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GAS DYNAMICS OF A ROCKET PLUME IN THE UPPER ATMOSPHERE*

Alexander G. Molchanov[1] **and Yulii V. Platov**[2†]

[1] *P. N. Lebedev Physical Institute, Russian Academy of Sciences*
Leninskii Prospect 53, Moscow 119991, Russia

[2] *N. V. Pushkov Institute of Terrestrial Magnetism, Ionosphere, and Radio Waves Propagation, Russian Academy of Sciences*
Troitsk, Moscow Region 142190, Russia

†Corresponding author e-mail: yplatov @ izmiran.ru

Abstract

We study the expansion of a rocket exhaust plume in the direction perpendicular to the rocket motion. We compare numerical calculations with a self-similar approximation for a strong cylindrically symmetric explosion and show that, if the rocket velocity V_∞ is high enough and exceeds the sum of the exhaust gas velocity V_e and the sound velocity V_s ($V_\infty > V_e + V_s$), a gasdynamic hole may be formed for a short time around the rocket flight trajectory in the upper atmosphere, where the plume-gas concentration is less than the ambient-gas concentration at a given altitude. We calculate the density and temperature profiles of the transversal plume expansion.

Keywords: upper atmosphere, rocket plume, expansion of combustion products, gas dynamics.

1. Introduction

Typical values of the pressure of combustion products on a cut of a nozzle of the rocket engine operating in the upper atmosphere are ∼0.005 MPa. Since the ambient-gas pressure on heights above 120 km is $< 10^{-2}$ Pa, the underexpansion factor $n = P_e/P_\infty$, i.e., the ratio of the gas pressure at the nozzle edge to the ambient-gas pressure may achieve a value $\sim 10^5$–10^7. Such a ratio of the gas pressures should lead to phenomena typical of the strong explosion in the direction perpendicular to the rocket-motion direction, namely, the formation of strong shock waves and the gas expansion with velocities up to 3–5 km/s.

A rocket plume may be schematically divided into three regions (Fig. 1). Region 1 is the closest to the rocket with a complex structure of the shock waves. This region can extend to several kilometers.

Region 2 is the region of cylindrically symmetric explosion where the pressure of fuel combustion products is still much higher than the ambient-gas pressure, $P_p \gg P_\infty$. It is characterized by an almost free gas expansion in the transversal direction and may occupy from tens to hundreds of kilometers.

In region 3, where the pressure of combustion products becomes comparable with the ambient pressure, $P_p \approx P_\infty$, the diffusive processes prevail along with the mixing of all gas components. This region may extend to hundreds of kilometers.

*The last paper of Alexander Molchanov.

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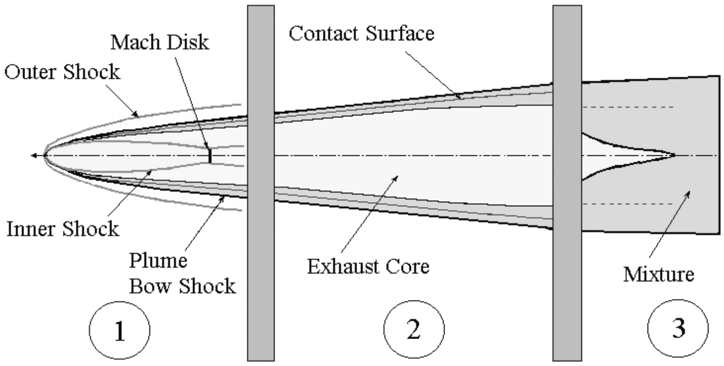

Fig. 1. Structure of a rocket plume in the upper atmosphere: region closest to the rocket (1), region of cylindrically symmetric explosion, $P_p \gg P_\infty$ (2), and region of the sound perturbations and diffusive mixing with the ambient gas, $P_p \approx P_\infty$ (3).

Region 1 was studied in detail in [1–3]. In this work, we investigate the rocket-plume structure in region 2. The plume radial expansion in this region was described in [4] with the help of self-similar solutions for a cylindrically symmetric point-like explosion. It should, however, be noted that the self-similar solutions are not appropriate for an accurate description of large central parts of the plume and large parts of the space where the ambient-medium pressure must be taken into account. Therefore, we must perform numerical calculations based on 3T 1D gas dynamic equations in a cylindrically symmetric geometry in order to describe in detail the motion of combustion products.

The nonstationary problem of cylindric explosion and the problem of stationary streamline flow of bodies are closely related by a common principle of the plane cross-sections [5]. If a body moves with Mach numbers $M\infty \gg 1$, the pulse that is transferred to the ambient gas is concentrated in the plane perpendicular to the body motion.

A supersonic jet in a wake hypersound underexpanded flow may also be considered as a streamline flow of a body with the characteristic transversal R and longitudinal X dimensions [6]:

$$R = r_e \frac{M_e}{M_\infty}\left(\frac{\gamma_e n}{\gamma_\infty}\right)^{1/2}, \qquad X = \frac{R}{\theta}, \qquad \theta = \left(\frac{2}{\gamma_e(\gamma_e+1)}\frac{1}{M_e^2} + \frac{1}{2}\sin^2\theta_e\right)^{1/2},$$

where r_e and θ_e are the nozzle radius and the tilt angle of the exit gas velocity vector, γ_e and γ_∞ are the exhaust specific heat ratio, and M_e and M_∞ are the exit and ambient Mach numbers.

At large distances from the vehicle downstream $x \gg X$ (Region 2 in Fig. 1), in a plane layer normal to the jet axis, one can consider the gas transversal motion as independent of the motion along the axis. To avoid gas overflow from the vehicle side along the jet axis, the longitudinal velocity of the combustion products must be directed toward the rocket, with respect to the ambient gas, and exceed the sound velocity, i.e., $M_\infty > M_e + 1$. This condition is usually fulfilled when the rocket enters the upper atmosphere; here the rocket velocity is ∼6 km/s, whereas the nozzle gas exhaust velocity is 2.5–3 km/s. In what follows, we assume that Eq. (2) is fulfilled.

2. Model of Cylindrically Symmetric Explosion

For a point-like cylindrically symmetric explosion, where the ambient-gas pressure can be neglected, the coordinate, velocity, and pressure of the gas immediately after the shock wave obey the self-similar equations [5]: $r = C_1 t^{1/2}$, $V = C_2 t^{-1/2}$, $P = C_3/t$, where $C_{1,2,3}$ are constants determined by the initial conditions. Time t of the explosion problem is related to the longitudinal coordinate x of the rocket exhaust plume problem through the equation $t = x/V_\infty$.

In order to estimate the fields of applicability of the self-similar solution, we compare it with numerical calculations of the transversal rocket-plume expansion in cylindric geometry. In numerical calculations, we assume that the exhaust plume is concentrated, initially, in a cylinder of the nozzle radius r_e, and the density is determined from the exit gas velocity V_e and the known value of the fuel-consumption rate. Figure 2 shows, as an example, the results of calculations of the position and velocity of the contact surface motion for the Saturn IVB rocket-exhaust plume at an altitude of 250 km. The composition of the combustion products was the same as in [7], i.e., 30% H_2 and 70% H_2O with an initial concentration of $2 \cdot 10^{-2}$ kg/m^3 and temperature 1400 K. The density and temperature of the ambient atmosphere were, respectively, $1 \cdot 10^{-10}$ kg/m^3 and 980 K.

From the calculations, one can see that the velocity of transversal plume expansion reaches ∼4 km/s during the first few seconds, and the motion itself is sufficiently accurately described by the self-similar approximation (3). But after 25 s of expansion, a strong difference in the numerical solution, which takes into account the ambient-gas pressure and the self-similar approximation, takes place. It should be noted that in our numerical calculations we did not consider the influence of the magnetic field of the Earth on the plume expansion. In principle, this influence can be taken into account for the ionosphere. The Reynolds magnetic number, in this case, is $R_M = 4\pi\sigma_\infty R_c V_c/c^2$, and it can exceed unity if the specific electrical conductivity of the ambient gas in this region during the plume expansion is $\sigma_\infty \sim 5 \cdot 10^{11}$ s^{-1}. This means that despite the small value of the specific electrical conductivity in the plume, $\sigma_p < \sigma_\infty$, the time of plume filling by the magnetic field $t_M = 2\pi(\sigma_\infty + \sigma_p)R_c^2 c^{-2}$ can be greater than the characteristic time of the plume expansion. Nevertheless, the magnetic field does not affect the qualitative picture of density and temperature distributions inside the plume because the magnetic-field pressure does not exceed a value of ∼10^{-3} Pa.

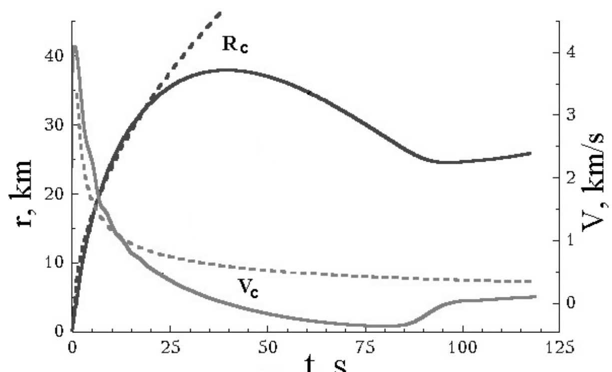

Fig. 2. Temporal dependence of the contact surface velocity V_c and coordinate R_c at an altitude of 250 km. The self-similar approximation is shown by dashes.

The results of calculations of the density and velocity transverse profiles of the exhaust plume and ambient gas motion at three time instants for the above-mentioned initial conditions are shown in Fig. 3. From the calculations, we see that about 10 s after the rocket transit moment, near the rocket's trajectory axis, a specific region with decreased gas concentration is formed. We call this region the gasdynamic hole. In contrast to the ionospheric hole, where the electron concentration decreases and can exist for a few hours, the gasdynamic hole exists for a few minutes and is not connected with the presence or absence of electrons.

Similar calculations were performed for the parameters of a plume produced by the second step of the Proton-K rocket at an altitude of 150 km. The main components are 27% H_2O, 31% CO_2, and 34% N_2 at the initial temperature 1500 K and nozzle dimensions $\rho_e = 3.6 \cdot 10^{-2}$ kg/m^3, $r_e = 1.5$ m, and $V_e = 2.5$ km/s, $\rho_\infty = 2.5 \cdot 10^{-9}$ kg/m^3, and $T\infty = 800$ K [8]. The qualitative distribution of densities and velocities proved to be the same as in Fig. 3, but the radial and temporal scales decrease about threefold. As in Fig. 3, one could observe, inside the gasdynamic hole near the contact surface, a comparatively narrow ring-like structure with a heightened gas density; inside the plume axis, a comparatively low temperature is maintained for quite a long time.

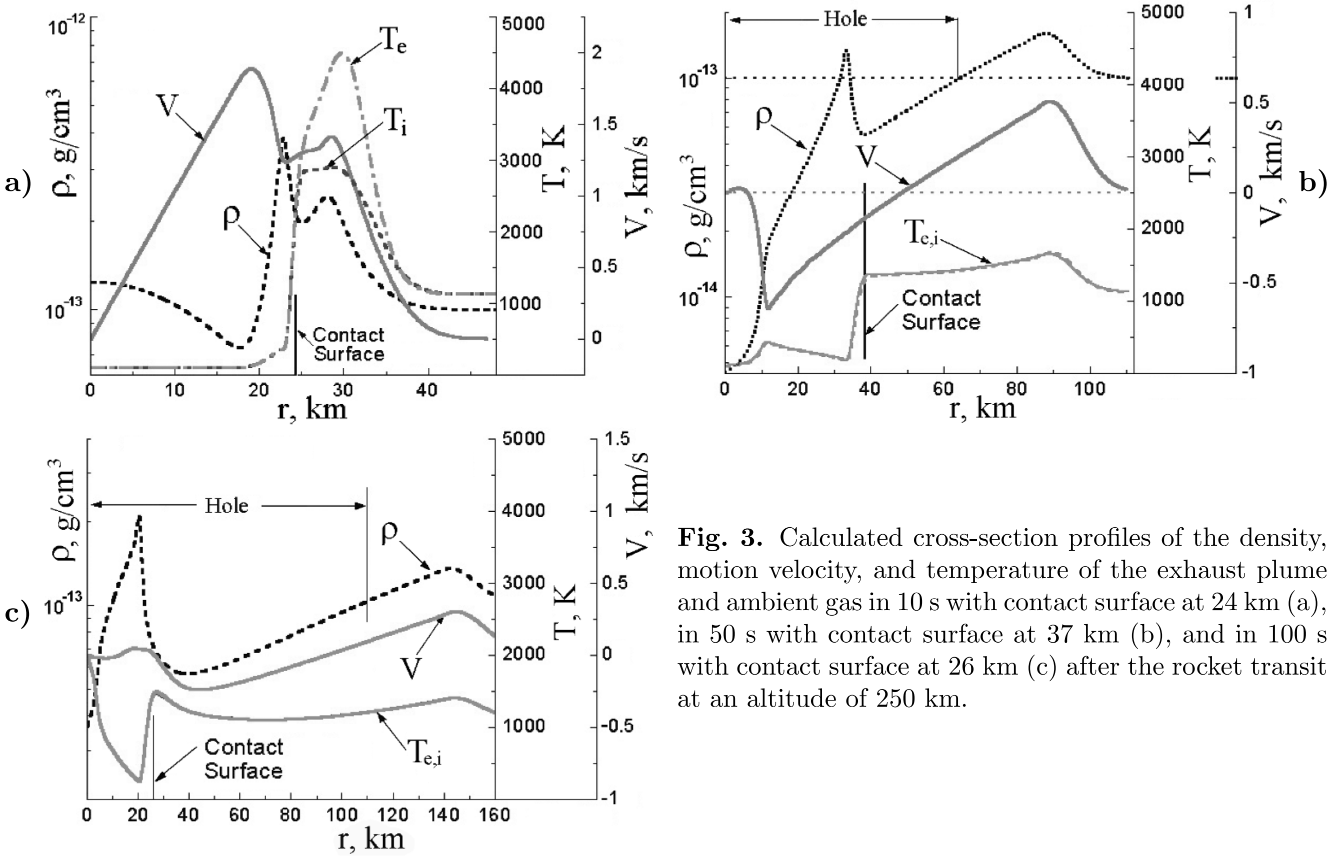

Fig. 3. Calculated cross-section profiles of the density, motion velocity, and temperature of the exhaust plume and ambient gas in 10 s with contact surface at 24 km (a), in 50 s with contact surface at 37 km (b), and in 100 s with contact surface at 26 km (c) after the rocket transit at an altitude of 250 km.

The cross-section of the gasdynamic hole increases with altitude and can achieve about a hundred kilometers. The plume cross-sections observed must be determined by the contact surface, but in many experiments, at altitudes above 100 km, one can observe a fast transverse plume expansion with velocities 2–3 km/s, far beyond the calculated contact surface.

The formation of the condensed phase upon sharp expansion of the combustion products occurs practically instantaneously behind the engine nozzle — at distances of 50–200 m from the nozzle [7, 12]. From our calculations, it follows that, if the condensed particles are in the combustion products, they can be accelerated to speeds of 2–3 km/s in the transversal direction with respect to the direction of the rocket movement only in the first seconds. The superposition of the rocket movement in the upper atmosphere with a speed of $\lesssim$6 km/s and expansion of particles lead to the formation of characteristic cone-shaped "clouds" that are observed under twilight conditions as a result of sunlight scattering on disperse components of the combustion products (see Fig. 4). It is obvious that the corner at the cone point is determined by the relation between the speed of the rocket movement and the radial speed of disperse components.

3. Conclusions

The performed calculations of transversal expansion of the rocket exhaust plume products demonstrate the complex internal structure of the plume in the upper atmosphere. In particular, we showed that near

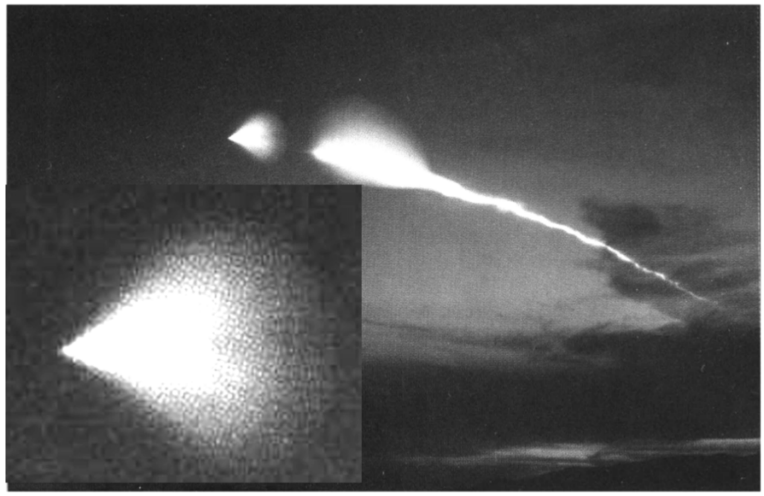

Fig. 4. Cone-shaped cloud of disperse particles observed in twilight conditions at a height of ∼140 km at the launch of the "Minotaur" on September 23, 2005. The characteristic size of the cloud is ∼30 km.

the rocket trajectory, gasdynamic holes can be formed, in which the gas concentration is less than the ambient-gas concentration, at a given altitude. In space, such regions should follow the rocket forming a gasdynamic bubble whose cross-section increased with altitude. The total volume of this region with decreased gas concentration can achieve 10^4–10^5 km^3. By considering the composition of the rocket exhaust plume and the conditions of its expansion in the second step of the Proton-K rocket, one can notice that this composition, excluding an excess of water molecules, is almost an ideal medium for the radiation amplification at a wavelength of 10.6 μm in a CO_2 molecule. The excess fraction of the absorbing water molecule can be compensated by the process of its condensation and withdrawal from the plume during transversal expansion. In any case, it is of interest to investigate the intensity of this line in the rocket plume.

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