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Estimating Mean Length of Stay in Prison: Methods and Applications

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Abstract This paper examines various approaches to studying the mean length of stay in prison. The literature contains a wide range of estimates of this quantity. The discrepancies that appear in these estimates and in the conclusions reached from them have been the subject of several reviews. We build upon that work, using the life table as the gold standard, to demonstrate the inaccuracy of common measures such as the ratio of the population size to the annual number of entrances or the mean length of time served by those exiting in a particular period. This demonstration is conducted in two parts. One part uses model populations with constant growth rates; the second part relies upon simulated prison populations with time-varying rates of entrance and exit. In addition, we introduce two new indirect measures that are more accurate than several existing indirect measures and that are relatively easy to use. The new measures are based on the entrance rate or the exit rate and adjust for the growth rate of the prison population.

Keywords Measurement · Prison population · Time served · Life table

Introduction

This paper assesses several methods for estimating the mean length of stay in prison. Techniques used to process data on prison populations are highly diffuse and their efficacy usually depends upon the validity of a particular set of assumptions. Most of the measures used assume the existence of a stationary population—a population in which the number of entrances equals the number of exits and duration-specific attrition rates are constant. Clearly, this assumption is not valid for the United States' prison population, which has

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been growing rapidly since the 1970s (Ruth and Reitz 2003). Violation of the assumption has led to flawed and inconsistent information in the criminological literature concerning the length of stay in prison.

The inadequacy of many present measurement approaches has been recognized by others. Lynch (1993) employed three methods to estimate the mean length of time served in prison and demonstrated that the two indirect methods based on stationary assumptions give values that are biased relative to those yielded by a life table, which is a direct embodiment of exit rates. Biderman (unpublished) reviewed several studies that examine mean length of stay in prison between 1988 and 1994. His paper critiques the methods used and advocates the use of life table methods employed by demographers and actuaries. Lynch and Sabol (1997) compare life table results to those using the most common indirect measure, the mean duration of time served by those leaving prison in any particular year. They show that this latter method provides estimates of the expected length of time to be served for someone incarcerated for murder that are too low by at least 4 years.

This paper provides a systematic analysis of existing measures and develops two new methods for estimating mean length of stay in prison. By this phrase, we refer to the mean length of time an individual will remain in prison from time of entrance to time of exit. We believe that the paper will contribute to the criminological literature by helping to foster accuracy and uniformity in measurement and more precise estimators of the impact of various programs and policies. Improved measures should prove especially useful in the deterrence literature, especially specific deterrence, where length of time spent in prison is often a key indicator (Tittle 1969; Chiricos and Waldo 1970; Logan 1975; Beck and Hoffman 1976; Orsagh and Chen 1988; Gendreau et al. 1999; Kleck et al. 2005).

The remainder of the paper proceeds as follows. First, we describe the best method for estimating mean length of stay, the life table, and then describe three other estimation methods, each of which relies upon the assumption of a stationary population. We show what biases result in these measures when a population is not stationary and we develop a method for correcting the biases in two of the three cases. Secondly, we demonstrate the performance of the four existing methods and two improved methods in populations that are not stationary. One set of such populations are termed "stable" populations in demographic parlance. Next, we use population simulations, allowing us to impose shocks stemming from growth or decline in prison entrances and in changes in the length of stay inside prison. Both of these approaches permit us to assess the sensitivity of the estimates to violation of assumptions and to error in data. We conclude with recommendations about the measures to be used for future examination of the length of stay inside prison.

Estimating Length of Stay in Prison

Life Tables

Methods for studying survival processes have been developed by demographers, biostatisticians, and actuaries. The principal device used to characterize survival, attrition, and length of stay is a life table (Keyfitz 1968; Preston et al. 2001). A life table presents many functions that describe particular features of the dying out of a cohort. A period life table, which is the most common variety, exposes a hypothetical cohort to a set of attrition rates recorded for that period. A period attrition rate has the number of exits during the period in the numerator and the number of person-years exposed to the risk of attrition in the denominator. These attrition rates are arrayed by duration in the state; if there is no duration-dependence, there is no reason to construct a life table. The cohort enters a state at duration zero and is then exposed to the set of duration-specific attrition rates until all members have exited. The classic life table was developed with respect to actual length of life, so that duration in the state (in this case, being alive) was simply one's age.

Table 1 presents a life table for all prison stays during 1997 in a group of 29 states.[1] Column 1 shows the duration of stay in prison at the beginning of a particular duration interval. The interval extends to the next value shown in the duration column. Column 2 presents the duration-specific attrition rates (i.e., exit rates) for various intervals.[2] Column 3 converts the attrition rates to probabilities of attrition during the interval for someone who stays in prison until the beginning of the interval.[3] Column 4 uses the values in column 3 to calculate the number of persons that remain in prison at the beginning of each duration interval. Column 5 presents the years expected to be spent by an arriving cohort in a particular duration category. Finally, column 6 shows the number of additional years expected to be spent in prison for someone who stays to a particular duration. Upon arrival, the expected length of stay based upon these attrition rates is 2.26 years. Table 2 presents the same information for the offense of murder, where the expected stay upon arrival is 20.01 years.

Stationary Population Measures

The format and calculations presented in Tables 1 and 2 are the most satisfactory means available for measuring the mean duration of stay in prison (the analog to life expectancy at birth in a classic life table). However, they require data on attrition rates by duration of stay, data that are often not available, in addition to requiring meticulous calculations. In the absence of appropriate data, several other means have been used to estimate mean duration of stay. The most common is the computation of the mean length of stay by people exiting prison in any particular period[4] (Biderman unpublished; Lynch and Sabol 1997; Lynch 1993). A second common method is to compute the ratio of the number of people in prison at any point in time to the annual number of entrances (Blumstein and Beck 1999; Lynch 1993). A third obvious method that has been suggested but not, to our knowledge, been employed is to compute the ratio of the number of people in prison at any point in time to the annual number of exits (Butts and Adams 2001). All of these surrogate measures rely, implicitly or explicitly, on the assumption that the population of prisoners is "stationary".

A stationary population is created when the annual number of entrances to a state has been constant for a long period and the set of duration-specific attrition rates from that state have also been constant over time. When these conditions prevail in a prison population, the number of prisoners at a particular duration will be constant and equal to the annual number of annual entrants times the probability of surviving to that duration. Accordingly, the total size of the prison population will be constant; the growth rate will be zero, and the number of annual entrances will equal the number of annual exits.

[1] See Appendix A for details on the data used to construct the life table.

[2] We assume that after the duration of 20 years, the duration-specific rates are constant.

[3] We assume that nobody survives past 50 years.

[4] A similar method combines information from both the entering cohort's mean length of sentence imposed and the exit cohort's mean percentage of sentence served (Beck, 1995; Greenfeld 1995; Sabol and McGready 1999).

Table 1 Life table for all prison stays, selected states, 1997

Duration of stay at beginning of interval (years) x	Exit rates at duration of stay ${}_nM_x$	Probability of leaving during interval ${}_nq_x$	Number residing in prison at beginning of interval l_x	Person-years spent in prison above duration T_x	Expected number of additional years to be spent in prison e_x
0.0	1.0220	0.4245	100000	226151	2.26
0.5	0.9293	0.3815	57547	184611	3.20
1.0	0.5543	0.2452	35596	160990	4.52
1.5	0.4693	0.2128	26869	145248	5.40
2.0	0.3415	0.1585	21152	133064	6.29
2.5	0.3214	0.1502	17799	123248	6.92
3.0	0.2432	0.1153	15127	114933	7.59
3.5	0.2538	0.1203	13383	107763	8.05
4.0	0.2011	0.0963	11773	101420	8.61
4.5	0.2260	0.1076	10640	95785	9.00
5.0	0.1348	0.5060	9495	90719	9.55
10.0	0.0795	0.3457	4653	57065	12.26
15.0	0.0520	0.2457	3044	38338	12.59
20.0	0.0782	0.3451	2296	25090	10.92
25.0	0.0782	0.3451	1504	15689	10.43
30.0	0.0782	0.3451	735	9533	9.68
35.0	0.0782	0.3451	645	5502	8.53
40.0	0.0782	0.3451	422	2862	6.77
45.0	0.1594	1.0000	277	1132	4.09

In a stationary population, the expected length of time to be spent in prison for a new arrival will equal the size of the total prison population divided by the number of annual arrivals. Thus, the expected length of time to be served is the reciprocal of the entrance rate. Since the entrance rate is equal to the exit rate, it is also the reciprocal of the exit rate. The expected length of stay for a new arrival will also equal the mean duration of time spent in prison by those exiting in any particular year, since the duration distribution of exits is constant over time. Thus, the exit distribution in any particular period will replicate the distribution of exits for any entering cohort.

So, a stationary population provides three handy indirect measures of the expected length of stay in a state:

- the ratio of the size of the population in the state at a moment in time to the annual number of entrances to that state (or, what is equivalent, the reciprocal of the entry rate);
- the reciprocal of the exit rate;
- the mean duration in the state of people exiting during any particular year or period.

While these measures are convenient, they can be misleading if the population under study is not stationary. For example, if the annual number of entrances has been growing and attrition constant, the prison population will be "younger" (in duration of time served) than the stationary population consistent with its attrition rates. The ratio of the number incarcerated for 2 years to the number incarcerated for 10 years will be greater than in a

Table 2 Life table for murder stays, selected states, 1997

Duration of stay at beginning of interval (years) x	Exit rates at duration of stay $_nM_x$	Probability of leaving during interval $_nq_x$	Number residing in prison at beginning of interval l_x	Person-years spent in prison above duration T_x	Expected number of additional years to be spent in prison e_x
0	0.1781	0.1622	100000	2001000	20.01
1	0.0337	0.0330	83777	1909915	22.80
2	0.0371	0.0364	81009	1827658	22.56
3	0.0360	0.0353	78059	1748109	22.39
4	0.0436	0.0427	75301	1671463	22.20
5	0.0469	0.0459	72084	1597743	22.16
6	0.0479	0.0467	68776	1527229	22.21
7	0.0317	0.0311	65566	1460159	22.27
8	0.0321	0.0316	63525	1395691	21.97
9	0.0507	0.0494	61520	1333213	21.67
10	0.0331	0.0326	58479	1273190	21.77
11	0.0307	0.0302	56574	1215656	21.49
12	0.0302	0.0297	54868	1160087	21.14
13	0.0316	0.0311	53239	1106135	20.78
14	0.0255	0.0252	51585	1053734	20.43
15	0.0232	0.1785	50287	1002833	19.94
20	0.0336	0.1785	43852	768317	17.52
25	0.0336	0.1785	36025	570319	15.83
30	0.0336	0.1785	29595	407662	13.77
35	0.0336	0.1785	24313	274038	11.27
40	0.0336	0.1785	19973	164264	8.22
45	0.2215	1.0000	16408	74084	4.52

stationary population because it comprehends not only attrition between durations two and ten but also growth in the number of entrants during the period 2–10 years ago. Since the population is "younger" than the stationary population, the mean duration of time served by people exiting in any year will be younger than in the stationary population and will provide a downwardly biased estimate of the mean length of stay for a newly incarcerated person. Likewise, the entrance rate will be higher than in a stationary population having the same attrition rates. Accordingly, the reciprocal of the entrance rate will be too low as an estimator of expected length of time served.

In order to gain a general sense of the likely amount of error in such estimates, we take advantage of another demographic model, that of a stable population. The stable model, like the stationary model, also assumes that duration-specific attrition rates are constant. But, rather than assuming a constant annual number of entrances, it assumes that the annual number of entrances grows or declines exponentially. The stationary model is a special case of the stable model, one for which the growth rate in the annual number of entrances is zero.

If attrition rates are constant and the growth rate of entrances has been constant for an extended period, then the population will become "stable": its entrance rate, exit rate, growth rate, and duration-composition will all be constant over time (Coale 1972; Preston et al. 2001). The basic equations representing the relations among variables are:

$$b = \frac{1}{\int e^{-ra} p(a) \mathrm{d}a} \tag{1}$$

$$c(a) = b e^{-ra} p(a) \tag{2}$$

where $c(a)$ = proportion of the population at duration a, $p(a)$ = probability of surviving from entrance to duration a, b = "entrance rate" of the population; annual number of entrances divided by person-years lived in that year and r = growth rate of the population.

In order to evaluate how the estimates that assume stationary conditions fare when the population is growing or declining, we compute the value of the three indirect measures cited above in stable populations. We continue to assume that attrition rates are constant but we allow systematic growth or decline in the annual number of entrances and thus in the total size of the prison population. Our growth rates vary from –0.05 to 0.2, which mirror the growth rates that occurred during the last two decades of the twentieth century. Using a spreadsheet published by the Bureau of Justice Statistics (Rice and Harrison 2000), we calculated that the annual growth rate of the prison population between 1980 and 1989 varied from –0.02 to +0.19; between 1990 and 1998 the range was –0.03 to +0.08.

Table 3 and Fig. 1 show the performance of the indirect measures for "all causes of incarceration" under various growth scenarios. Of course, when the growth rate is zero and the population is stationary, all of the indirect measures are completely accurate. As conditions diverge from stationarity, the measures become more and more biased. At an annual growth rate of .05, the entrance rate estimator is 25% too low; at a growth rate of –0.05, it is 68% too high.

One important result is that the estimated mean length of stay using the reciprocal of the exit rate is always closer to the correct mean length of stay than the other two estimates. This result seems ironic in view of the fact that it is the only one of the indirect measures that has not been employed in the literature. The mean duration of time served by those exiting in any particular year, the most commonly employed measure, is by far the worst estimator in all circumstances. Thus, over a broad range of conditions, the reciprocal of the exit rate is the best estimator of the mean duration of stay in prison for a new entrant.[5] Below, we show that this result has broad applicability.

To test the performance of these measures under a quite different pattern of exit rates, we examine those imprisoned for murder, an offense with much longer sentences. Table 4 presents the indirect estimates of length of stay for those convicted of murder. In this case, the entrance rate estimator performs even more poorly than in Table 3. At a growth rate of .05, the entrance rate estimator is less than half of its true value. At a growth rate of –.05, it is more than three times the correct value. The entrance rate performance is worse for murder than for "all causes" because there is a longer set of durations over which the growth-bias can manifest itself. The same increase in error is apparent for the mean duration of time served at exit. On the other hand, in the wide range of growth rates from –.025 to +.05, the exit rate estimator is within 10% of the correct value. So when nothing else is known, the size of the prison population divided by the annual number of exits provides the best estimate of the mean length of stay in prison among the three methods (Fig. 2).

[5] It is important to recognize that the denominator of the entrance (or exit) rate is not the total population but the number of person-years lived in the population during the period under study. Person-years provide an estimate of the amount of exposure to the event given a particular year. Person-years lived in a population can be approximated in several ways, the most common of which is by the product of the mid-period population and the length of the time interval in years.

Table 3 Various estimators of the length of stay in prison in stable populations (in years), all offenses, e_0 = 2.262

	Growth rate of population						
	–0.05	–0.025	0	0.025	0.05	0.1	0.2
Assuming stationary population							
$\frac{1}{b}$	3.791	2.805	2.262	1.929	1.708	1.427	1.126
$\frac{1}{d}$	3.188	2.621	2.262	2.028	1.868	1.666	1.458
Mean duration of exits (A_D)	5.439	3.277	2.262	1.753	1.471	1.180	0.930
Using stable population approximations							
$\frac{1}{b[1-rA_P]}$	2.214	2.239	2.262	2.238	2.181	2.042	1.822
$\frac{1}{d[e^{-r(A_D-A_P)}]}$	2.053	2.210	2.262	2.227	2.156	2.000	1.774
Other parameters							
Mean duration of prison population (A_P)	14.246	10.107	7.315	5.510	4.339	3.011	1.910

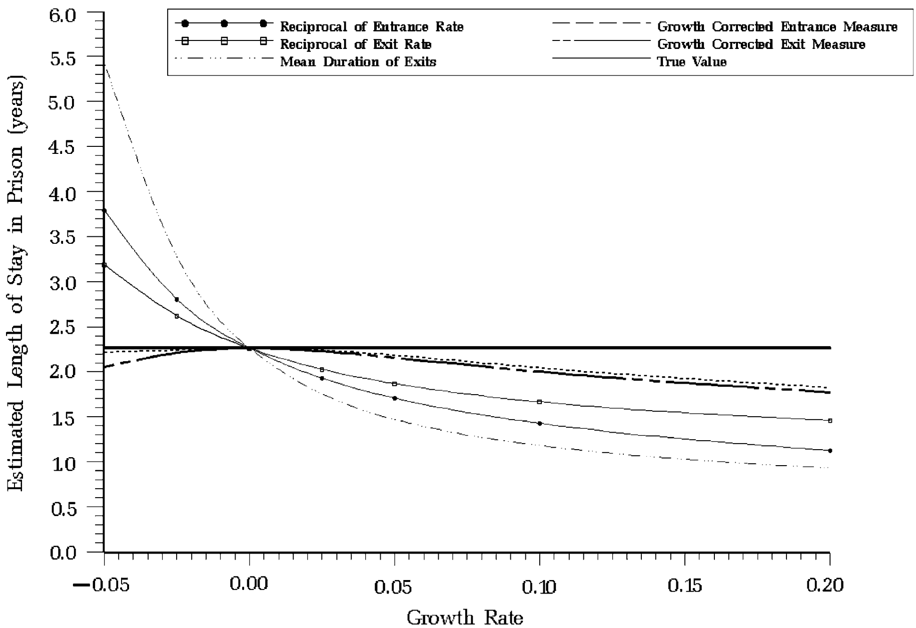

Fig. 1 Various estimators of length of stay inside prison, all offenses

Improved Indirect Measures

Even without complete information on attrition rates, analysts can often do a better job of estimating the mean length of stay in prison than by using the indirect measures that we have described. Improvements are possible because, as shown in Tables 3 and 4, the biases in these measures are related to the growth rate of the prison population, a readily observable datum. In order to provide better indirect measures of mean length of stay, we offer two equations that are derived in Appendix B. One is based upon the observed entry rate (3) and the other upon the observed exit rate (4):

Table 4 Various estimators of the length of stay in prison in stable populations (in years), murder, $e_0 = 20.01$

	Growth rate of population						
	–0.05	–0.025	0	0.025	0.05	0.1	0.2
Assuming stationary population							
$\frac{1}{b}$	61.618	32.994	20.010	13.529	9.961	6.401	3.721
$\frac{1}{d}$	15.052	18.066	20.010	20.433	19.814	17.729	14.486
Mean duration of exits (A_D)	35.997	28.286	20.010	13.350	9.028	4.908	2.449
Using stable population approximations							
$\frac{1}{b[1-rA_P]}$	25.925	21.138	20.010	20.646	21.794	23.094	22.022
$\frac{1}{d[e^{-r(A_D-A_P)}]}$	22.980	20.912	20.010	20.653	21.712	22.361	20.376
Other parameters							
Mean duration of prison population (A_P)	27.535	22.435	17.684	13.788	10.859	7.228	4.155

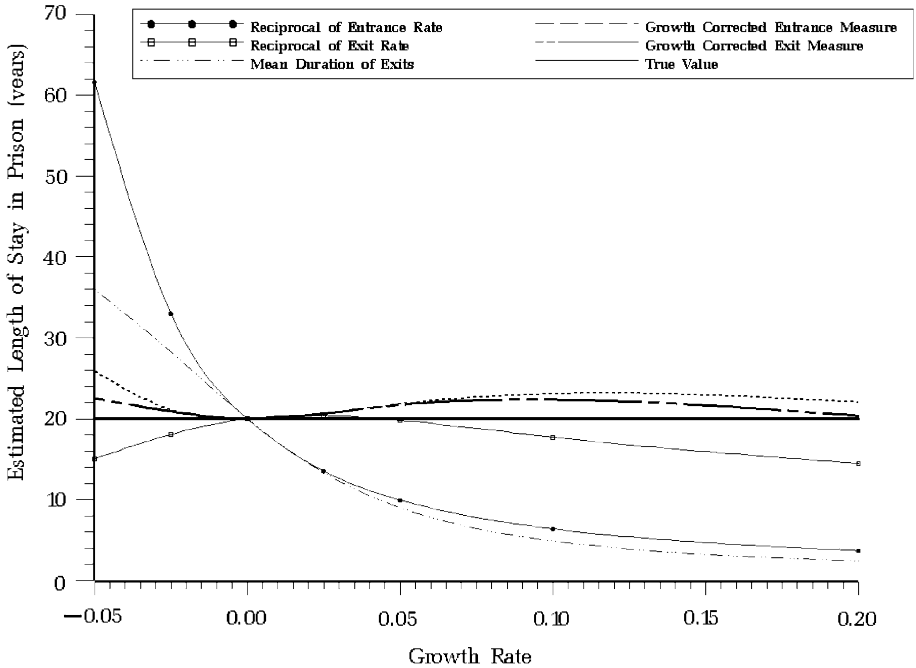

Fig. 2 Various estimators of length of stay inside prison, murder

$$e_0^0 \approx \frac{1}{b[1 - rA_P]} \tag{3}$$

$$e_0^0 \approx \frac{1}{d[e^{-r(A_D - A_P)}]}, \tag{4}$$

where d is the exit rate of the stable population and b is the entrance rate of the stable population. There are two expressions rather than three because the indirect procedure involving the mean duration of time served at exit does not lend itself to a simple expression but rather involves variances whose values are infrequently observable.

The formulas involving corrections of the entrance rate and exit rate are straightforward. To estimate the entrance rate in a stationary population—the reciprocal of life expectancy at entrance—it is necessary to perform a growth correction on the observed entrance rate. This correction involves the rate of growth and the mean age of the population. The expression shows that the reciprocal of the entrance rate will underestimate life expectancy in a growing population, and overestimate it in a declining one. This pattern is clearly revealed in Tables 3 and 4. Likewise, to estimate the exit rate in a stationary population (also the reciprocal of life expectancy at entrance) it is necessary to perform a growth-correction on the observed exit rate. This correction involves the rate of growth and the *difference* between the mean age at exit and the mean age of the population.

These expressions are relevant to the common practice of assuming stationary conditions and using the reciprocal of either the entrance rate or exit rate to estimate years spent in prison. They indicate why the error made by using the entrance rate will nearly always be larger than the error made by using the exit rate, as shown in Tables 3 and 4. The reason is that the difference between the mean age and exit and the mean age of the population, $A_D - A_P$, will almost never exceed the value of the mean age of the population itself. If exit rates are constant with age, then A_D will equal A_P and there will be no bias whatever in the exit rate estimator. At the other extreme, everyone exits at the same duration. In this case, A_P is approximately equal to $A_D/2$ and the biases in the exit rate estimator and entrance rate estimators will be equal to one another. In between these extremes, where most populations reside, A_P will be greater than $A_D/2$ and less than A_D, so that the bias in the exit rate estimator will be smaller than that in the entrance rate estimator. This result is also relevant to the sensitivity of estimates to errors in the estimated value of r. Because the multiplier of r is almost always larger in the estimator using b than in the estimator using d, the estimator using d will be less sensitive to an error in r than will the estimator using b.

Tables 3 and 4 show that these new expressions perform much better than the uncorrected indirect estimators. In contrast to the uncorrected estimates, there is now little to choose between entrance-based and exit-based estimates. Both give estimates of mean length of stay that are well within 10% of its true value in a range of r from −.05 to + .05 for "all offenses". For murder, the range of below-10% error extends from $r =$ −.025 to $r = .05$; both are also within 10% at a value of r as high as 20% per year.

Simulations

The preceding analysis assumed that the prison population was stable and compared two sets of estimators under these somewhat artificial conditions. We now relax the assumption of stability and permit random variability in

- the time sequence of entrances;
- the set of attrition rates;
- both the time sequence of entrances and the set of attrition rates.

To introduce variability into the number of entrances, we assume that "entrances" are expected to grow at an exponential rate, but that the rate is subject to random variation:

$$r(t) = r[1 + v(t)], \tag{5}$$

where $v(t)$ is a normally distributed random variable with mean of zero and standard deviation of .05. We conducted 100 simulations at each of the expected growth rates, using as expectations the same set of growth rates employed in the stable population analysis.

To introduce a stochastic element into the attrition rates, we assume that

$$m(x,t) = m(x)[1 + w(t)], \tag{6}$$

where $w(t)$ is a normally distributed random variable with a mean of zero and a standard deviation of 0.10.

Table 5 shows how the "all offenses" estimators perform under these conditions.

Table 6 presents the same information for murder. With one exception out of 24 comparisons, the uncorrected exit rate always outperforms the uncorrected entrance rate as a predictor of mean duration of stay. The mean duration of time served at exit is almost always the worst performer. With two exceptions out of 32 realizations, corrected entrance rates and exit rates give mean values of estimated mean stay that are within 10% of the actual values in a range of growth rates from −.025 to .05. This accuracy applies regardless of whether the source of variation is entrances or exit rates.

Although the mean of corrected entrance rate and exit rate estimates are satisfactory and similar to one another within this range of growth rates, the standard deviation of corrected exit rate estimates is smaller than that of corrected entrance rate estimates 75% of the time (12 of 16 comparisons). Thus, the estimate based on the corrected exit rate is less likely to be seriously wrong. This result holds despite the fact that we assumed twice as high a standard deviation in the annual exit series as in the annual entrance series. The estimates based on the corrected entrance rate are particularly erratic (i.e., have a high standard deviation) when the growth rate in entrances is at least 10%/year and variability is introduced into the entrance sequence. When the growth rate is highly negative ($r = -.05$), all "corrected" estimators work well for the short-lived process and poorly for murder.

Finally, we combine variability in the entrance sequence with variability in exit rates, using the same parameters previously described. The results in Table 7 indicate that uncorrected estimates based on exits always have a lower standard deviation than those based on entrances. And with one exception, uncorrected estimates based on exits have a mean value closer to the true mean than uncorrected estimates based on entrances (ignoring cases where $r = 0$, where the estimates are essentially indistinguishable). The mean duration at exit is consistently the worst performer. The corrected exit rate estimator has a lower standard deviation than the corrected entrance rate estimator in all but two circumstances. When mean values for the two estimators differ by more than .03, the mean value of the exit rate estimator is closer to the true value in 12 cases out of 13.

Thus, the simulations show the same pattern of results as the stable population analysis. When no growth-correction is possible, the best estimator is the reciprocal of the exit rate and the worst estimator is the mean length of time served by those exiting in a particular year. When a correction for growth can be made, exit rate and entrance rate estimators are more similar in performance but the exit rate estimator has a lower standard deviation, is less sensitive to errors in data, and is usually closer to the correct value.

Conclusion

This paper compares various estimates of the mean length of stay in prison for someone newly admitted. One estimate is provided by the life table, which yields precise

Table 5 Various estimators of the mean length of stay (in years) in prison in destabilized populations, all offenses (100 simulations), e_0 = 2.262

	Expected growth rates						
	–0.05	–0.025	0	0.025	0.05	0.1	0.2
Stochastic births							
$\frac{1}{b}$	3.929 (0.80)	2.830 (0.28)	2.268 (0.11)	1.948 (0.19)	1.772 (0.34)	1.650 (0.64	1.997 (1.78)
$\frac{1}{d}$	3.204 (0.19)	2.628 (0.08)	2.263 (0.03)	2.027 (0.06)	1.871 (0.10)	1.688 (0.16)	1.548 (0.26)
Mean duration of exits (A_D)	5.450 (0.45)	3.285 (0.14)	2.264 (0.05)	1.751 (0.07)	1.474 (0.12)	1.205 (0.19)	1.03 (0.30)
$\frac{1}{b[1-rA_P]}$	2.413 (10.22)	2.416 (0.64)	2.309 (0.28)	2.477 (0.92)	1.288 (14.22)	1.267 (3.89)	2.418 (17 .74)
$\frac{1}{d[e^{-r(A_D-A_P)}]}$	2.272 (0.98)	2.274 (0.49)	2.270 (0.23)	2.250 (0.38)	2.225 (0.65)	2.134 (0.91)	1.921 (0.94)
Stochastic survival rates							
$\frac{1}{b}$	3.704 (0.05)	2.753 (0.05)	2.228 (0.05)	1.907 0.05)	1.691 (0.05)	1.418 (0.04)	1.126 (0.04)
$\frac{1}{d}$	3.153 (0.08)	2.598 (0.07)	2.246 (0.07)	2.017 (0.07)	1.860 (0.07)	1.662 (0.06)	1.457 (0.07)
Mean duration of exits (A_D)	5.346 (0.149)	3.232 (0.06)	2.238 (0.04)	1.739 (0.03	1.462 (0.02)	1.174 0.02)	0.926 (0.02)
$\frac{1}{b[1-rA_P]}$	2.285 (0.51)	2.398 (0.74)	2.473 (0.87)	2.451 (0.86)	2.369 (0.79)	2.178 (0.63)	1.911 (0.46)
$\frac{1}{d[e^{-r(A_D-A_P)}]}$	2.104 (0.50)	2.270 (0.55)	2.309 (0.52)	2.260 (0.45)	2.177 (0.38)	2.009 (0.28)	1.775 (0.19)

Table 6 Various estimators of the mean length of stay (in years) in prison in destabilized populations, murder (100 simulations), $e_0 = 20.01$

	Expected growth rates						
	–0.05	–0.025	0	0.025	0.05	0.1	0.2
Stochastic Births							
$\frac{1}{b}$	65.63 (19.04)	33.57 (4.74)	20.06 (1.07)	13.72 (1.91)	10.60 (2.98)	8.29 (4.92)	10.52 (14.699)
$\frac{1}{d}$	15.27 (0.88)	18.13 (0.375)	20.02 (0.18)	20.43 (0.51)	19.86 (1.25)	18.16 (2.75)	16.57 (4.94)
Mean duration of exits (A_D)	35.68 (4.00)	28.168 (0.576)	20.01 (0.28)	13.38 (0.50)	9.05 (0.82)	5.04 (1.12)	2.98 (1.33)
$\frac{1}{b[1-rA_P]}$	27.55 (6.85)	21.46 (1.82)	20.06 (0.21)	21.459 (2.64)	19.879 (29.44)	19.62 (41.55)	18.33 (68.14)
$\frac{1}{d[e^{-r(A_D-A_P)}]}$	23.12 (1.68)	20.97 (0.83)	20.02 (0.27)	20.70 (0.32)	22.02 (0.71)	23.66 (3.50)	23.03 (5.91)
Stochastic survival rates							
$\frac{1}{b}$	61.60 (0.16)	32.93 (0.08)	19.95 (0.06)	13.48 (0.04)	9.921 (0.03)	6.374 (0.03)	3.703 (0.02)
$\frac{1}{d}$	15.25 (0.62)	18.11 (0.51)	19.94 (0.45)	20.32 (0.52)	19.69 (0.61)	17.62 (0.72)	14.41 (0.77)
Mean duration of exits (A_D)	35.71 (0.597)	28.08 (0.67)	17.66 (0.03)	13.31 (0.35)	9.000 (0.25)	4.887 (0.17)	2.444 (0.02)
$\frac{1}{b[1-rA_P]}$	25.93 (0.33)	21.10 (0.27)	19.94 (0.312)	20.53 (0.438)	21.62 (0.61)	22.84 (0.94)	21.95 (1.22)
$\frac{1}{d[e^{-r(A_D-A_P)}]}$	22.97 (0.38)	20.87 (0.39)	19.94 (0.43)	20.54 (0.45)	21.57 (0.50)	22.22 (0.64)	20.27 (0.77)

Table 7 Various estimators of the mean length of stay (in years) in prison in destabilized populations, stochastic births and stochastic survival rates (100 simulations)

	Expected growth rates						
	–0.05	–0.025	0	0.025	0.05	0.1	0.2
All offenses (e_0=*2.262*)							
$\frac{1}{b}$	3.840 (0.78)	2.776 (0.28)	2.236 (0.13)	1.927 (0.19)	1.759 (0.34)	1.648 (0.65)	2.011 (1.80)
$\frac{1}{d}$	3.167 (0.21)	2.603 (0.12)	2.247 (0.08)	2.018 (0.09)	1.868 (0.12)	1.692 (0.18)	1.559 (0.28)
Mean duration of exits (A_D)	5.374 (0.49)	3.243 (0.16)	2.240 (0.06)	1.739 (0.08)	1.468 (0.12)	1.204 (0.19)	1.030 (0.03)
$\frac{1}{b[1-rA_P]}$	2.131 (10.75)	1.833 (7.72)	2.619 (1.46)	2.291 (3.13)	2.324 (4.57)	2.814 (15.57)	5.015 (25.63)
$\frac{1}{d[e^{-r(A_D-A_P)}]}$	2.316 (1.17)	2.326 (0.76)	2.320 (0.58)	2.287 (0.61)	2.251 (0.78)	2.142 (0.95)	1.923 (0.92)
Murder (e_0=*20.010*)							
$\frac{1}{b}$	65.372 (18.95)	33.457 (4.72)	19.992 (1.08)	13.685 (1.90)	10.568 (2.97)	8.264 (4.90)	10.467 (14.58)
$\frac{1}{d}$	15.31 (1.08)	18.128 (0.65)	19.942 (0.47)	20.317 (0.74)	19.728 (1.42)	18.015 (2.87)	16.426 (4.98)
Mean duration of exits (A_D)	35.552 (1.09)	28.034 (0.78)	19.900 (0.60)	13.304 (0.64)	9.003 (0.87)	5.016 (1.12)	2.962 (1.32)
$\frac{1}{b[1-rA_P]}$	27.462 (6.86)	21.393 (1.83)	19.985 (0.40)	21.363 (2.67)	17.354 (45.78)	32.300 (141.82)	14.408 (36.00)
$\frac{1}{d[e^{-r(A_D-A_P)}]}$	23.048 (1.74)	20.898 (0.93)	19.950 (0.49)	20.617 (0.54)	21.907 (0.85)	23.492 (3.49)	22.797 (5.61)

information about survival when duration-specific attrition rates are available. We also considered three indirect indicators—the mean number of years served at exit for people exiting in a particular year, the ratio of the total population to the number of entrances, and the ratio of the total population to the number of exits. These measures are accurate when the population is stationary, i.e., constant in size with constant attrition and constant annual numbers of entrances and exits. Unfortunately, the prison population is not stationary, and we demonstrate the biases that result in these measures from implementing the stationary assumption in a non-stationary population. We also develop two new formulas that provide a growth-adjustment to the reciprocal of the exit or entrance rate so that they give values that are much closer to the true value contained in the life table.

Our findings indicate a hierarchy in the success of the estimates. Estimates based on the reciprocal of the attrition rate perform best both when a growth-correction can be made and when it cannot. When no growth-correction can be made, it is the only one of the three indirect measures that provides satisfactory results over a substantial range of growth rates. When a growth correction can be made, estimates using the exit rate are generally closer to the mark than those based on the entrance rate, and they show substantially less variability in most circumstances. We recommend the use of this measure, which to date has not been exploited in the criminological literature.

Uncorrected entrance rate-based estimates are subject to considerable error even when prison populations exhibit only mild deviations from stationary conditions. However, the corrected measure of the entrance rate works well in stable populations and in population with simulated variability in entrances and in exit rates. Nevertheless, in comparison to corrected exit-based estimates, those based on the corrected entrance rate are more sensitive to errors in the growth rate and have higher standard deviations.

The measure that performed the worst in all applications was the mean duration of time served among those exiting in a particular year, which is also the most commonly used measure in the literature.

Given our findings, every effort should be made to improve estimates of length of stay. Existing measures are often quite misleading and can produce bad policies or flawed policy evaluations. For example, in 1999 a New York Times article summarizing a report from the Bureau of Justice Statistics stated that: "the average time served in state prison by violent criminals rose to 49 months in 1997 from 43 months in 1993" (Butterfield 1999). The article went on to conclude that legislation imposing longer sentences was working. However, the observation could also be a reflection of the observed decline in the growth rate of the prison population between these years, a factor unrelated to sentencing practices. Without a proper analysis, no policy conclusion is warranted. Because these policies affect both liberty and safety, we cannot afford to be cavalier in our calculations.

Appendix A: Sources and Processing of Data

The data used come from the Bureau of Justice Statistics. Using the 1995 Adult Correctional Census, in conjunction with the National Correctional Reporting Program's admissions and exits, we projected the prison population of the 29 states—Alabama, California, Colorado, Hawaii, Illinois, Kentucky, Maryland, Michigan, Minnesota, Mississippi, Missouri, Nebraska, New Hampshire, New Jersey, New York, North Carolina, North Dakota, Ohio, Oklahoma, Oregon, Pennsylvania, South Carolina, Tennessee, Texas, Utah, Virginia, Washington, West Virginia, and Wisconsin—forward to mid-1997. Because no data exist concerning the duration-specific structure of the current population for each state, we adopted the duration-distribution of persons housed inside prison from the 1997 National Survey of Inmates in State and Federal Correctional Facilities. This distribution was national and not restricted to the 29 states. We excluded those housed in federal facilities and women.

After obtaining the duration-specific distribution of the population, we used the 1997 National Corrections Reporting Program Release to produce duration-specific exit rates for men. We exclude persons whose exit was a result of death or escape. Additionally, our estimates do not include time served prior to the current admission. That is, if the person spent time in jail for the current offense and it was applied to the sentence length, we do not consider that. We do this to avoid confusion in the denominator and numerator. Since the survey provides the admission date to prison of the sentenced person, the numerator should reflect the same.

We calculated exit rates for all offenses as well as one offense-specific category—murder. The calculation of the duration-specific rate, ${}_nM_x$, was the quotient of the number of exits in the duration group x to $x + n$ and the mid-year population. The mid-year population was used to approximate the number of persons exposed to the risk of an exit. Thus the following equation represents the duration specific rate:

$$ {}_nM_x = \frac{{}_nD_x}{{}_nN_x}, \tag{A.1} $$

where ${}_nD_x$ = the number of exits occurring in the duration group of x to $x + n$ and ${}_nN_x$ = the mid-year population for the duration category of x to $x + n$, our estimation of person-years lived in this portion of the analysis.

The same process was used for murder. The 1997 National Survey of Inmates in State and Federal Correctional Facilities was used to find the duration-specific structure of those in prison for murder, and then the release file was used to select those who were released for murder (includes murder and non-negligent manslaughter).

Appendix B: Derivations

In this section of the appendix we derive the new indirect estimators of the mean length of stay in prison based on stable population assumptions. Because it allows for population growth or decline, this class of populations is far more general than the class of stationary populations typically assumed in indirect indicators. Thus, the indirect estimators should be applicable to a much wider range of populations.

Because the entrance rate, exit rate, growth rate, and duration-composition are constant over time in a stable population, we can make use of several basic equations to express the relationships between various population components (Coale 1972; Preston et al. 2001). The expression for the entrance rate is

$$ b = \frac{1}{\int_0^\infty \mathrm{e}^{-ra} p(a) \mathrm{d}a} \tag{B.1} $$

where r = growth rate of the stable population and $p(a)$ = probability of surviving from entry to duration a.

We expand e^{-ra} in a Taylor expansion through the first term and substitute into Eq. (B.1), giving

$$b \approx \frac{1}{\int_0^\infty [1 - ra]p(a)\mathrm{d}a} \tag{B.2}$$

Recognizing that the integral of $p(a)$ is mean length of stay upon entrance, e_0^0, we rearrange this expression to give

$$\mathrm{e}_0^0 \approx \frac{1}{b[1 - rA_P]}, \tag{B.3}$$

where A_P is the mean duration of the stable population. Equation B.3 is the "corrected" entrance rate estimator of the mean length of incarceration. The correction involves the growth rate of the prison population and the mean length of imprisonment to date for those currently in prison.

The expression for the exit rate of a stable population is

$$d = \frac{\int_0^\infty \mathrm{e}^{-ra}p(a)\mu(a)\mathrm{d}a}{\int_0^\infty \mathrm{e}^{-ra}p(a)\mathrm{d}a}, \tag{B.4}$$

where $\mu(a)$ = exit rate at duration a.

Differentiating this expression with respect to r and simplifying gives

$$\frac{d}{\mathrm{d}r}d \approx d[A_D - A_P], \tag{B.5}$$

where A_D is the mean duration at exit for those exiting prison in a particular year or period. Alternatively,

$$\frac{d}{\mathrm{d}r}(\ln\ d) \approx A_D - A_P. \tag{B.6}$$

Thus, the proportionate error in d as an estimator of d_0, the exit rate of the stationary population and the reciprocal of mean length of stay upon entrance, is $r[A_D - A_P]$. So the corrected estimator of mean length of stay upon entrance is

$$\mathrm{e}_0^0 \approx \frac{1}{d\left[\mathrm{e}^{-r(A_D - A_P)}\right]}. \tag{B.7}$$

Equation B.7 is the corrected exit rate estimator of the mean length of incarceration. The correction involves the growth rate of the population as well as the difference between the mean duration at exit for those exiting in any particular period and the mean length of imprisonment for those currently in prison.

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