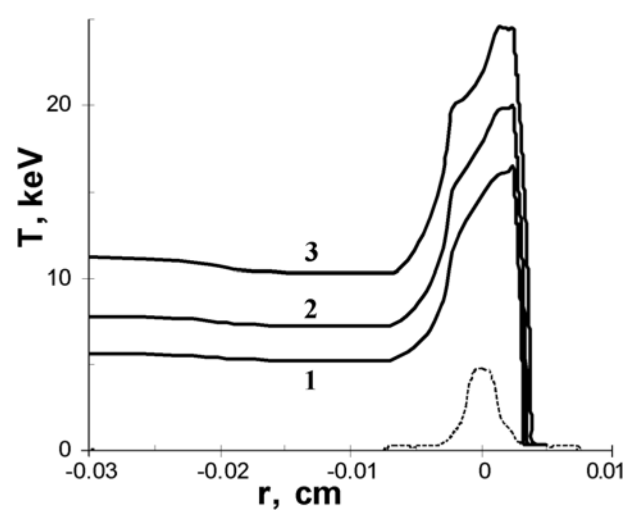

Fig. 6. Spatial distribution of temperature in a fast-ignition BeDT target during heating by a vanadium ion beam of energy $E_{j0} = 12$ GeV with various specific energies $Q = 5$ (1), 7 (2), and 10 GJ/cm^2 (3). The initial distribution of temperature (dashed curve).

Figure 6 presents the results of calculations of the heating of a target by a vanadium ion beam of energy $E_{j0} = 12$ GeV ($E_{j0}/\mu_j \cong 230$ MeV/u). Here the Bragg peak is much more pronounced and, correspondingly, twice less energy of the beam is required. Even better result is obtained using beams of heavy ions. Figure 7 presents the results of calculations for the heating of a target by a gold ion beam of energy $E_{j0} = 80$ GeV ($E_{j0}/\mu_j \cong 400$ MeV/u). In this case, virtually half of the beam energy gets to the igniter region.

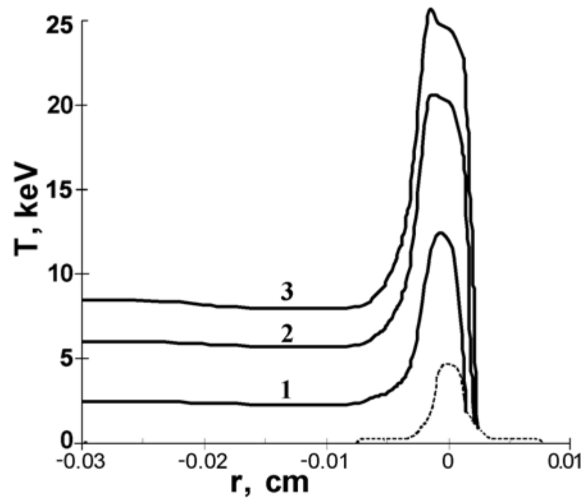

Fig. 7. Spatial distribution of temperature in a fast-ignition BeDT target during heating by a gold ion beam of energy $E_{j0} = 80$ GeV with various specific energies $Q = 2$ (1), 5 (2), and 7 GJ/cm^2 (3). The initial distribution of temperature (dashed curve).

4. Conclusions

The results of numerical simulation for the heating of a fast-ignition target by a beam of monochromatic ions demonstrate the possibility of developing a central igniter with parameters sufficient to ignite targets by beams of ions with energies $E \sim 100 - 400$ MeV/u and specific energies of the beam $Q \sim 5 - 20$ GJ/cm^2. The specific energy of the beam required for ignition drops with increase in the ion energies. However, larger-charge ions have to be used due to increased path lengths.

Acknowledgments

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