

BOUND STATES OF A PARTICLE IN MOVING δ -POTENTIALS IN THE PRESENCE OF A LINEAR FIELD

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Abstract

The problem of bound states in scattering point centers in the presence of a field described by a non-stationary quadratic potential is studied. One-dimensional and three-dimensional cases are considered.

Keywords: Schrödinger equation, scattering point center, moving potential, linear nonstationary oscillator.

1. Introduction

Zero-range potentials or point potentials or δ -potentials are widely used in atomic and nuclear physics (see [1–3]).

In this context, the possibility of finding the exact solution to nonstationary Schrödinger equation for such potential is of special interest. For scattering δ -centers, the exact solution for the bound state described by exponentially decreasing functions has been presented in [4].

A well-known example of the exact solution of the nonstationary Schrödinger equation is that of the equation for an oscillator with the frequency dependent explicitly on time. The equation for one-dimension parametric oscillator was solved by Husimi [5] (see, for example, also [2, 6]). In this case, the potential depends smoothly on coordinates.

The aim of the present work is to consider the possibility of exact solutions of the Schrödinger equation with a combined potential of a linear nonstationary oscillator and scattering δ -centers in both one-dimensional and three-dimensional problems.

2. A Particle in the Quadratic Nonstationary Potential and Classical Integral of Motion

The Schrödinger equation with a quadratic nonstationary potential in the one-dimensional problem has the following form:

$$i \frac{\partial \psi}{\partial t} + \frac{1}{2} \frac{\partial^2 \psi}{\partial x^2} + \frac{\Omega^2(t)x^2}{2} \psi = 0, \quad (1)$$

where ψ is the wave function, x and t are the coordinate and time, respectively, and $\Omega(t)$ is the time-dependent imaginary frequency of the oscillator. We take dimensionless units and assume that $m = \hbar = 1$.

In order to solve Eq. (1), we use the change of variables $x, t \rightarrow y, \tau$ given by

$$y = \frac{x}{\eta(t)}, \quad t = \xi(\tau).$$

Then one obtains the Schrödinger equation in the form

$$\frac{i}{\dot{\xi}} \frac{\partial \psi}{\partial \tau} - iy \frac{\partial \psi}{\partial y} \frac{\eta'}{\eta} + \frac{1}{2\eta^2} \frac{\partial^2 \psi}{\partial y^2} + \frac{\Omega^2 y^2 \eta^2}{2} \psi = 0,$$

where

$$\dot{\xi} = \frac{d\xi}{d\tau} = \frac{dt}{d\tau}, \quad \eta' = \frac{d\eta}{dt}.$$

Then let us assume that

$$\psi = \frac{1}{\sqrt{\eta}} \phi(y, \tau) \exp \frac{iy^2 \eta' \eta}{2}.$$

We arrive at the equation for ϕ in the following form:

$$\frac{i}{\xi} \frac{\partial \phi}{\partial \tau} + \frac{1}{2\eta^2} \frac{\partial^2 \phi}{\partial y^2} + \frac{y^2 \eta}{2} (\Omega^2 \eta - \ddot{\eta}) \phi = 0. \quad (2)$$

Let us assume now that

$$\dot{\xi} = \frac{dt}{d\tau} = \eta^2, \quad \text{i.e.,} \quad \tau = \int \frac{dt'}{\eta^2(t')}.$$

Equation (2) can be solved in two different ways.

Suppose that the auxiliary function $\eta(t)$ satisfies the equation

$$\eta'' - \Omega^2(t)\eta = 0. \quad (3)$$

Then Eq. (2) has a solution in the form of a plane wave in variables y and τ , namely,

$$\phi = \exp \left\{ \frac{ip^2 \tau}{2} + ipy \right\}$$

and it leads to the following solution of Eq. (1):

$$\psi_p(x, t) = \exp \left\{ ip \frac{x}{\eta} - \frac{ip^2}{2} \int \frac{dt'}{\eta^2(t')} + \frac{ix^2 \eta'}{2\eta} \right\}. \quad (4)$$

The classical equation of motion corresponding to Eq. (1) reads

$$\ddot{x} - \Omega^2(t)x = 0. \quad (5)$$

It is known that Eq. (5) has an integral of motion that is linear in \dot{x} and x . This linear integral of motion is not directly related to the space-time symmetry properties. It can be written as follows:

$$I^{(1)} = -\eta' x + \eta \dot{x}.$$

Calculation of the derivative $dI^{(1)}/dt$ taking into account (3) and (5) demonstrates the conservation of $I^{(1)}$.

For the quantum-mechanical problem, the momentum \hat{x} has to be replaced by the operator $-i\partial/\partial x$. In this case, one has the eigenvalue equation

$$\left(-i\eta\frac{\partial}{\partial x} - \eta'x\right)\psi_p = p\psi_p, \quad (6)$$

where p is the eigenvalue of the operator [6, 7]

$$\hat{I}^{(1)} = -i\eta\frac{\partial}{\partial x} - \eta'x.$$

Linear integrals of motion were recently used to solve some nonstationary problems in [8, 9].

The function $\phi_p(x, t)$ defined by Eq. (4) satisfies Eq. (6).

The second way to solve Eq. (2) is as follows.

Instead of Eq. (3) let us require that $\eta(t)$ satisfy the following equation:

$$\eta'' - \Omega^2(t)\eta = \frac{\Omega_0^2}{\eta^3(t)}, \quad (7)$$

where $\Omega_0 \equiv \text{const}$. This means that Eq. (2) is reduced to the oscillator equation with a constant frequency Ω_0 . Besides the linear integral of motion of Eq. (5), there exists also a quadratic in the x, \dot{x} integral of motion

$$I^{(2)} = (\eta'x - \dot{x}\eta)^2 + \Omega_0^2\frac{x^2}{\eta^2}.$$

Calculation of the derivative $dI^{(2)}/dt$ taking into account Eqs. (3) and (7) demonstrates that $dI^{(2)}/dt \equiv 0$. After replacing \dot{x} by the operator $-i\partial/\partial x$, one obtains the eigenvalue equation

$$\left(x^2\eta'^2 + 2i\eta\eta'x\frac{\partial}{\partial x} + i\eta\eta' - \eta^2\frac{\partial^2}{\partial x^2} + \Omega_0^2\frac{x^2}{\eta^2}\right)\psi_a = a\psi_a, \quad (8)$$

where a is the eigenvalue of the operator $\hat{I}^{(2)}$ (see, [6, 10]). The corresponding function $\phi(y, \tau)$ satisfies the equation

$$\left(\Omega_0^2y^2 - \frac{\partial^2}{\partial y^2}\right)\phi_a = a\phi_a. \quad (9)$$

Comparing (9) with (2) one can see that the parameter $a = 2E$, where E plays the role of the harmonic-oscillator energy. It is worthy noting that Eq. (1) describes a situation where the energy is not preserved. Therefore, the operator $\hat{I}^{(2)}$ can be considered as a generalization of the energy operator for a nonstationary problem.

The solutions of Eq. (2) described by different auxiliary functions $\eta(t)$ are mutually related. The connection between the problems of a harmonic oscillator and a free particle has been analyzed in [11].

3. A Particle in the Potential of Point Centers Superposed with a Quadratic Potential

Let us add on the right-hand side of Eq. (1) a term describing scattering δ -centers, whereby we have

$$i\frac{\partial\psi}{\partial t} + \frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + \frac{\Omega^2x^2}{2}\psi = -\alpha\left[\delta(x - x_0(t)) + \delta(x + x_0(t))\right]\psi. \quad (10)$$

Using the change of variables employed in the previous section, one arrives at the following equation:

$$i\frac{\partial\phi}{\partial\tau} + \frac{1}{2}\frac{\partial^2\phi}{\partial y^2} + \frac{y^2\eta^3}{2}(\Omega^2\eta - \eta'')\phi = -\alpha\eta\left[\delta\left(y - \frac{x_0}{\eta}\right) + \delta\left(y + \frac{x_0}{\eta}\right)\right]\phi. \quad (11)$$

Equation (11) can be solved if $\frac{x_0}{\eta} \equiv \text{const}$ and $\alpha\eta \equiv \text{const}$. Under these conditions, the variables τ and y are separated. In addition, it is possible to assume the equality

$$\eta'' - \Omega^2\eta = \frac{\Omega_0^2}{\eta^3};$$

then

$$\alpha\alpha'' - 2\alpha'^2 + \Omega^2(t)\alpha^2 = -\Omega_0^2.$$

In this case, after separating the variables τ and y , one obtains a more complicated equation for the function of y , which is the oscillator equation with a constant frequency in the presence of two stationary δ -centers. Situations where the separation of variables is impossible are the most interesting.

The case where $\alpha \equiv \text{const}$, $\eta(t) = c_*t$, and $x_0 = vt$ is widely known [4, 12]. Here $\eta'' \equiv 0$ and $\Omega \equiv 0$. If one assumes $\eta = \tau^{-2}$, then

$$t = -\frac{1}{3\tau^3}, \quad \eta = \frac{1}{(3t)^{2/3}}, \quad \eta'' = \frac{10}{(3t)^{2/3}}$$

and, in order to exclude the term proportional to y^2 in Eq. (11), the initial equation (10) should have an external force described by a quadratic potential $\Omega^2 x^2/2$, where $\Omega^2 = 10/3t^2$.

The equation for ϕ is reduced to the following one:

$$i\frac{\partial\phi}{\partial\tau} + \frac{1}{2}\frac{\partial^2\phi}{\partial y^2} = -\frac{\alpha}{\tau^2}\left[\delta(y - y_0) + \delta(y + y_0)\right]\phi. \quad (12)$$

For $t \rightarrow -\infty$, i.e., at $\tau = 0$ and $\phi = 0$, Eq. (12) can be presented in the integral form, in view of the retarded Green function, as follows:

$$\phi(y, \tau) = \frac{i\alpha}{\sqrt{2\pi i}} \int_0^\tau \frac{d\tau'}{\sqrt{\tau - \tau'}} \frac{1}{(\tau')^2} \phi(y_0, \tau') \left(\exp\left[\frac{i(y - y_0)^2}{2(\tau - \tau')}\right] + \exp\left[\frac{i(y + y_0)^2}{2(\tau + \tau')}\right] \right). \quad (13)$$

Let us consider now the symmetric case where $\phi(y, \tau) \equiv \phi(-y, \tau)$.

Introducing the notation

$$\frac{\phi(y, \tau)}{\tau^2} = c(\tau),$$

one has the following equation:

$$\tau^2 c(\tau) = \frac{i\alpha}{\sqrt{2\pi i}} \int_0^\tau \frac{d\tau' c(\tau')}{\sqrt{\tau - \tau'}} \left(1 + \exp\left[\frac{2iy_0^2}{\tau - \tau'}\right] \right), \quad (14)$$

which is a consequence of Eq. (13).

Assuming that

$$H(p) = \int_0^{\infty} \exp(-p\tau)c(\tau)d\tau,$$

Eq. (14) implies that

$$\frac{d^2H}{dp^2} = \frac{\alpha H}{\sqrt{-2ip}} \left[1 + \exp\left(-2y_0\sqrt{-2ip}\right) \right].$$

By introducing the variable $q = \sqrt{-2ip}$, one obtains

$$\frac{d^2H}{dq^2} - \frac{1}{q} \frac{dH}{dq} = -\alpha H q \left[1 + \exp(-2y_0q) \right]. \quad (15)$$

As a result, the solution to Eq. (13) may be rewritten in the form of a two-fold integral, with $c(\tau)$ being expressed in terms of H using the inverse Laplace transform, which should be substituted into the integral in expression (13). Equation (15) is unlikely to have a compact analytical solution but finding the numerical solution is rather simple.

Another case providing the solution of the Shrodinger equation is the case where $\eta = \tau$, $t = \tau^3/3$. If the external force in Eq. (10) is described by the frequency $\Omega^2 = -2/9t^2$, then the equation for $\phi(y, \tau)$ reads as follows:

$$i \frac{\partial \phi}{\partial \tau} + \frac{1}{2} \frac{\partial^2 \phi}{\partial y^2} = -\alpha \tau \left[\delta(y - y_0) + \delta(y + y_0) \right] \phi, \quad (16)$$

where $\phi = 0$ at $t = +\infty$, i.e., at $\tau = 0$.

In view of the advanced Green function, the above equation can be written in the integral form

$$\phi(y, \tau) = \frac{\alpha}{\sqrt{2\pi i}} \int_{\tau}^{\infty} \frac{\tau' d\tau'}{\sqrt{\tau' - \tau}} \phi(y_0, \tau') \left(\exp \left[\frac{i(y - y_0)^2}{2(\tau' - \tau)} \right] + \exp \left[\frac{i(y + y_0)^2}{2(\tau' - \tau)} \right] \right). \quad (17)$$

While considering the symmetrical case, let us assume that

$$\phi(y_0, \tau) = \phi(-y_0, \tau) = c(\tau).$$

From (17) follows

$$c(\tau) = \frac{\alpha}{\sqrt{2\pi i}} \int_{\tau}^{\infty} \frac{\tau' d\tau' c(\tau')}{\sqrt{\tau' - \tau}} \left(1 + \exp \left[\frac{2iy_0^2}{\tau' - \tau} \right] \right). \quad (18)$$

Since one of the limits of the integral in (18) equals ∞ , the solution can be presented (similar to [1]) in the form

$$c(\tau) = \int_L \exp(ip\tau) g(p) dp, \quad (19)$$

where the path of integration L is determined by the condition of integral convergency. For $g(p)$, it is possible to obtain the equation

$$g(p) = i \frac{dg}{dp} \frac{\alpha}{\sqrt{p}} \left[1 + \exp(-2y_0\sqrt{2p}) \right]. \quad (20)$$

Assuming $p = q^2/2$, the solution to Eq. (20) reads

$$g(q) = C \exp\left(-\frac{i}{\alpha\sqrt{2}} \int \frac{q dq}{1 + \exp(-2y_0q)}\right), \quad (21)$$

where C is a constant.

It is possible to calculate the integral in Eq. (18), and the expression for ϕ can be found in the following form:

$$\begin{aligned} \phi = C \int_{L_0} g(q) dq \exp\left(\frac{iq^2\tau}{2}\right) & \left\{ \left(\tau + \frac{i}{q^2} + i\frac{|y-y_0|}{q}\right) \exp(-q|y-y_0|) \right. \\ & \left. + \left(\tau + \frac{i}{q^2} + i\frac{|y+y_0|}{q}\right) \exp(-q|y+y_0|) \right\}. \end{aligned} \quad (22)$$

The path of integration L_0 contains two lines in the complex plane, i.e., $-(1+i\varepsilon)\infty, 0$ and $0, +\infty$, where the constant ε satisfies the condition $1 \gg \varepsilon > 0$.

4. Three-Dimensional Problem of a Particle in Combined Potential

The problem of scattering δ -centers for the three-dimensional case has been considered in a number of papers (see, for example, [12–15], where [12–14] deal mainly with the scattering states in the nonstationary problem and [15] considers the bound states in the presence of δ -centers). In this section, we study the bound states in the system of scattering centers in the presence of a linear force described by a quadratic potential.

In this case, the Schrödinger equation has the following form:

$$\begin{aligned} i\frac{\partial\psi}{\partial t} + \frac{1}{2}\Delta\psi + V(r,t)\psi = \frac{2\pi}{\chi_0} & \left\{ \left[\psi(1 - i\vec{r}'(\vec{r} - \vec{r}_0(t))) + (\vec{r} - \vec{r}_0(t))\nabla\psi \right] \delta(\vec{r} - \vec{r}_0(t)) \right. \\ & \left. + \left[\psi(1 + i\vec{r}'(\vec{r} + \vec{r}_0(t))) + (\vec{r} + \vec{r}_0(t))\nabla\psi \right] \delta(\vec{r} + \vec{r}_0(t)) \right\}, \end{aligned} \quad (23)$$

where $V = \Omega^2(t)r^2/2$, χ_0 is the depth of the bound level, and $\pm\vec{r}_0$ are the positions of δ -centers at the time moment t . At $V \equiv 0$ and $\vec{r}_0 = \vec{v} = \text{const}$, Eq. (23) describes the scattering point potentials (see [12, 13]).

In accordance with [12], in order to solve Eq. (23), it is necessary to introduce time-dependent coordinates

$$\vec{\rho} = \frac{\vec{r}}{|\vec{r}_0(t)|}, \quad t = \xi(\tau).$$

We assume that

$$\psi = r_0^{3/2} \exp\left[\frac{ir^2 r'_0}{2 r_0}\right] \phi(r, t),$$

where $r'_0 = dr_0/dt$. If $\vec{r}_0(t)$ is varying only in magnitude while preserving the direction, then from Eq.

(23) follows

$$\begin{aligned} & \frac{i}{\xi} \frac{\partial \phi}{\partial \tau} + \frac{1}{2r_0^2} \frac{\partial^2 \phi}{\partial \bar{\rho}^2} + \phi \frac{\rho^2}{2} \left(\frac{\ddot{\xi}}{\xi^3} r_0 \dot{r}_0 - \frac{r_0 \ddot{r}_0}{\xi^2} \right) + \frac{\Omega^2(\xi(\tau)) \rho^2 r_0^2}{2} \phi \\ & = \frac{2\pi}{\chi r_0^3} \left\{ \left[\phi + (\bar{\rho} - \bar{\rho}_0) \frac{\partial \phi}{\partial \bar{\rho}} \right] + \left[\phi + (\bar{\rho} + \bar{\rho}_0) \frac{\partial \phi}{\partial \bar{\rho}} \right] \delta(\bar{\rho} + \bar{\rho}_0) \right\}, \end{aligned} \quad (24)$$

where

$$\dot{\xi} = \frac{dt}{d\tau}, \quad \dot{r}_0 = \frac{dr_0}{d\tau} = r_0' \dot{\xi}.$$

Let us consider now the case where

$$V + \frac{\rho^2}{2} \left(\frac{\ddot{\xi}}{\xi^3} r_0 \dot{r}_0 - \frac{r_0 \ddot{r}_0}{\xi^2} \right) \equiv 0 \quad \text{and} \quad \dot{\xi} \equiv r_0'^2.$$

Under these conditions, it is suitable to use the Green function of the Schrödinger equation. In the symmetric case under consideration, the boundary conditions read

$$\left(\phi + (\bar{\rho} - \bar{\rho}_0) \frac{\partial \phi}{\partial \bar{\rho}} \right)_{\bar{\rho} \rightarrow \bar{\rho}_0} = \left(\phi + (\bar{\rho} + \bar{\rho}_0) \frac{\partial \phi}{\partial \bar{\rho}} \right)_{\bar{\rho} \rightarrow -\bar{\rho}_0} = -\chi_0 c(\tau). \quad (25)$$

In view of the advanced Green function, Eq. (24) can be written in integral form:

$$\phi = -\frac{2\pi}{(2\pi i)^{3/2}} \int_{\tau}^{\infty} \frac{d\tau' c(\tau')}{(\tau' - \tau)^{3/2} r_0(\tau')} \left\{ \exp \left[\frac{i(\bar{\rho} - \bar{\rho}_0)^2}{2(\tau' - \tau)} \right] + \exp \left[\frac{i(\bar{\rho} + \bar{\rho}_0)^2}{2(\tau' - \tau)} \right] \right\}. \quad (26)$$

Using Eq. (26) with the boundary condition (25) one can obtain the following equation for $c(\tau)$:

$$\chi_0 c(\tau) = \frac{2\pi}{(2\pi i)^{3/2}} \left\{ 2 \int_{\tau}^{\infty} \left[\frac{d}{d\tau'} \frac{c(\tau')}{r_0(\tau')} \right] \frac{d\tau'}{\sqrt{\tau' - \tau}} + \int_{\tau}^{\infty} \frac{d\tau'}{(\tau' - \tau)^{3/2}} \left[\frac{c(\tau')}{r_0(\tau')} \right] \exp \left[\frac{2i\rho_0^2}{\tau' - \tau} \right] \right\}. \quad (27)$$

While deriving Eq. (27), which describes the bound state, it is supposed that τ increases with t and at $t \rightarrow \infty$, $\phi \equiv 0$. In addition, the operation of “subtraction of infinities” (see [1, 15]) is applied.

Now we consider the case where $r_0 = a\tau$ and

$$t = \frac{a^2 \tau^2}{3}, \quad \tau = \left(\frac{3t}{a^2} \right)^{1/3}, \quad r_0 = (3at)^{1/3}.$$

The potential of the external force reads

$$V(r, t) = -\frac{r^2}{9t^2}.$$

In addition to the force attracting the particles to δ -centers, there exists a nonstationary force attracting them to the point $\vec{r} = 0$.

The solution to Eq. (27) can be found in the following form:

$$\frac{c(\tau)}{r_0(\tau)} = \int_L u(p) \exp(ip\tau) dp, \quad (28)$$

where the path of integration L has to be chosen from the condition of integral convergency.

From (27), the following equation for $u(p)$ can be derived:

$$a\chi_0 \frac{du}{dp} = -i\sqrt{2p}u(p) + \frac{i}{2\rho_0}u(p) \exp(-2\rho_0q) \quad (29)$$

and one has

$$u(q) = C \exp \left\{ -\frac{i}{a\chi_0} \left(\frac{q^3}{3} + \frac{q \exp(-2q\rho_0)}{4\rho_0^2} + \frac{\exp(-2q\rho_0)}{8\rho_0^3} \right) \right\}. \quad (30)$$

The following expression for $\phi(\rho, \tau)$ can be obtained from (28), (30), and (26):

$$\phi = \tilde{C} \int_{i\infty}^{\infty} q dq u(q) \exp \left(\frac{iq^2\tau}{2} \right) \left\{ \frac{\exp(-q|\rho - \rho_0|)}{|\vec{\rho} - \vec{\rho}_0|} + \frac{\exp(-q|\rho + \rho_0|)}{|\vec{\rho} + \vec{\rho}_0|} \right\}. \quad (31)$$

It is not difficult to check that the integral in (31) converges at both $q \in [+i\infty, 0]$ and $q \in [0, +\infty]$.

The solution for $\psi(\vec{r}, t)$ reads

$$\psi = \frac{1}{r_0^{3/2}} \exp \left(\frac{i}{2} \frac{r^2 r_0'}{2r_0} \right) \phi \left[\frac{\vec{r}}{(3at)^{1/3}}, \left(\frac{3t}{a^2} \right)^{1/3} \right].$$

Thus, in this paper we presented partial solutions of the nonstationary Schrödinger equation, which can be important in analyzing particular experiments.

Some methods of solution were presented where the problem under consideration was reduced to solving the standard differential equations and evaluating the integrals.

References

1. Yu. N. Demkov and V. N. Ostrovskii, *Zero-Range Potentials and Their Application in Atomic Physics*, Plenum Press, New York (1988).
2. A. I. Baz', Ya. B. Zel'dovich, and A. M. Perelomov, *Scattering, Reactions, and Decays in Nonrelativistic Quantum Mechanics* [in Russian], 2nd ed., Nauka, Moscow (1971).
3. G. Breit, *Ann. Phys. (N.Y.)*, **34**, 377 (1965).
4. S. K. Zhdanov and A. S. Chikhachev, *Dokl. Akad. Nauk SSSR*, **218**, 1323 (1974) [*Sov. Phys. – Dokl.*, **19**, 696 (1975)].
5. K. Husimi, *Progr. Theor. Phys.*, **9**, 381 (1953).
6. I. A. Malkin and V. I. Man'ko, *Dynamical Symmetries and Coherent States of Quantum Systems* [in Russian], Nauka, Moscow (1979).
7. I. A. Malkin, V. I. Man'ko, and D. A. Trifonov, *Phys. Lett. A*, **30**, 414 (1969); *Phys. Rev. A*, **2**, 1371 (1970).

8. R. Fedele and V. I. Man'ko, *Phys. Rev. E*, **58**, 992 (1998); **60**, 6042 (1999).
9. R. Fedele, F. Galluccio, V. I. Man'ko, and G. Miele, *Phys. Lett. A*, **209**, 263 (1995).
10. H. R. Lewis, *Phys. Rev. Lett.*, **18**, 510 (1967).
11. E. A. Solov'ev, *Yad. Phys.*, **35**, 242 (1982).
12. E. A. Solov'ev, *Teor. Mat. Fiz.*, **28**, 240 (1976) [*Theor. Math. Phys.*, **28**, 757 (1976)].
13. J. H. Macek, S. Yu. Ovchinnikov, and E. A. Solov'ev, *Phys. Rev. A*, **60**, 1140 (1999).
14. J. Wang, J. Burgdörfer, and A. Barany, *Phys. Rev. A*, **43**, 4036 (1991).
15. A. S. Chikhachev, *Zh. Éksp. Teor. Fiz.*, **125**, 1012 (2004) [*JETP*, **98**, 882 (2004)].