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A FAIR SAMPLING TEST FOR EPR–BELL EXPERIMENTS

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Abstract

The low efficiency of detectors in all EPR experiments with optical photons makes the use of the fair sampling assumption unavoidable. This assumption is reputed to be both reasonable and impossible to test experimentally. We argue that there is, in fact, little evidence supporting the fair sampling assumption, and we propose an experiment capable of putting this crucial hypothesis to a test.

Keywords: EPR experiments, Bell inequalities, fair sampling assumption.

1. Introduction

The seminal EPR discussion on the completeness of quantum mechanics [1–3] and the shift induced by Bell's theorem [4, 5] towards experimental realizations of local realism tests [6, 7] has seen over the past few years a renewed interest motivated by the emergence of the quantum-information fields [7–22].

Experiments with optical photons have offered the closest realization of the ideal EPR–Bell scheme by enforcing locality between the measuring stations but have unfortunately suffered from a low detection efficiency.[a] This detection loophole has been exploited by local realistic models to reproduce experimental data, using a sample selection bias in the statistics [24–29].

Hence, in order to assess that a violation of Bell's inequality has been observed, all EPR–Bell experiments with optical photons are interpreted assuming that the sample of detected pairs is not biased and represents fairly the population of emitted pairs. This *fair sampling assumption* is taken to be both reasonable and impossible to test experimentally. The purpose of this paper is to question the reasonability of this assumption and actually to propose a test capable of falsifying it.

[a]An experiment with massive particles [23] did reach perfect detection efficiency. However, this experiment was unfortunately incapable of fulfilling the locality condition, which made it powerless to challenge local realism the way EPR–Bell experiments are meant to.

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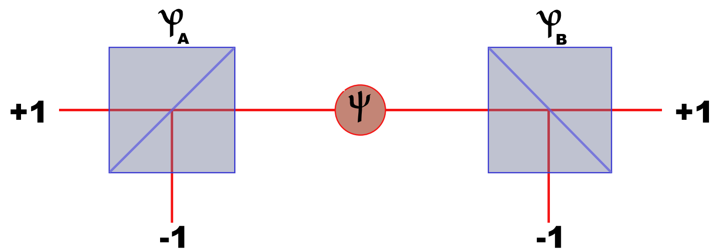

Fig. 1. Two-channel EPR–Bell experiment with photons. The source is the rotationally invariant singlet state $|\psi\rangle$ and the measurement is made by two polarizing beam splitters with orientation settings φ_A and φ_B monitored by a fourfold coincidence setup (not shown).

2. A Simple Local Hidden-Variable Model

The scheme of EPR–Bell experiment is presented in Fig. 1. It requires a source of entangled photons in a singlet state $|\psi\rangle$. For each pair of photon produced, photon 1 is sent to an experimenter named Alice, while photon 2 is sent to another experimenter named Bob. The distance between Alice and Bob is arbitrary, but it should be large enough to insure that the locality condition is fulfilled. They perform each a polarization measurement with a polarizing beam splitter (PBS) oriented respectively along φ_A and φ_B. Each photon can either be detected in the transmitted $(+1)$ or reflected (-1) channel of a PBS.

The source produces many pairs of photons per second. We assumed throughout this paper that the source is stable and that all runs are recorded during the same fixed time. The number of the four possible coincident detections is recorded and labeled as N_{++}, N_{+-}, N_{-+}, and N_{--}.

Let N_T be the total number of detected pairs defined as

$$N_T = N_{++} + N_{+-} + N_{-+} + N_{--}. \tag{1}$$

It is then possible to define the measured correlation function

$$E = \frac{N_{++} - N_{+-} - N_{-+} + N_{--}}{N_T}, \tag{2}$$

which can be compared to the correlation predicted by quantum mechanics if one assumes that the set of detected pairs is an unbiased sample of the set of emitted pairs. Provided that this *fair sampling assumption* holds, experiments with pairs of optical photons [7, 30] have shown a good agreement with the $\cos\left[2(\varphi_B - \varphi_A)\right]$ predicted by quantum mechanics.

From a local realist standpoint, there are not many ways to get such an EPR-like correlation using a model taking advantage of the detection loophole. This point has been stressed by Clauser and Shimony as a case against such local hidden-variables models [31], but we would like to emphasize that it may arguably be used as a case *in favor* of such models. Since they all actually point in the same conceptual direction, there might be some relevant physical truth behind the idea of a sample selection bias [29]. In any case, the remarkable convergence of all such models is important for our present purpose since it ensures the generality of our demonstration.

The model we use here is indeed a simple variant of the one that has been used many times to show explicitly that a local realistic model based on detection loophole can be in agreement with the experimental results [24–26, 28, 32]. In our model, each particle is provided with a supplementary parameter λ

– a polarization – that is randomly drawn from a uniform distribution on the interval $[0, 2\pi]$. The perfect correlation for identical measurement is ensured by particles issued from one pair having the same polarization. The fate of a particle impinging on an analyzer oriented along the direction φ is determined by the difference $\alpha = |\lambda - \varphi|$:

- If α is close to 0 modulo π, the particle goes into the channel labeled +1.
- If α is close to $\pi/2$ modulo π, the particle goes into the channel labeled −1.

As such, the model is incapable of providing anything better than the sawtooth for the correlation function (with maxima reaching 1 for identical measurements) and is thus incapable of violating any of Bell's inequality, independently of what is meant exactly by "close" in the model, and whatever the polarization distribution of the pairs of particles.

Now, if there exists a non-detection channel (labeled 0), pairs that would normally reduce the correlation can be discarded, leading to an apparent violation of Bell's inequalities on the sample of detected pairs. The photons that must remain undetected for this purpose are the ones for which $\alpha \simeq \pi/4$ modulo $\pi/2$ (see Fig. 2). All other type of sampling would tend to reduce the correlation, resulting in no apparent violation of Bell's inequalities. This sampling process is unfair in the sense that the probabilities $\eta_u^+(\varphi, \lambda)$ and $\eta_u^-(\varphi, \lambda)$ for a particle to be detected in a specific channel (respectively, +1 and −1) depends explicitly on its hidden parameters λ and on the measurement settings φ. In our model, these probabilities are always equal either to 0 or 1.

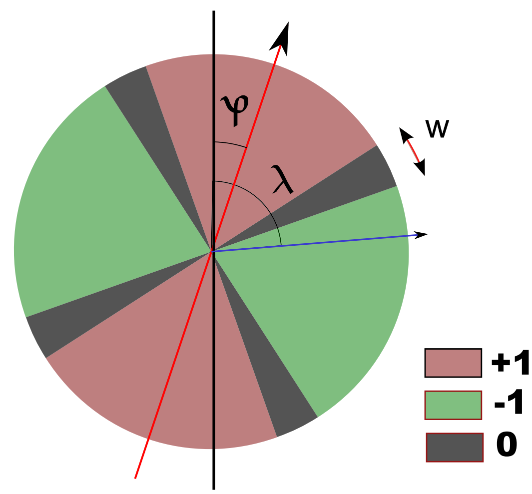

Fig. 2. Detection pattern. The measurement setting of the polarizer is represented by φ, and the polarization of the particle (its hidden variable) is represented by λ.

If both particles from a pair were sharing exactly the same polarization λ, the correlation obtained would show unrealistic sharp edges. This can be smoothed very near the cosine predicted by quantum mechanics simply by breaking the polarization alignment of the particles from the same pair. For this purpose, we add a random deviation to the polarization of one of the particles, the distribution of the deviation being a Gaussian centered on zero.[b] The correlation then shows a good agreement with the predictions of quantum mechanics (see Fig. 3).

It may be worthwhile to note that, contrary to the usual understanding of entanglement as being a stronger correlation than classically possible, we obtain a very good agreement with the EPR–Bohm statistics using loosely correlated particles. It is, however, possible to proceed with exactly correlated particles and with detection probability assuming values between 0 and 1, and similarly get an apparent violation of Bell's inequalities [26–28].

[b]As a matter of detail, we set the standard deviation of the Gaussian distribution to $\pi/16.80$, and the size of each of the four non-detection domains was $\pi/13.39$.

Note also that this process is fully local. It mimics nonlocality only because of the necessary coincidence relation — each time one particle remains undetected at one location, the *whole* pair is necessarily rejected because no correlation can be computed. This naturally introduce a *contextuality* in the measurement process, which is known to preclude the possibility of deriving Bell's theorem [33]. For each context, that is, each particular choice of measurement settings (φ_A, φ_B), a different and specific part of the probability space associated to the random variable λ is discarded from the statistics because of non-detection. The ensemble of detected pairs $S_{\varphi_A\varphi_B}$ belongs to a specific probability spaces $\Omega_{\varphi_A\varphi_B}$ which does depend on the measurement settings φ_A and φ_B. Bell's theorem, which is derived by using a single fixed probability space [21], is therefore no longer valid, and multi-context framework generalizations of Bell's and CHSH inequalities are required [18, 21, 34].

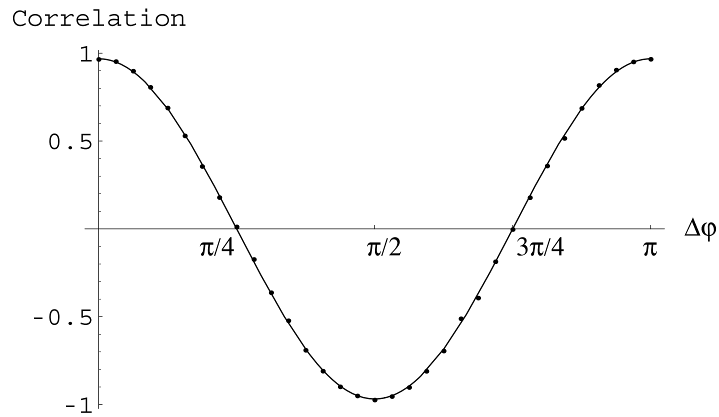

Fig. 3. The polarization of each particle forming a pair differs slightly. The difference is characterized statistically by a Gaussian distribution. The result is an apparent raw violation of CHSH–Bell's inequality by $S = 2.76$, for less than 30% pairs rejected and a visibility of 0.97. The solid curve is not a fit to the data but the prediction of quantum mechanics.

3. Testing the Fair Sampling Assumption for Local Hidden Variable Models

We will not discuss here the possible physical causes leading to such a sample selection bias,[c] but instead we focus on the observable consequences of such a sampling and discuss the feasibility of a fair sampling test.

Fair sampling is reputed to be a reasonable assumption. The trouble is that 'reasonable' is not a scientific criterion. It rather denotes a compliance with common sense, sound judgement, and perhaps general consensus. The lack of any better assessment probably comes from the alleged impossibility to test this assumption experimentally, the justifications of which being rather scarce in the literature.[d] What is probably meant by 'reasonable' is that the fair sampling assumption is not blatantly in contradiction with observed experimental facts or with known theories (in particular, with quantum theory). However, the fact that an assumption does not contradict known experiments and theories is by no means a confirmation of this assumption, as tempting as such an inductive leap might be.

To take a closely related example, the strength of Bell's theorem is that once it is accepted, experiments on EPR pairs can, in principle, "falsify" local realism, in Popper's sense. It would be more difficult, if not impossible, to design an experiment meant to *confirm* local realism. One could always conceive models in which particles correlated in a nonlocal way would hide their nonlocal character whenever one tries

[c]This important question will be discussed in an upcoming article.

[d]The impossibility of a test is justified for the one-channel type experiment [5] but, to the best of our knowledge, there is no such a justification for the two-channel type experiment.

to uncover it. From the experimental point of view, an EPR test failing to show a violation of Bell's inequalities would not challenge the predictions of quantum mechanics for entangled states. One could simply consider that the experiment was badly performed and the state under investigation was, in fact, not entangled.[e] Hence, local realism can be falsified by experiments, but it cannot be confirmed.

Similarly, the fair sampling assumption can, in principle, be falsified, but it probably cannot be confirmed. Just as it would be problematic to conceive a test that would confirm local realism, it is probably always possible to come up with a model in which particles detected following an unfair sampling process would conspire to hide this biased character whenever one tries to uncover it. It does not mean, however, that there is no point in performing a fair sampling test. Obviously, experiments meant to falsify local realism demanded a certain amount of faith in the correctness of quantum-mechanical predictions and the existence of entangled states over space-like distances. Similarly, investigations on the fair sampling assumption would require a certain amount of faith in the possibility that the sampling could actually be biased. Unfortunately, the general consensus is that this possibility is not worth investigating; so strong is the confidence and belief in the predictions of quantum mechanics. Before the advent of Bell's inequality tests, local realism would most certainly have been deemed a very reasonable assumption. The great amount of literature published every year on Bell's theorem and local realism testifies that it is still considered a very reasonable assumption by many. Yet, it would clearly have been unfortunate if local realism had been considered too reasonable an assumption to be worth testing. Just like local realism demanded to be tested, regardless of how reasonable an assumption it was, similarly fair sampling demands a test.

Many new EPR tests are proposed each year, but all are oriented towards a falsification of local realism. It is unfortunate that at the same time no test is endeavored to falsify fair sampling, because it renders the game unfair — no experiment is challenging this simple prediction of quantum mechanics, namely, that the sampling in an optical EPR experiment is fair.[f]

3.1. A Passive Fair Sampling Test

The main existing argument in favor of the fair sampling assumption is based on experimental data.[g] It consists in investigating whether the total rate number of coincidences N_T is independent of measurement settings.

The idea to test the fair sampling assumption is to vary the angle between the polarizing beam splitters $\Delta\varphi = |\varphi_B - \varphi_A|$ and check whether a variation of N_T is observed. A numerical simulation using the model described above shows that the size of N_T depends on $\Delta\varphi$ (see Fig. 4). This is due to the fact that the non-detection events are dependent — when both analyzers are oriented in the same direction modulo $\pi/2$, the chances that only one particle remains undetected are smaller than for other

[e] For instance, the experiment performed by Holt and Pipkin in 1973 at Harvard University [35] that showed violation of Bell's inequalities was never considered a threat to the predictions of quantum mechanics and was, in fact, never even published.

[f] By definition, quantum mechanics does not have any supplementary parameters (or hidden variable) according to which a sample bias could be introduced — the sampling must be fair since all photons described by the same quantum state are, in principle, indistinguishable.

[g] A second argument is based on the design of two-channel EPR experiments: the symmetrical scheme of the experiments in the sense that the number of single counts in the $+1$ and -1 channels are balanced at each station (Alice and Bob). It has been mentioned as an argument in favor of fair sampling [30], particularly, as a comparison with one-channel-type experiments in which a -1 result is not distinguishable from a non-detection. However, existing local realistic models exploiting the detection loophole for the two-channel design fulfill this requirement as well.

relative angles; since the total number of single counts registered at each detector is independent of the measurement settings, the total number of detected pairs is larger.

This is indeed a valid fair sampling test, but we would like to argue that it has never been thoroughly tested experimentally. Indeed, this modulation of the total number of coincidences requires the detection pattern on both sides to be strictly identical. If they are not (for instance, if the detection efficiency of one of the detector is lowered with random non-detection events [25]) then the non-detections can be made independent, and the total number of coincidences no longer depends on the measurement settings [28].

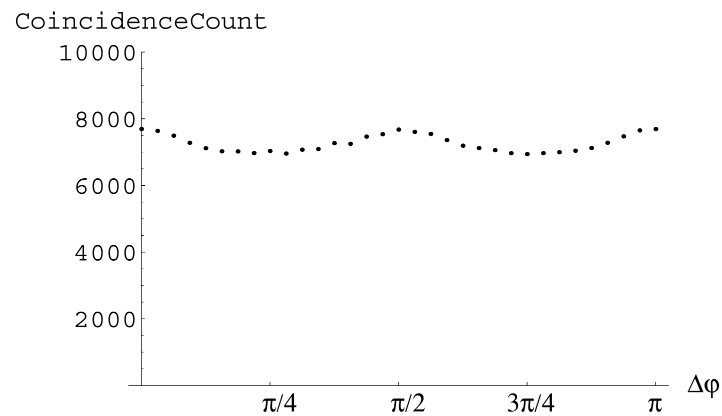

Fig. 4. Total number of coincidences for the apparent violation of CHSH–Bell's inequality in the case of identical detection pattern.

From the experimental point of view, this identical detection pattern condition cannot be enforced, but one should, however, make sure that this condition is not blatantly violated — the experiment should be such that Alice and Bob do record the same number of counts. True enough, EPR experiments are necessarily set with carefully balanced detection efficiencies η^+ and η^- at each station. However, to the best of our knowledge, a balanced efficiency of the two stations has never been sought as an experimental feature worth achieving, probably because it is not required to observe an apparent violation of Bell's inequalities. It should, nevertheless, be stressed that it is a required feature for this fair sampling test to be valid.

3.2. An Active Fair Sampling Test

Nevertheless, another approach to testing fair sampling is possible by varying not only φ_A and φ_B but also the polarization distribution ρ of the source.

The idea is that, in all known local realistic models based on a sample selection bias, the probabilities of detection $\eta_u^+(\varphi,\lambda)$ and $\eta_u^-(\varphi,\lambda)$ crucially depend not only on the measurement setting φ of the polarizer, but also on a hidden variable λ, thus, identifiable as the polarization of the particle. Moreover, in order to lead to an apparent violation of Bell's inequality, the model must be such that $\eta_u^+(\varphi,\lambda)$ and $\eta_u^-(\varphi,\lambda)$ reach a minimum together[h] for a fixed φ. Hence, we propose to break the rotational invariance of the source of entangled photons and feed instead the coincidence circuitry with a state having a probability distribution ρ_θ centered on a particular polarization angle θ. In the case of a sample selection bias, varying ρ_θ allows us to hit specific regions of the detection patterns, in particular, those for which the probability of detection reaches its extrema, which will be directly visible in the total rate of coincidences.

In order to control the source experimentally, our proposal is, therefore, to insert two aligned polarizers oriented along the same angle θ in the coincidence circuitry, just after the rotationally invariant source ρ of entangled photons and before the two-channel measurement devices (see Fig. 5). This test is, therefore,

[h]It is the essence of the sample selection bias — some particles with a specific λ must have a much smaller probability to be detected than others in a given context, if an apparent violation of Bell's inequalities is to be obtained.

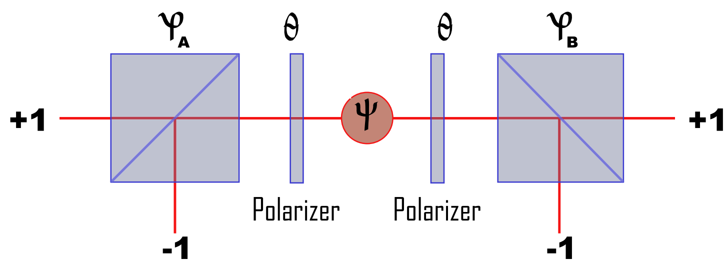

Fig. 5. Scheme of controlled EPR–Bell experiment to actively test fair sampling. The only difference from the usual EPR–Bell experiment (shown in Fig. 1) is the source, which is controlled by the parameter θ. In the case of unfair sampling, this setup should exhibit oscillations for the total number of coincidences as in Fig. 6.

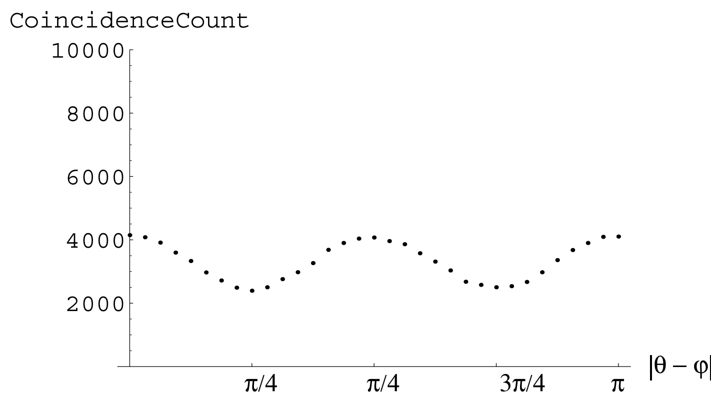

Fig. 6. Total number of coincidences for a source controlled with aligned polarizing beam splitters at angle 0 and for realistic settings (as in Figs. 3 and 4). The variation of R_d due to unfair sampling appears sharply.

easy to implement once the EPR–Bell setup is ready — it requires simply the insertion of two polarizers, as it was done in early single-channel EPR experiments.

From a local realistic point of view, the procedure to simulate our active fair sampling test is the following:

1. All models based on a sample selection bias provide a detection pattern for the couple PBS + Detector. This detection pattern processes an *input*, dispatching each particle in the proper channel in order to obtain the EPR–Bell statistics, when a specific rotationally invariant source impinges onto it. The model usually says nothing as to the *output* polarization distribution ρ_θ of each channel, and that is what is needed in our test.

2. This output polarization distribution ρ_θ must be set consistently with the known behavior of two successive polarizers, that is, consistently with Malus' law. One should also keep in mind that a polarizer does not only act like a filter, but it also rotates the output polarization of an incoming beam, as can be seen by inserting a polarizer between two crossed polarizers.

3. Once the output polarization distribution ρ_θ is known, we simply proceed with the test itself, by inserting two aligned polarizing beam splitters oriented along an angle θ in the coincidence circuitry, just before the two-channel measurement devices, both oriented along the same angle φ.

Note that this procedure is quite general and need not apply solely to our model. Although local realist models based on the detection loophole are always concerned only with reproducing the EPR correlations (Step. 1), it is straightforward in most cases to complete the model so as to get the Malus law (Step. 2), and thereof perform our fair sampling test (Step. 3). We have actually followed this procedure both with our model and Larsson's symmetrized model and found that the output necessary to reproduce Step. 2 is the same, and that the result of our test in Step. 3 shows exactly the same behavior as exhibited in Fig. 5.

To be more specific, we gave a Gaussian distribution to the *shape output* polarization of the photons in the considered channel (+1 in this case). In other words, the output state of each particle dispatched in channel +1 was modified randomly according to the context of the encountered polarizing beam splitter so that this output has statistically the form of a Gaussian distribution centered on the main polarization direction of the polarizer.[i] In a quantum formulation, this modification of the state is nothing but the collapsing of the initial vector state, from the rotationally invariant singlet state to $|++\rangle_\theta$. By doing so, we have obtained the Malus' law behavior without in any way modifying our detection pattern for the EPR–Bell experiment.

The numerical simulation of our active fair sampling test shows a much clearer characteristic of unfair sampling (see Fig. 6) with very clear oscillations — the contrast is roughly five times better than for the passive fair sampling test of Fig. 4. It must be noted that these oscillations cannot be made arbitrarily small without hindering the observed EPR–Bell correlations — the higher the violation of Bell's inequalities, the higher the oscillations of the total number of coincidences in our fair sampling test.

Our test can, therefore, rule out fair sampling if these oscillations are observed experimentally, since no fair sampling process could account for such contrasting oscillations thus connected to an apparent violation of Bell's inequalities.[j]

4. Conclusions

Decades after the first attempts to put the ideal EPR–Bohm scheme to an experimental test, the detection loophole remains a weakness that keeps hindering the interpretational reach of existing local realism tests. We have argued here that the available EPR–Bell experiments would be far more convincing, if fair sampling was thoroughly tested each time and an apparent violation of Bell's inequalities is obtained experimentally. The fair sampling test we have proposed here is simple to implement in standard experimental setups, and it is as necessary a test as local realism tests are.

[i] The Gaussian distribution leading to a good agreement with Malus' law has a standard deviation of $\pi/9$. The choice of the Gaussian distribution is quite arbitrary, and another distribution might provide a slightly better fit to Malus' law. The standard deviation would, nevertheless, have to be similar, and the qualitative result obtain under our test would thus remain unchanged.

[j] Note that it is possible to introduce an additional hidden variable that tells each photon impinging on a PBS whether it should behave according to a fair sampling process or to an unfair one depending on whether the photon comes from the output of another polarizer or from the output of a source of entangled photons. This would allow for the model to reproduce both Malus' law and EPR–Bell correlation, yet with an invariant total number of coincidences, even under our test (we are grateful to Jan-Åke Larsson for pointing out this idea). This illustrates the fact that no test is likely to be capable of *confirming* fair sampling. Yet, as discussed above, the status of a fair sampling test is similar to that of local realism tests — both hypotheses can only be falsified, and they both need to be tested nevertheless.

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