

# Adaptive Auction Framework for Spectrum Market in Cognitive Radio Networks

Muhammad Shafiq<sup>1</sup>  · Jin-Ghoo Choi<sup>1</sup>

Received: 7 September 2016 / Revised: 4 September 2017 / Accepted: 11 September 2017 /  
Published online: 14 September 2017  
© Springer Science+Business Media, LLC 2017

**Abstract** For the recent decade, cognitive radio networks have received much attention as an alternative to the traditional static spectrum allocation policy since the licensed spectrum channels are not being used efficiently. The most critical issue of the cognitive radio networks is how to distribute the idle spectrum channels to the secondary users opportunistically. The auction-based market is desirable for the trade of idle spectrum channels since the secondary users can purchase a channel in timely manner and the licensed primary users can earn the additional profit while not using the channels. Among the auction algorithms proposed for the spectrum market, we focus on the TASG framework, which consists of two nested auction algorithms, because it enables the group-buying of spectrum channels for the secondary users with limited budgets, and possesses many positive properties such as budget-balance, individual rationality and truthfulness. However, the TASG framework is not very attractive to the market participants since the seller earns the small revenue and the buyer has the low utility. In this paper, we propose a new auction framework for the spectrum markets, called aDaptive and Economically robust Auction-based Leasing (DEAL), that keeps all the benefits of TASG while improving the utility (or revenue) of the participants. To this end, we develop an enhanced inner-auction algorithm, called the Global Auction algorithm in our DEAL framework, and adapt the involved parameters dynamically based on the previous bids from the potential buyers. Simulation results demonstrate that our framework significantly outperforms the previous TASG.

---

✉ Jin-Ghoo Choi  
jchoi@yu.ac.kr

Muhammad Shafiq  
shafiq.pu@gmail.com

<sup>1</sup> Yeungnam University, 280 Daehak-Ro, Gyeongsan, Gyeongbuk 38541, Republic of Korea

**Keywords** Cognitive radio technology · Dynamic spectrum access · Spectrum auction · Group-buying · Reserve price

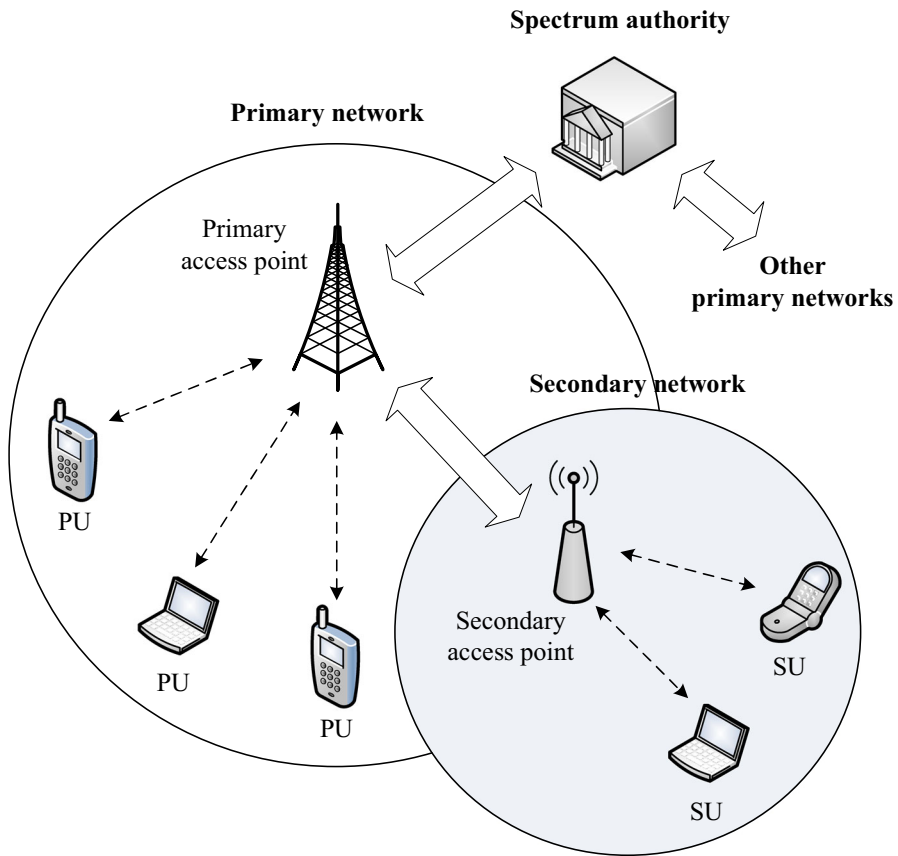
## 1 Introduction

Radio spectrum is a fundamental resource for wireless communications, which becomes more precious than any other times since the traffic from various mobile devices is exploding tremendously. Traditionally, the government of each country has leased the radio spectrum to telecommunication companies or broadcast companies under the long term contract. Unfortunately, the static spectrum allocation is not efficient to accommodate the highly dynamic demands on the radio spectrum. Indeed, most of the allocated spectrum channels are being underutilized or not utilized at all in the USA, according to the spectrum utilization survey conducted by Federal Communication Commission (FCC) in 2002 [1]. The situation is anticipated to be worse due to the growing mobility of information devices.

During the past decade, the Cognitive Radio (CR) technology has attracted much attention as a promising alternative to the static spectrum allocation strategy [2]. Under the CR technology, a network consists of three entities, i.e., spectrum authority, primary network and secondary network, as shown in Fig. 1. The spectrum authority refers to the national regulatory agency like FCC, which possesses the ultimate ownership of radio spectrum, develops the spectrum policy and controls the spectrum licenses in each country. Primary networks such as telecommunication companies and/or broadcast companies lease the radio spectrum from the spectrum authority and serve the licensed Primary Users (PUs). The Secondary Users (SUs), equipped with spectrum sensors, are served by secondary networks in the opportunistic manner when the adjacent PUs are idle.

In CR networks, the radio spectrum or spectrum channels should be shared by PUs and SUs. For the channel sharing, two kinds of channel access schemes have been proposed, i.e., cooperative schemes [3–5] and non-cooperative schemes [6–8]. In non-cooperative schemes, PUs do not recognize SUs at all. It is the SUs' responsibility not to cause the substantial interference to PUs. To this end, SUs perceive the activities in channels continuously through the various spectrum sensing techniques such as energy detection, cyclo-stationary feature detection, and so on [9–11]. SUs access the channel if all the adjacent PUs are inactive, but should evacuate the channel immediately once a PU returns. Unfortunately, every spectrum sensing technology has the fundamental performance issues [12]. The spectrum sensor of an SU may fail to identify the channel access opportunity due to the ambient noise (false positive), which causes the loss of spectrum utilization. The sensor may declare a busy channel as idle (false negative) and makes the SU disturb the ongoing communications of PUs. Meanwhile, the cooperative schemes can avoid all these problems since the primary networks explicitly announce the channel access opportunity to the secondary networks. Hence the SUs do not rely on the error-prone spectrum sensing techniques to share the radio spectrum with PUs.

The cooperative channel access schemes can maximize the spectrum utilization of CR networks. However, they should address the key issue of how to distribute



**Fig. 1** Architecture of CR networks

idle channels to multiple secondary networks first. One of the promising solutions is the open market, where PUs sell the chances to access the channel for additional profits and SUs purchase the access opportunity for communications. We call this market as the secondary market, recalling that spectrum channels are traded between the spectrum authority and primary networks in so-called primary markets. For the efficient design of the secondary market, we need to have the salient features of the spectrum channels in mind. First, the spectrum channel is imperishable. Channels sold to a user can be reused by other buyers after some time. Moreover, a channel can be used simultaneously by multiple buyers only if the buyers are separated in distance. Second, each user can evaluate the same channel very differently since the channel quality substantially depends on the Signal-to-Interference-and-Noise-Ratio (SINR), measured at the receiver. Third, the value of a channel fluctuates with time quickly due to the dynamic demands on the channel. For example, a statistics shows that the traffic in peak hours can be more than ten times of that in valley hours [13, 14].

There are various pricing mechanisms for markets. Among them, the auction, where PUs act as the seller (or auctioneer) and SUs become buyers (or bidders), is the most appropriate for secondary markets due to the following reasons. First, in auctions, the price of an item is determined mainly by the buyers. In secondary markets, it is the buyers (SUs) that can evaluate the spectrum channels fairly by measuring SINR. Second, the items are traded quickly in auctions with the minimal interaction between the auctioneer and bidders. Hence a spectrum channel can be sold before its value changes. Third, auctions are suitable for scarce items like the spectrum channels since the more urgent bidders are liable to purchase them with high probability. Fourth, we can design the auctions to control the excessive competition between bidders and improve the social welfare [15], which is desirable since the spectrum channels are public goods.

In addition to these general merits, an auction can have distinctive properties according to the algorithm design. In the literature, we can find many desirable economic properties of the auction algorithms such as truthfulness, budget-balance, individual rationality, system-efficiency, usable-efficiency, etc., [16–18]. An auction is said to be truthful if each buyer cannot improve its utility by the fake bid. Hence, in the truthful auctions, bidders are encouraged to behave honestly. An auction is budget-balanced if the utility of the seller is non-negative, which is the minimal condition for the auction to be self-sustainable. In the individually rational auctions, a winning buyer is not charged more than the submitted bid. An auction is system-efficient if the winning bidders maximize the total valuation of the item sold. Furthermore, an auction is usable-efficient if the item can be shared by multiple buyers. We note that an auction algorithm cannot satisfy all these properties at the same time [19]. A tradeoff can exist between some properties, e.g., a truthful auction is hard to be system-efficient.

The main advantage of using auction mechanism in cognitive radio networks is to create incentives for PUs releasing spectrum channels and at the same time, to enhance channel availability for SUs. This in turn increases the spectrum utilization while generating more revenues for PUs. In this paper, we propose a new auction algorithm for managing the spectrum channels efficiently. Specifically, we summarize our main contributions as follows.

- We propose a novel auction-oriented radio spectrum purchasing (or leasing) framework for secondary markets of CR networks, which is called an aDaptive and Economically robust Auction-based Leasing (DEAL).
- The proposed DEAL framework adapts the parameters in auction algorithms for better performance. A parameter, called the reserve price, values the highly volatile prices of spectrum channels and produces more revenues for PUs. Another parameter, called the maximum number of sacrificed users, improves the bargaining power of SUs by increasing the group budget.
- The DEAL framework supports the group-buying such that the SUs with limited budget can purchase the expensive spectrum channels. Furthermore, our framework is shown to possess three desirable properties of auctions: individual rationality, budget-balance, and truthfulness.

The rest of this paper is organized as follows. In Sect. 2, we overview the previous literature related to our work. In Sect. 3, the system model is described. In Sect. 4, we present the DEAL framework and prove its economic properties in Sect. 5. We provide the simulation results and discuss the performance of our proposed system in Sect. 6. Finally, the conclusion follows in Sect. 7.

## 2 Related Works

There are many spectrum auction algorithms proposed for the secondary markets of CR networks. We briefly overview the related spectrum auction schemes proposed for secondary markets in the following.

The VERITAS system has been proposed for eBay-like dynamic spectrum markets [20], where the spectrum is evaluated by the buyers only. So the channels can be sold at much lower price than the seller's expectation. In [17], the authors suggest the TRUST system that considers the channel evaluations of both the seller and the buyer at the same time. The system exhibits the properties of truthfulness, budget-balance and individual rationality, but has the weakness of sacrificing many buyers by the McAfee mechanism. Furthermore, it does not support heterogeneous spectrum channels. In [21], an auction mechanism, called TASC, has been proposed for heterogeneous channels. TASC is budget-balanced and individually rational. It can prevent any participant from placing the fake bid. However, the system is not efficient mainly because it keeps a single price for all the channels. The SMALL is an auction framework for multichannel networks [22]. It has been designed to ensure the non-negative utility to sellers. For that, multiple buyers submit the sealed bids simultaneously and if all the bids are less than the predefined threshold, the channel is not sold at all. The SMALL framework supports the spatial reusability, but does not consider the channel heterogeneity and loses the truthfulness when extended to the double auction framework.

We can also find auction algorithms based on the game theory in [23–26]. The players therein perform a set of strategies to achieve a co-win situation in a given problem. In [26], for example, the most urgent users are chosen by the game theoretic approach, in CR networks. When the spectrum channels are allocated at random, users are hard to catch the channel access chances in a timely manner. This scheme helps the urgent users to win the channel access opportunity and increases the spectrum efficiency, while maintaining fairness. The channel allocation scheme is evaluated in terms of throughput, blocking probability, spectral efficiency, etc. However, the authors do not consider the various design issues of auction algorithms, which are of interest to make them economically robust.

In [27], the SMASHER scheme is proposed for the lease of heterogeneous spectrum channels in non-cooperative wireless networks. It possesses truthfulness, budget-balance and individual rationality, but has the drawback of restricting bidders to value all the spectrum channels uniformly. In [28], the TAHES scheme is proposed to enable spectrum trades between multiple buyers and sellers. The auction algorithm satisfies the economic properties but assumes the trading of non-divisible spectrum, which is impractical. The authors of [29] suggest the

TAMES scheme to address the spectrum reusability issue among multiple buyers. This is a collusion-free auction algorithm that supports virtual grouping of users. That is, the users can create a group to participate in the spectrum auction and if the group purchases a channel successfully, the involved users can use it together. Other auction schemes supporting the user-grouping follow in [30–32]. We note that the users in the secondary market are hard to purchase the whole spectrum channel due to their intermittent needs and limited budgets. So the user-grouping concept is very meaningful practically.

In [33], the authors propose a new concept of the group-buying for spectrum auctions. In reality, a spectrum channel is tremendously expensive. For example, in the AWS-3 spectrum auction of the USA in 2014, six blocks of 65 MHz spectrum have been leased for twelve years at \$44.9 billion [34]. Therein, 31 out of 70 eligible bidders were big telecommunication companies. It is practically impossible for small companies or individuals to purchase the spectrum channels in the auction due to the limited budget. To resolve this issue, in the TASG framework [33], an agent gathers the budgets of multiple SUs and places a bid on behalf of them in the spectrum auction. In this way, a group of SUs can increase their purchasing power and obtain the spectrum channels successfully.

Some other previous schemes create a group of SUs and share the purchased channel among the members [17, 22, 35], but they do not boost the purchasing power by gathering the budgets of individual users. Inspired by TASG, in this paper, we propose an auction-based radio spectrum purchasing (or leasing) framework for the secondary markets of CR networks, which is called an aDaptive and Economically robust Auction-based Leasing (DEAL). The TASG framework has many positive economic properties such as truthfulness, budget-balance and individual rationality, in addition to supporting the group-buying. However, it results in the small revenue for the spectrum seller and the low utility for the spectrum buyer since it sacrifices the channel access opportunity of many SUs. In our proposed framework, we improve the auction algorithm of TASG and adapt the involved parameters dynamically to achieve better performance. Our DEAL framework outperforms the TASG system significantly, while preserving its positive economic properties.

To the authors' best knowledge, our scheme is the first auction algorithm to adapt the reserve price for fair valuation of spectrum channels. With the adaptive reserve price, the proposed system enables the seller to fairly evaluate the highly volatile prices of spectrum channels. At the same time, our system encourages the buyers to maximize their winning chances by adapting another parameter, i.e., maximum number of sacrificed users, in the group-buying. As a result, sellers release more underutilized spectrum channels and buyers lease more spectrum channels, which leads to better spectrum utilization and social welfare. In Table 1, we briefly compare our DEAL framework with the existing auction schemes in the qualitative manner.

**Table 1** Comparison of the proposed DEAL framework with existing schemes

Properties	Auctions										
	VERITAS	TRUST	TASC	SMALL	SMASHER	TAMES	TAHES	TASG	DEAL		
Truthfulness	✓	✓	✓	x	✓	✓	✓	✓	✓		
Budget balance	✓	✓	✓	✓	✓	✓	✓	✓	✓		
Individual rationality	✓	✓	✓	✓	✓	✓	✓	✓	✓		
Heterogeneous reserve price	x	x	x	x	✓	✓	✓	✓	✓		
Group-buying and budget limitation	x	x	x	x	x	x	x	x	✓		
Adaptive reserve price	x	x	x	x	x	x	x	x	✓		
Utility maximization for the Agents	x	x	x	x	x	x	x	x	✓		

### 3 System Model

We consider a CR network with one primary network and  $N$  secondary networks, as shown in Fig. 2. The primary network leases the idle spectrum channels to the secondary networks through the auction. An agent in the primary network plays the role of the Spectrum Auctioneer (SA) and each access point in secondary networks becomes the Secondary Bidder (SB). Indeed, SBs do not need the spectrum channels for their own uses. However, during the auction process, each SB  $i$  in the  $i$ -th secondary network acts on behalf of its serving Secondary Users (SUs), whose number is denoted as  $N_i$ .

The system time is divided into consecutive periods with the identical interval. At the beginning of each period, SUs purchase (or lease) the spectrum channels for the communication. We propose a four-stage auction mechanism, which consists of two nested auction algorithms, for the efficient trade of spectrum channels. The Local Auction is carried out first between each SB and its serving SUs, where SBs sell the *fictitious* channels not purchased yet from the SA, to SUs in advance. Then the Global Auction follows between the SA and SBs. If an SB succeeds to buy some channels, the channels are allocated to the winners of the Local Auction. At the final stage, the parameters involved in the auction algorithms are adapted to improve the system performance. We assume that the system has  $C$  spectrum channels. Each SB can purchase at most one channel every period, and the channel can be shared by multiple SUs. The characteristics of a spectrum channel vary according to time and space in terms of SINR, availability, maximum allowed transmission power, etc. Hence, the evaluation on a channel can be different substantially in each SB and/or period. Moreover, we allow the seller (SA) to incorporate its evaluation on the channel in the auction process. Specifically, the SA sets the *reserve price*  $r^c(t)$  for

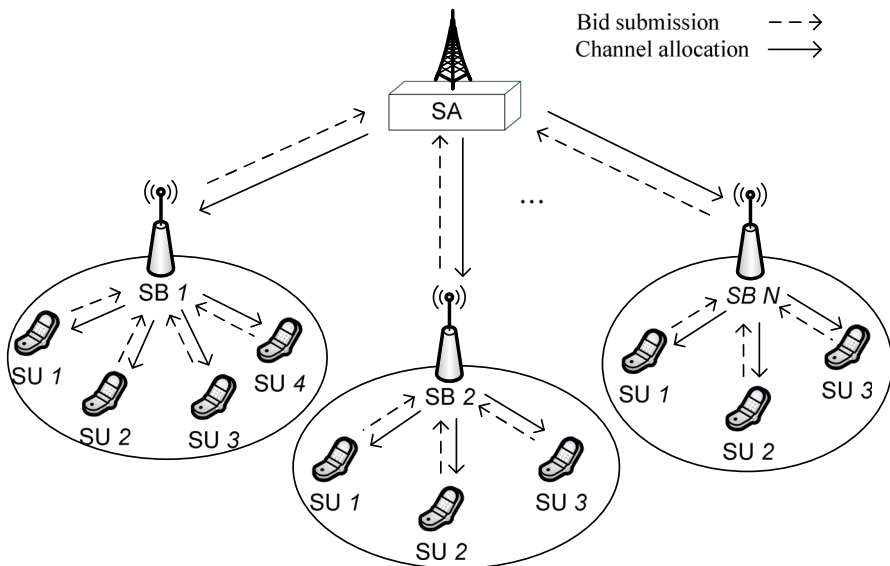


Fig. 2 System model



spectrum channel  $c$  at period  $t$ , and does not sell the channel below the reserve price. The reserve price is not open to SBs. And, the SA can adjust the reserve price dynamically, considering the past bids from the SBs on the channel. From now on, we remove the period index  $t$  in every notation if there is no confusion.

Every period  $t$ , the SA initiates the auction process by announcing the channel information to the SBs, e.g., the availability and maximum allowed transmission power. Each SB disseminates the information to its serving SUs, and the Local Auction starts. In SB  $i$ , every SU  $j$  evaluates the channels in the buyer’s stance and assigns the budget  $b_{ij}^c$  for channel  $c$ , where  $c = 1, \dots, C$ . The budget indicates how much the SU can pay for the channel at the highest. We denote the evaluation of SU  $j$  on the channel  $c$  as  $v_{ij}^c$ . Then, SU  $j$  submits the bid  $\{(v_{ij}^c, b_{ij}^c)\}_{c=1, \dots, C}$  to the serving SB  $i$ . The SB  $i$  collects the bids from all the serving SUs and determines the set of winners  $\mathbf{W}_i^c$  for each channel  $c$ . In addition, SB  $i$  decides the group budget  $B_i^c$  for channel  $c$  as the bid of the following Global Auction.

The Global Auction begins when all the SBs submit the bids to the SA. The bid of SB  $i$  is given as  $\{B_i^c\}_{c=1, \dots, C}$ , i.e., the group budgets for all the channels. Based on the bids, the SA determines the selling price  $\sigma^c$  of channel  $c$  as well as the winner set  $\mathbf{W}$ . We define the indicator function  $a(i, c)$  to represent which SB has purchased each channel, which is 1 if SB  $i$  obtains channel  $c$ , and 0 otherwise. The SB  $i \in \mathbf{W}$  now announces to the winning SUs in  $\mathbf{W}_i^c$  that it has purchased a spectrum channel  $c$ . We can also denote the set of winning SUs by  $\mathbf{W}_i$ , which is  $\mathbf{W}_i^c$  for the channel  $c$  with  $a(i, c) = 1$ .

The SA issues a bill of  $\beta_i$  to each winning SB  $i \in \mathbf{W}$  to harvest the revenue. If the SB  $i$  has purchased channel  $c$ , the bill  $\beta_i$  is equal to the selling price  $\sigma^c$  of the channel, which becomes the revenue of the SA. We denote the SA’s revenue from channel  $c$  as  $V^c$ . In cascade, SB  $i$  issues a bill of  $\beta_{ij}$  to the SU  $j \in \mathbf{W}_i$  as it enjoys the benefit from the spectrum channel. Specifically, the SU  $j$  gains  $v_{ij}/|\mathbf{W}_i|$  by equally sharing the purchased channel with other winners, where  $v_{ij} \triangleq \sum_c v_{ij}^c a(i, c)$  and  $|\mathbf{X}|$  denotes the cardinality of a set  $\mathbf{X}$ . Hence, the utility of SU  $j$  can be written as

$$U_{ij} = \begin{cases} \frac{v_{ij}}{|\mathbf{W}_i|} - \beta_{ij}, & \text{if SB } i \in \mathbf{W} \text{ and SU } j \in \mathbf{W}_i \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

SB  $i$  earns the revenue of  $\sum_{j \in \mathbf{W}_i} \beta_{ij}$  from the winning SUs, and pays  $\beta_i$  to the SA. So, it has the utility of

$$U_i = \begin{cases} \sum_{j \in \mathbf{W}_i} \beta_{ij} - \beta_i, & \text{if SB } i \in \mathbf{W} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

We notice that an SU or an SB has the zero utility if it fails to buy a channel in the period. Finally, the utility of the SA is defined as the total revenue since it does not pay for the channels to anyone, i.e.,

$$U = \sum_c V^c. \tag{3}$$

We aim to improve the utility of each SU and the utility of the SA. We do not care about SBs because they are just agents for the group-buying. However, the utility of SBs should not be negative since then, the proposed system will not be sustainable. We define one more performance metric here, i.e., the social welfare  $\omega$ , which measures how efficiently the spectrum channels are utilized in the system. The social welfare is the total valuation of all the winning SUs and can be written as

$$\omega \triangleq \sum_{i \in \mathbf{W}, j \in \mathbf{W}_i} v_{ij}. \tag{4}$$

We summarize the defined notations in Table 2.

**Remark** In our auction framework, each SU can evaluate the spectrum channel arbitrarily according to its local policy. However, one of the reasonable policies is to incorporate the channel capacity in the evaluation. Given channel  $c$ , we denote the channel bandwidth as  $W^c$ , the maximum allowed transmission power as  $P^c$ , the path loss from the serving SB  $i$  to SU  $j$  as  $G_{ij}$ , interference to SU  $j$  from the primary network as  $I_{ij}$  and the background noise as  $P_n$ . Then, we suggest SU  $j$  in SB  $i$  to evaluate channel  $c$  as

$$v_{ij}^c = W^c \log_2 \left( 1 + \mu_{ij}^c \frac{P^c G_{ij}}{I_{ij} + P_n} \right), \tag{5}$$

where  $\mu_{ij}^c$  is the weighting factor of channel  $c$  placed by SU  $j$  strategically.

**Table 2** Summary of notations

Notation	Description
$N$	Number of SBs
$N_i$	Number of SUs in SB $i$
$C$	Number of channels in the system
$S^c$	Maximum number of sacrificed SUs for channel $c$
$r^c$	Reserve price of channel $c$
$b_{ij}^c$	SU $j$ 's budget on channel $c$ (at SB $i$ )
$v_{ij}^c$	SU $j$ 's evaluation on channel $c$ (at SB $i$ )
$B_i^c$	Group budget on channel $c$ at SB $i$
$\sigma^c$	Selling price of channel $c$
$\mathbf{W}$	Winner set of the Global Auction
$\mathbf{W}_i$	Winner set of the Local Auction at SB $i$
$\mathbf{W}_i^c$	Winner set of the Local Auction at SB $i$ (for channel $c$ )
$\beta_i$	Bill to the SB $i$ from the SA
$\beta_{ij}$	Bill to the SU $j$ from SB $i$
$V^c$	SA's revenue from channel $c$
$U$	Utility of the SA
$U_i$	Utility of SU $i$
$U_{ij}$	Utility of SU $j$ served by SB $i$
$\omega$	Social welfare

## 4 DEAL Framework

Our proposed auction framework consists of the following four stages.

### 4.1 Stage I: Local Auction (SUs to SB)

---

#### Algorithm 1 Local Auction algorithm in SB $i$ .

---

```

1: for  $c \leftarrow 1$  to  $C$  do
2:   Choose  $s \in \{1, \dots, S^c\}$  randomly;
3:    $\kappa_i^c \leftarrow (N_i - s + 1)$ -th largest element of  $\{b_{i1}^c, \dots, b_{iN_i}^c\}$ ;
4:    $\mathbf{W}_i^c \leftarrow \emptyset$ ;
5:   for  $j \leftarrow 1$  to  $N_i$  do
6:     if  $b_{ij}^c \geq \kappa_i^c$  then
7:        $\mathbf{W}_i^c \leftarrow \mathbf{W}_i^c \cup \{j\}$ ;
8:     end if
9:   end for
10:  while  $v_{ij}^c \leq \kappa_i^c \times |\mathbf{W}_i^c|$  for some  $j \in \mathbf{W}_i^c$  do
11:     $\mathbf{W}_i^c \leftarrow \mathbf{W}_i^c \setminus \{j \mid \operatorname{argmin}_{j \in \mathbf{W}_i^c} b_{ij}^c\}$ ;
12:  end while
13:   $B_i^c \leftarrow \kappa_i^c \times |\mathbf{W}_i^c|$ ;
14: end for

```

---

The Local Auction is performed between each SB and its serving SUs. Generally, the auction process can be divided into the bid phase and the item transfer phase. In the DEAL framework, we conduct the Global Auction between the two phases of the Local Auction in time. The first stage of DEAL, i.e., Local Auction (SUs to SB) is on the bid phase of the Local Auction. We discuss the item transfer phase of the Local Auction in the third stage of the DEAL, i.e., Local Auction (SB to SUs).

We now discuss the design of the Local Auction algorithm, which determines how an SB chooses the winning SUs and sets the *clearing prices* of channels. The algorithm also computes the group budget of each channel, which becomes the bid of the SB in the subsequent Global Auction. The Local Auction algorithm is described in Algorithm 1 for an SB  $i$ . By this algorithm, we choose the SUs with the large budget and the high evaluation as the winners. Note that the same procedure is repeated for each channel.

- Determine the clearing price (lines 2–3): We preclude the SUs with small budgets from the winner set. We call such an SU as a sacrificed SU, and the survivors as the potential winners. We choose the number of sacrificed SUs,  $s$ , randomly among the integers from 1 to  $S^c$ , where the parameter  $S^c (< N_i)$  represents the maximum number of sacrificed SUs for channel  $c$ . We then set the largest budget of the sacrificed SUs (or the  $(N_i - s + 1)$ -th largest budget) as the clearing price  $\kappa_i^c$  of channel  $c$ . The clearing price is closely related to the group budget, as shown in line 13.
- Initialize the winner set (lines 4–9): The winner set is initialized to be empty (line 4). We then put all the potential winners into the winner set, i.e., SU  $j$  is placed in the winner set  $\mathbf{W}_i^c$  only if its budget  $b_{ij}^c$  is larger than the clearing price

$\kappa_i^c$ . However, the winner set is not fixed yet since an SU should satisfy one more condition on the channel evaluation to be a winner.

- Shrink the winner set (lines 10–12): We require each SU’s portion on the valuation of the channel to be larger than or equal to the clearing price when it becomes the winner. In other words,  $\frac{v_{ij}^c}{|W_i^c|} \geq \kappa_i^c$  since the channel is shared by all the winners equally. If this condition is not satisfied for all the SUs in the current  $W_i^c$ , we shrink the winner set by removing the SU with the smallest budget one by one. In line 11, the set difference from  $X$  to  $Y$  is defined as  $X \setminus Y \triangleq \{ \forall x | x \in X \text{ and } x \notin Y \}$ .
- Set the group budget (line 13): We set the group budget of channel  $c$  as

$$B_i^c = \kappa_i^c \times |W_i^c|. \tag{6}$$

Our Local Auction algorithm is similar to the SAMU algorithm of TASG [33]. However, it is distinguished from SAMU in the choice of the number of sacrificed SUs. In SAMU, the maximum number of sacrificed SUs,  $S^c$ , is fixed to  $N_i - 1$  for all the channels, while our algorithm allows the parameter to have any value from 1 to  $N_i - 1$ . With a large  $S^c$  like  $N_i - 1$ , the number of potential winners is small with high probability, and so is the winner set. Then the group budget is likely to be small although the clearing price can be high. The small group budget reduces the SB’s winning chance in the Global Auction, and negatively affects the utility of SUs as well as the social welfare.

**Example** We consider an example to explain the operation of the proposed Local Auction algorithm. We assume that an SB  $i$  serves 5 SUs and there is only one spectrum channel in the system. We list the evaluations and budgets of all the SUs in Table 3, in the decreasing order of the budgets.

We assume that  $s = 1$  has been chosen as the number of sacrificed SUs, for instance. Then SU 4 with the smallest budget is sacrificed and the clearing price is set to its budget, 8. In addition, the winner set is set as  $W_i^1 = \{1, 5, 3, 2\}$ . Among the SUs in  $W_i^1$ , the evaluation of SU 2, 30, is less than  $\kappa_i^1 \times |W_i^1| = 8 \times 4 = 32$ . So, the winner set shrinks to  $W_i^1 = \{1, 5, 3\}$ . Now, the evaluation of every SU in  $W_i^1$  is larger than  $8 \times 3 = 24$ . Hence, it becomes the final winner set and the group budget is set to 24. In Table 4, we provide the clearing price, the number of winners and the group budget, for each choice of  $s$ . We observe that as the number of sacrificed SUs

**Table 3** Example of the Local Auction algorithm

SU	Evaluation	Budget
1	60	22
5	42	20
3	48	14
2	30	12
4	40	8

increases, the group budget increases at first but decreases soon even if the clearing price grows up monotonically.

Using this example, we now show that if an SB behaves *deterministically* to maximize the group budget, the serving SUs are motivated to submit the fake bids. In Table 4, we observe that the SB can maximize the group budget by choosing  $s = 2$ . Then, SU 3 of our concern has the utility of  $\frac{v_3}{|W_1^1|} - \kappa_i^1 = \frac{48}{3} - 12 = 4$ . Suppose that the SU 3 submits the fake bid (48, 7) instead of the true bid (48, 14). It is straightforward to check that the SB can obtain the maximum group budget of 28 when  $s = 1$ , with the number of winners 4 and the clearing price 7. In that case, the utility of SU 3 increases to  $\frac{48}{4} - 7 = 5 (> 4)$ . To prevent this selfish behavior, we randomize the number of sacrificed SUs in the Local Auction algorithm, accepting the potential loss of the group budget.

## 4.2 Stage II: Global Auction

We now present our Global Auction algorithm, which is conducted by the SA to allocate the idle spectrum channels to SBs. We recall that in our model, each SB can purchase at most one channel. So, once an SB obtains a channel, it is not considered for the further allocation.

---

### Algorithm 2 Global Auction algorithm in the SA.

---

```

1:  $\mathbf{N} \leftarrow \{1, \dots, N\}$ ;
2:  $\mathbf{W} \leftarrow \emptyset$ ;
3:  $a(i, c) \leftarrow 0$  for  $i = 1, \dots, N$  and  $c = 1, \dots, C$ ;
4: for  $c \leftarrow 1$  to  $C$  do
5:    $\sigma^c \leftarrow 0$ ;
6:    $\hat{\mathbf{N}} \leftarrow \{i \in \mathbf{N} | B_i^c \geq r^c\}$ ;
7:   if  $|\hat{\mathbf{N}}| = 1$  then
8:      $\mathbf{W} \leftarrow \mathbf{W} \cup \hat{\mathbf{N}}$ ;
9:      $a(i, c) \leftarrow 1$  for  $i$  such that  $\hat{\mathbf{N}} = \{i\}$ ;
10:     $\sigma^c \leftarrow r^c$ ;
11:     $\mathbf{N} \leftarrow \emptyset$ ;
12:   else if  $|\hat{\mathbf{N}}| > 1$  then
13:     Choose two elements  $p, q \in \hat{\mathbf{N}}$  randomly;
14:      $w \leftarrow \operatorname{argmax}_{i \in \{p, q\}} B_i^c$ ;
15:      $\mathbf{W} \leftarrow \mathbf{W} \cup \{w\}$ ;
16:      $a(w, c) \leftarrow 1$ ;
17:      $\sigma^c \leftarrow \min(B_p^c, B_q^c)$ ;
18:      $\mathbf{N} \leftarrow \mathbf{N} \setminus \{w\}$ ;
19:   end if
20: end for

```

---

We describe our Global Auction algorithm in Algorithm 2. For simplicity, we assume that all the channels are idle in the given period.

- Initialize the sets and indicator functions (lines 1–3): We denote the set of remaining SBs, not allocated a channel yet, as  $\mathbf{N}$ . All the SBs initially belong to  $\mathbf{N}$  and if an SB leases one channel, it is removed from the set. The winner set  $\mathbf{W}$

**Table 4** Results of the Local Auction algorithm for various choices of  $s$

	$s = 1$	$s = 2$	$s = 3$	$s = 4$
$\kappa_i^1$	8	12	14	20
$ \mathbf{W}_i^1 $	3	3	2	1
$B_i^1$	24	36	28	20

is initialized to be empty. Further, the indicator function  $a(i, c)$  is set to 0 since SB  $i$  has not purchased any channel  $c$  currently.

- Construct the set of eligible SBs (line 5–6): The selling price  $\sigma^c$  is set to 0 for each channel  $c$ . We construct the set of eligible SBs,  $\hat{\mathbf{N}}$ , by collecting every remaining SB  $i \in \mathbf{N}$  when its bid or group budget  $B_i^c$  is larger than or equal to the reserve price  $r^c$ .
- One eligible SB case (lines 7–11): If there is only one eligible SB, we choose the SB as the winner of channel  $c$  (lines 8–9) and the selling price is set to the reserve price of the channel (line 10). Since the last SB has purchased one channel, the set of remaining SBs becomes empty.
- Multiple eligible SBs case (lines 12–19): If there are more than one eligible SBs, we pick up two SBs randomly. Among the two SBs chosen, the SB with the larger bid becomes the winner (line 14–16) and the smaller bid itself becomes the selling price of the channel (line 17). The winner is then removed from the set of remaining SBs,  $\mathbf{N}$ .

Once the channels are allocated to SBs, the SA issues the bills to the winning SBs to reap the revenue. In our DEAL framework, the bill to each SB should be equal to the selling price of its purchased channel. That is, SB  $i \in \mathbf{W}$  receives the bill of

$$\beta_i = \sigma^c \tag{7}$$

when it has purchased the channel  $c$ .

Our Global Auction algorithm has been inspired by the DCP algorithm of TASG [33]. The DCP algorithm selects the winner of a channel randomly among the eligible SBs and sets the smallest bid above the reserve price as the selling price. Without collusion, an SB cannot improve its utility since the winner is chosen randomly and the selling price of the channel is independent of the winner’s bid. However, the selling price is minimal and thus, the SA should accept the small revenue. Our Global Auction algorithm produces the higher revenue for the SA since the selling price is just the relatively smaller bid among the two eligible SBs (rather than the smallest bid above the reserve price). Further, an SB alone cannot improve its utility due to the same reason to the DCP algorithm. If a winner tries to decrease the selling price by lowering its bid, it may not be chosen as the winner any longer and the utility drops to zero.

**Example** We consider the SA with two idle channels, and the bids of three SBs are shown in Table 5. We assume that the reserve prices are 75 for channel 1 and 45 for channel 2, respectively.

We conduct the allocation for the channel 1 first. All the SBs have submitted the

**Table 5** Example of Global Auction algorithm

	Channel 1 ( $r^1 = 75$ )	Channel 2 ( $r^2 = 45$ )
SB 1	90	40
SB 2	85	48
SB 3	78	42

bids larger than  $r^1 = 75$ . So, the set of remaining SBs is  $\mathbf{N} = \{1, 2, 3\}$ , and the set of eligible SBs is  $\hat{\mathbf{N}} = \{1, 2, 3\}$  as well. Suppose that SB 1 and SB 3 are chosen by the random procedure. Then, SB 1 with the larger bid is chosen as the winner and the smaller bid, 78, becomes the selling price of channel 1. Next, the allocation of channel 2 begins. The set of remaining SBs is  $\mathbf{N} = \{2, 3\}$ , and the set of eligible SBs is given as  $\hat{\mathbf{N}} = \{2\}$  since the bid of SB 3 is less than  $r^2$ . Then, SB 2 becomes the winner and the selling price of channel 2 is set to the reserve price, 45.

### 4.3 Stage III: Local Auction (SB to SUs)

This stage is the item transfer phase of the Local Auction. Each winning SB of the Global Auction distributes the purchased spectrum channel to the winning SUs of the channel. Every SB  $i$  has already constructed the winner set  $\mathbf{W}_i^c$  for each channel  $c$ , in the stage I. So, the winner set  $\mathbf{W}_i$  is specified as soon as the SA notifies the purchased channel id to the SB  $i \in \mathbf{W}$ .

The SB  $i$  should reap the revenue from the winning SUs and pay the bill to the SA for the purchased channel  $c$ . In DEAL, the SB  $i$  charges the amount of

$$\beta_{ij} = \frac{B_i^c}{|\mathbf{W}_i|} = \kappa_i^c \quad (8)$$

to SU  $j \in \mathbf{W}_i^c$ , regardless of the bill from the SA. In other words, an SB charges the equal portion of the group budget to all the winning SUs. It can be more reasonable for each winner to pay in proportion to either its budget or its evaluation on the channel. However, in that case, a winning SU is motivated to submit the fake bid to reduce the charged amount. Moreover, the equal split is fair since all the winners use the channel for the identical time.

### 4.4 Stage IV: Parameter Adaption

Our proposed auction algorithms have the configurable parameters, i.e., the maximum number of sacrificed SUs,  $S^c$ , in the Local Auction algorithm and the reserve price,  $r^c$ , in the Global Auction algorithm for  $c = 1, \dots, C$ . In this stage, we adapt the parameters dynamically to improve the system performance. Each SB  $i$  maximizes the group budget  $B_i^c$  by adjusting the parameter  $S^c$ , and the SA maximizes its utility or total revenue  $V$  by choosing the reserve prices,  $r^c$ . We notice that the bigger group budget can increase the utility of each SU and the social welfare indirectly. We here discuss about the adaption of the reserve prices only

since the similar procedure can be applied for the maximum number of sacrificed SUs also.

We should consider two points in choosing the reserve prices. First, the reserve price is the evaluation on the channel in the SA’s viewpoint. If the reserve price is higher than the SBs’ expectation, the channel is not likely to be sold, and the revenue of the SA decreases. On the other hand, if the reserve price is very low, the revenue of the SA is small although the channel is sold out. Second, the SA should set the reserve prices based on the bids of SBs to maximize the revenue. However, in that case, SBs are motivated to submit the fake bids to affect the reserve prices (and accordingly, the results of the auction). As a compromise, we adapt the reserve prices using the previous bids only rather than the current bids.

We recall that the reserve price of channel  $c$  is denoted as  $r^c(t)$  in the period  $t$ . Given bids  $\{B_i^c(t)\}$  for  $i = 1, \dots, N$ , the SA obtains the revenue of  $V^c(t)$  from each channel  $c$ . Since the revenue is affected by the reserve price, we denote the revenue from channel  $c$  as  $V^c(t; r^c)$ .

We emulate  $K$  systems in parallel for each channel  $c$ , where the  $k$ -th system has the reserve price  $r_k^c$ . As shown in Fig. 3, at period  $t$ , each emulated system takes the bids  $\{B_i^c(t)\}$  for  $i = 1, \dots, N$  as the input and outputs the revenue  $V^c(t; r_k^c)$  as well as the long term revenue  $\tilde{V}^c(t; r_k^c)$ . We use the following iteration to calculate the long term revenue,

$$\tilde{V}^c(t; r_k^c) = (1 - \epsilon(t))\tilde{V}^c(t - 1; r_k^c) + \epsilon(t)V^c(t; r_k^c), \tag{9}$$

where the weighting factor  $\epsilon(t)$  is chosen as a small constant  $\epsilon (> 0)$ .

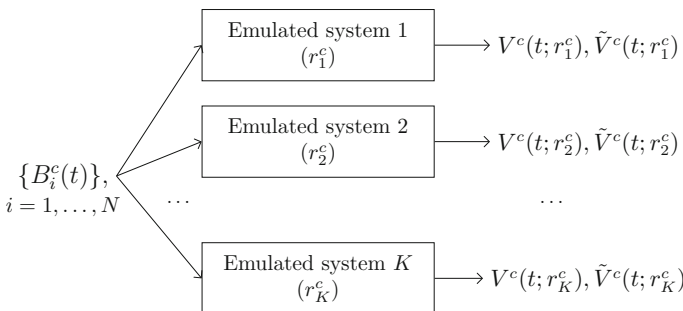
Then, for channel  $c$ , we can denote the system with the largest long term average revenue as

$$k^* = \operatorname{argmax}_{k \in \{1, \dots, K\}} \tilde{V}^c(t; r_k^c). \tag{10}$$

Finally, we set the reserve price of the next period  $t + 1$  as

$$r^c(t + 1) = r_{k^*}^c \text{ for } c = 1, \dots, C. \tag{11}$$

We choose the reserve price of channel  $c$  among  $\{r_1^c, \dots, r_K^c\}$  and, in this sense, we call them the candidate reserve prices. We can increase the SA’s revenue by



**Fig. 3** Emulated systems for adapting the reserve price of channel  $c$



considering more candidate reserve prices or emulating more systems, with the cost of computational complexity.

We lastly discuss the convergence of reserve price  $r^c(t)$ . Given the bids from SBs, i.e.,  $\{B_i^c(t)\}$ ,  $i = 1, \dots, N$ , the  $k$ -th emulated system produces the revenue of  $V^c(t; r_k^c)$ . We note that for fixed  $r_k^c$ , the revenue  $V^c(t; r_k^c)$  is determined by two independent uniform random variables  $p$  and  $q$  only (See line 13 of Algorithm 2). So  $V^c(t; r_k^c)$  is i.i.d. random variable across time  $t$  and then, its long term average  $\bar{V}^c(t; r_k^c)$  converges if the weighting factor  $\epsilon(t)$  satisfies the following conditions:  $\epsilon(t) > 0$ ,  $\lim_{t \rightarrow \infty} \epsilon(t) = 0$ , and  $\sum_t \epsilon(t) \rightarrow \infty$  [36]. Practically, the long term average converges for a small constant weighting factor. If  $\bar{V}^c(t; r_k^c)$  converges for every  $k$ , the optimal index  $k^*$  of Eq. (10) also converges. As a result, the system-wise reserve price follows that of the  $k^*$ -th emulated system according to Eq. (11).

## 5 Properties of DEAL

In this section, we show that our DEAL framework possesses the three properties: budget-balance, individual rationality, and truthfulness.

An auction is budget-balanced if the auctioneer has the non-negative utility. We prove that the auctioneers of the Local Auction and the Global Auction both have the non-negative utility.

**Proposition 1** *DEAL is budget-balanced.*

*Proof*

1. Local Auction: The auctioneer of the Local Auction is each SB. If an SB  $i$  fails to purchase a spectrum channel in the Global Auction, its utility is 0. Hence, we assume that the SB  $i$  has obtained a channel  $c$ . Then its utility is given as  $U_i = \sum_{j \in \mathbf{w}_i} \beta_{ij} - \beta_i = B_i^c - \sigma^c$  since  $\beta_{ij} = \frac{B_i^c}{|\mathbf{w}_i|}$  from Eq. (8) and  $\beta_i = \sigma^c$  from Eq. (7). In the following, we refer to Algorithm 2 for the line numbers in parentheses.

If the SB  $i$  is the only eligible SB, its bid is larger than or equal to the reserve price (line 6) and the selling price is equal to the reserve price (line 10). That is,  $B_i^c \geq r^c$  and  $\sigma^c = r^c$ , which leads to  $B_i^c \geq \sigma^c$ . If there are other eligible SBs, there exists other SB  $q$  which has been chosen as two candidate winners (line 13) but fails to be the winner. The bid of SB  $i$  is larger than or equal to that of SB  $q$  (line 14), which becomes the selling price (line 17). That is,  $B_i^c \geq B_q^c = \sigma^c$  or  $B_i^c \geq \sigma^c$ .

In both cases,  $U_i = B_i^c - \sigma^c \geq 0$ .

2. Global Auction: The auctioneer of the Global Auction is the SA, whose utility is given as  $U = \sum_c V^c$  from Eq. (3). Since the revenue  $V^c$  from each channel  $c$  is larger than or equal to 0, the utility  $U$  is non-negative.  $\square$

We now move to the next property of the DEAL framework. An auction is individually rational if the winning buyer is not charged more than its bid always.

**Proposition 2** *DEAL is individually rational.*

*Proof*

1. Local Auction: The buyer of the Local Auction is each SU. We assume that SU  $j$  is one of the winners of channel  $c$  in SB  $i$ . The SU  $j$  has submitted the budget  $b_{ij}^c \geq \kappa_i^c$  (lines 6-8 of Algorithm 1) in the bid, and is charged by  $\beta_{ij} = \kappa_i^c$  from Eq. (8). So,  $b_{ij}^c \geq \beta_{ij}$ .
2. Global Auction: The buyer of the Global Auction is each SB. If SB  $i$  purchases channel  $c$ , it has submitted the budget  $B_i^c$  in the bid, and is charged by  $\beta_i = \sigma^c$  from Eq. (7). Since  $B_i^c \geq \sigma^c$  (see the proof of Proposition 1),  $B_i^c \geq \beta_i$ .  $\square$

We recall that an auction is truthful if a buyer cannot increase its utility by submitting the fake bid, without collusion.

**Proposition 3** *DEAL is truthful, given parameters  $S^c$ s and  $r^c$ s.*

*Proof*

1. Local Auction: We can apply the proof for the truthfulness of SAMU algorithm when the parameter  $S^c$ , the maximum number of sacrificed SUs, is given for channel  $c = 1, \dots, C$ . Hence we refer to the proof of Lemma 3 in [33].
2. Global Auction: The buyer of the Global Auction is each SB  $i$ . We consider the following five cases, one by one.
  - i. SB  $i$  is not eligible: The utility of SB  $i$  is 0 since it cannot purchase a channel. If the SB submits a fake bid  $\tilde{B}_i^c$  larger than or equal to the reserve price, i.e.,  $\tilde{B}_i^c \geq r^c (> B_i^c)$ , it can be chosen as the winner of a channel  $c$ . However, it is billed then  $\beta_i \geq r^c$ . So, the utility becomes  $U_i = B_i^c - \beta_i < r^c - \beta_i \leq 0$ , and SB  $i$  cannot improve the utility.
  - ii. SB  $i$  is the only eligible SB: The SB  $i$  becomes the winner of channel  $c$  and the selling price of the channel is  $r^c$ . The utility of SB  $i$  is given as  $U_i = B_i^c - r^c$ . Note that  $B_i^c$  does not change although SB  $i$  submits the fake bid, and the selling price is fixed to  $r^c$ . So, the SB  $i$  cannot increase its utility.
  - iii. SB  $i$  is eligible but not chosen as candidate winners: The utility of SB  $i$  is 0 since it is not a winner. Even if the SB submits a fake bid, it cannot be chosen as the candidate winners since the selection process is purely random. So, the utility is still 0.
  - iv. SB  $i$  is chosen as candidate winners, but fails to be the winner: The utility of SB  $i$  is 0 since it is not a winner. Denoting the other candidate winner as SB  $q$ ,  $B_q^c \geq B_i^c$  since SB  $q$  is the winner. If the SB  $i$  submits a fake bid  $\tilde{B}_i^c$  larger than the true bid  $B_q^c$ , it can be chosen as the winner and the selling price becomes  $B_q^c$ . Then, the utility of SB  $i$  is  $U_i = B_i^c - B_q^c \leq 0$ , so the utility decreases.
  - v. SB  $i$  is the winner while there exist more than one eligible SBs: Denoting the other candidate winner as SB  $q$ , the utility of SB  $i$  is given as

$U_i = B_i^c - B_q^c \geq 0$ . If the SB  $i$  submits the fake bid  $\tilde{B}_i^c < B_q^c$ , it can decrease the selling price to  $\tilde{B}_i^c$ . However, it is not a winner of the channel any more such that the utility becomes 0. Hence, SB  $i$  cannot improve its utility by the fake bid.  $\square$

## 6 Numerical Analysis

In this section, we investigate the performance of our proposed DEAL framework by the simulations, and compare it with TASG of [33]. We also consider the DEAL without the parameter adaption (i.e., stage IV), to evaluate the contribution of adaptive parameters to the overall performance. This scheme is referred to as Static Economically robust Auction-based Leasing (SEAL) in this section. In SEAL, each SB  $i$  sets the parameter of the Local Auction algorithm as  $S^c = N_i - 1$  for every channel  $c$  such that it matches SAMU of the TASG framework.

### 6.1 Simulation Environment

We have developed the simulation code by the MATLAB, incorporating all the details of the proposed DEAL framework. In simulations, we set the number of SBs,  $N$ , as 15 and the number of channels,  $C$ , as 10. Each SB  $i$  serves 25 SUs, i.e.,  $N_i = 25$ , if not stated otherwise. SU  $j$  in SB  $i$  selects the budget for channel  $c$  among the integers from 3 to 20, and the evaluation on the channel  $c$  among the integers from 0 to 215 according to the uniform distribution. We set the candidates of the optimal maximum number of sacrificed SUs for channel  $c$ ,  $S^c$ , as the 50 arbitrary values chosen from  $[0, 25]$ . Similarly, the candidates of the optimal reserve price for channel  $c$ ,  $r_k^c$ , are set as the 50 values chosen from  $[0, 150]$ , thus  $K = 50$ . We run each simulation for 1000 periods to obtain one result. To put in a nutshell, we summarize the default parameters in Table 6.

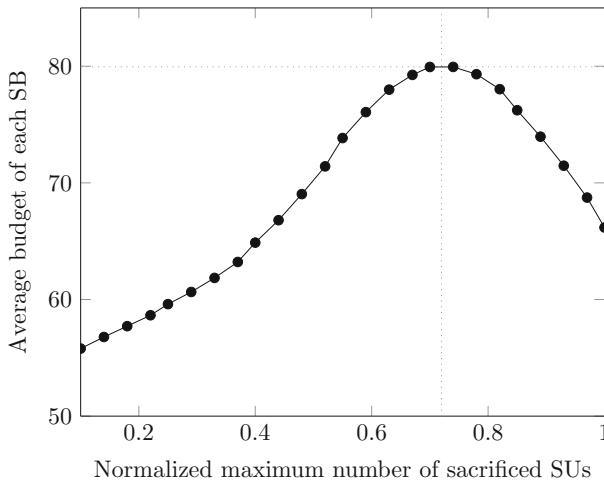
### 6.2 Simulation Results

#### 6.2.1 Effect of Maximum Number of Sacrificed Users on Group Budget

In Fig. 4, we present the average group budget of SBs (for a channel 1 chosen arbitrarily) for various maximum number of sacrificed SUs,  $S^1$ , which has been normalized by the number of SUs in each SB, i.e.,  $N_i = 25$ . We observe that as the parameter  $S^1$  increases, the group budget increases initially but decreases after reaching the maximum. When  $S^1$  is small, the number of sacrificed SUs is likely to be small, so the number of potential winners is large conversely. Although the winner set is large, the resulting group budget is small since the clearing price is low. On the other hand, when  $S^1$  is large, the number of sacrificed SUs is large and

**Table 6** Default parameter values used in simulations

Parameter name	Value
Number of channels ( $C$ )	10
Number of SBs ( $N$ )	15
Number of SUs in each SB $i$ ( $N_i$ )	25
SU $j$ 's budget on channel $c$ ( $b_{ij}^c$ )	3 – 20
SU $j$ 's evaluation on channel $c$ ( $v_{ij}^c$ )	0 – 215
Candidates of maximum number of sacrificed SUs for channel $c$	0 – 25
Candidates of reserve price for channel $c$	0 – 150

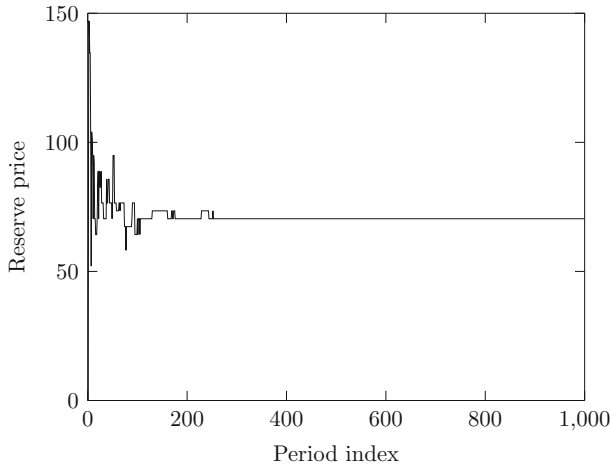


**Fig. 4** Average budget of each SB vs. Normalized maximum sacrificed SUs

the number of potential winners is small. Although the winner set is small, the group budget is large due to the high clearing price. However, if  $S^1$  exceeds a certain threshold, i.e., 0.72 in Fig. 4, the group budget can decrease again since the winner set size becomes a dominant factor. In summary, there exists the optimal  $S^c$  for the maximum group budget. The DEAL framework adapts the parameter of the Local Auction algorithm to find the optimal value, as explained in Sect. 4.4.

6.2.2 Convergence of Reserve Price in Global Auction

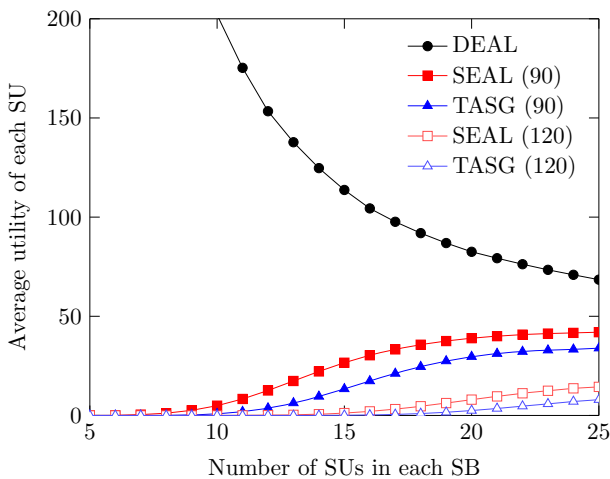
We show the variation of the reserve price (for a channel 1 chosen arbitrarily) in Fig. 5. In the early periods, the reserve price fluctuates searching for the optimum, and converges to a certain value quickly. We use the same approach to adapt the parameter of the Local Auction algorithm, i.e., the maximum number of sacrificed SUs, so the parameter also behaves similarly in time.



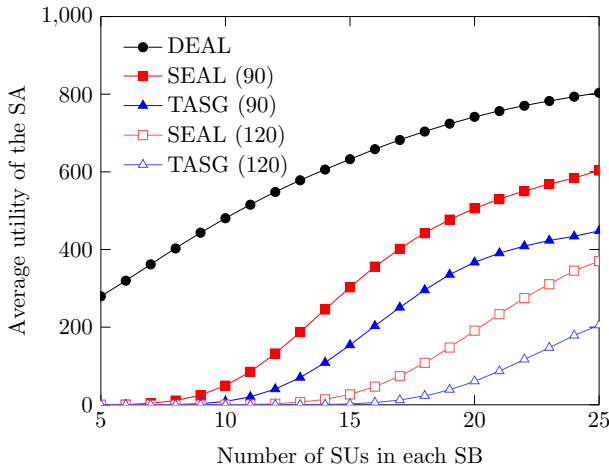
**Fig. 5** Convergence of the adaptive reserve price

6.2.3 Utility of Involved Entities

In Fig. 6, we provide the average utility of each SU for various numbers of SUs in each SB. For SEAL and TASG, we consider two different reserve prices of 90 and 120 for all the channels, and present them in the legends of the figure. We can see that the utility of each SU varies similarly in SEAL and TASG when the reserve prices are equal, because both schemes run the same auction algorithm between each SB and the serving SUs and choose the winning SBs randomly. However, the SEAL shows better performance than TASG since its winning SB has the larger group budget than the winning SB of TASG. In the two schemes, for the small



**Fig. 6** Average utility of each SU vs. Number of SUs in each SB



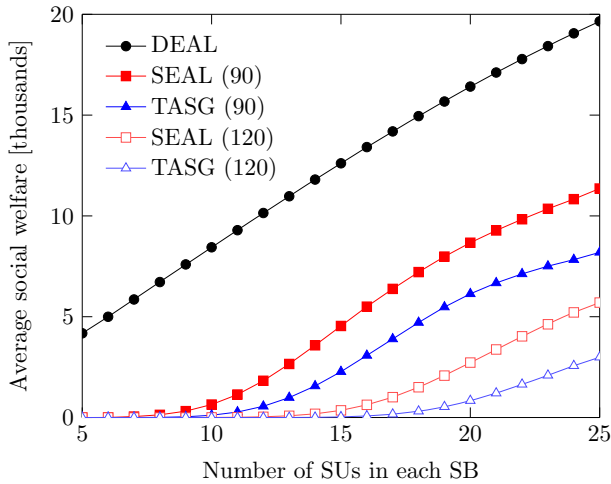
**Fig. 7** Average utility of the SA vs. Number of SUs in each SB

number of SUs, the utility of each SU is close to zero since then, each SB has so small a group budget such that it is less than the (fixed) reserve price. Hence, the SB cannot purchase a channel and its serving SUs have the zero utility. If the number of SUs increases, each SB has the larger group budget. The SB is liable to purchase a channel and the serving SUs have higher utilities. On the contrary, we observe that in DEAL, the utility of each SU decreases as the number of SUs increases. In DEAL, even an SB with a small group budget can purchase a channel with high probability since the reserve price is adapted based on the previous bids. Indeed, it is the number of winners in the Local Auction that affects the SU’s utility dominantly. The utility of each SU decreases inversely with the number of winning SUs since the channel is shared by the winners equally.

Figure 7 shows the average utility or revenue of the SA for different numbers of SUs in each SB. We can see that the DEAL always provides the higher revenue to the SA, compared to SEAL and TASG. Moreover, DEAL shows positive revenues even with a small number of SUs while the other schemes have almost zero revenues then. In SEAL and TASG, if there are only a few SUs, each SB is likely to have a smaller group budget than the fixed reserve price. As the number of SUs increases, the group budget exceeds the reserve price and the revenue of the SA grows up also since the channels are sold. It is notable that the SEAL has the larger revenue than TASG, given the same reserve price, because our Global Auction algorithm ensures the higher selling price for each channel.

### 6.2.4 Social Welfare

We present the average social welfare in Fig. 8, varying the number of SUs in each SB. As the number of SUs increases, the number of winning SUs and each winner’s evaluation also increase at the same time. Hence, we obtain the higher social welfare with more SUs in each SB. Our DEAL outperforms SEAL and TASG since



**Fig. 8** Average social welfare vs. Number of SUs in each SB

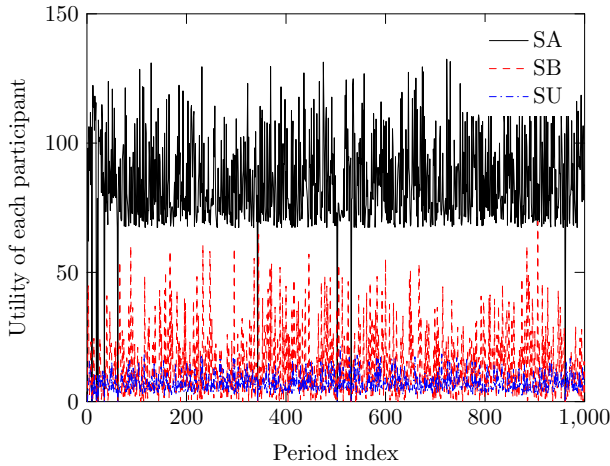
the DEAL sells the channels to SBs with higher probability, due to the parameter adaption. SEAL and TASG show the similar variation of the social welfare as the number of SUs increases, since they have the same auction algorithm between each SB and the serving SUs and pick up the winning SBs randomly. However, the SEAL has the larger social welfare than TASG since it chooses the SBs with higher group budgets as winners.

### 6.2.5 Variation of Utility over Time

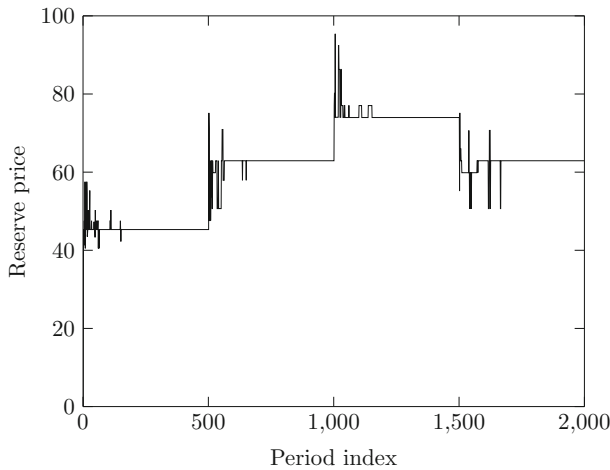
In Fig. 9, we demonstrate the utility of each participant (i.e., SA, SB and SU) in DEAL, according to the period index. We can see that the utility of the SA is always non-negative, which shows that our Global Auction algorithm is budget-balanced. The SA's utility sometimes drops to zero since the channels are not sold to any SBs. The utility of an SB is also non-negative, which implies the budget-balance of the Local Auction. Lastly, an SU shows the non-negative utility so that it is motivated to participate in the auction voluntarily.

### 6.2.6 Variation of Utility in Dynamic Environment

Until now, we have assumed that the number of SUs in each SB  $i$ , denoted as  $N_i$ , is fixed. We here evaluate the performance of our proposed scheme in the dynamic environment with time-varying number of SUs. Specifically, we change  $N_i$  every 500 periods;  $N_i$  is 10 for the periods from 0 to 499. It increases to 20 for the periods from 500 to 999 and increases further to 30 for the periods from 1000 to 1499. Finally,  $N_i$  steps down to 20 for the periods from 1500 and 1999. In Fig. 10, we present the variation of the adaptive reserve price. We can see that the reserve price fluctuates right after the number of SUs changes and then converges to a certain



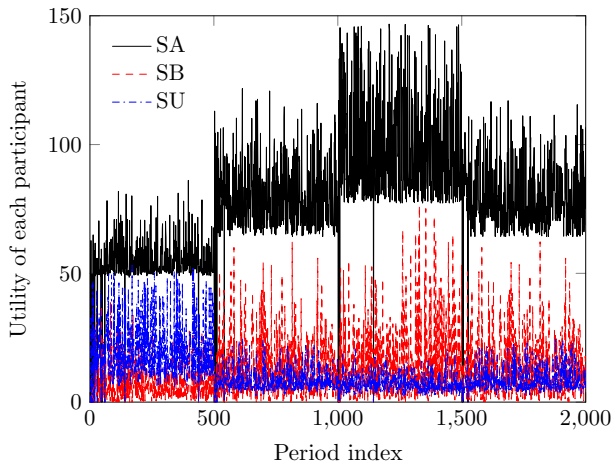
**Fig. 9** Dynamics of utilities of SA, SB and SU in DEAL



**Fig. 10** Convergence of the adaptive reserve price in a dynamic environment with time-varying  $N_i$

value quickly. Figure 11 shows the variation of utilities of SA, SB, and SU under the same scenario. We first note that all the participants have non-negative utility even in the dynamic environment. The utility of each SB grows up with more SUs since the bargaining power of SBs increases then. Similarly, the SA’s utility increases as the number of SUs increases. It can be explained as follows. When the number of SUs increases, bids from SBs increase as well. Accordingly, the SA raises the reserve price and obtains more revenue (or larger utility). On the contrary, the utility of each SU is inversely proportional to the number of SUs. If there are



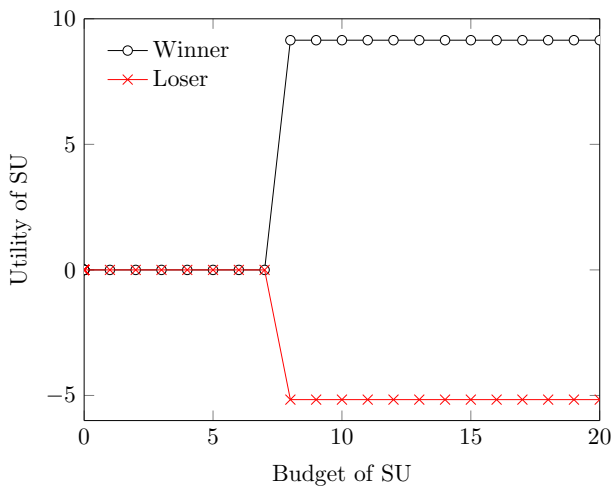


**Fig. 11** Dynamics of utilities of SA, SB and SU in a dynamic environment with time-varying  $N_i$

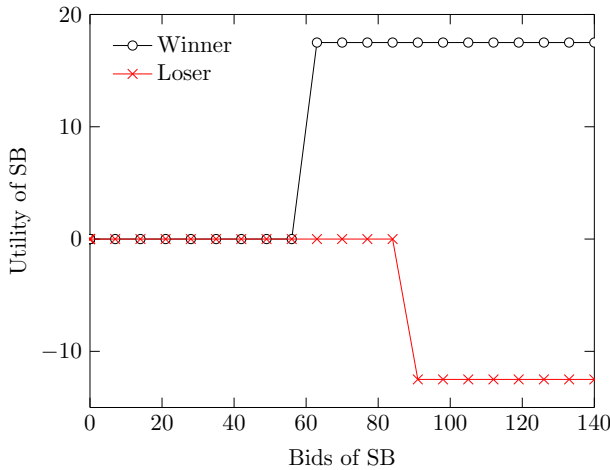
many SUs, more users are liable to be winners. Each SU’s benefit then decreases significantly since the purchased channel should be shared by all the winners.

6.2.7 Truthfulness of DEAL

We now show the truthfulness of our proposed scheme by simulations. We choose one winner and one loser arbitrarily in the Local Auction and in Fig. 12, we examine how their utilities change assuming that they submit the fake bids. We observe that the winning SU has positive utility with the true budget of 13. The utility of the winner does not change if the SU submits the fake budget larger than



**Fig. 12** Truthfulness of the Local Auction



**Fig. 13** Truthfulness of the Global Auction

the true value. However, if the SU sends the budget below 7.5, it loses the auction and its utility decreases to zero. Meanwhile, the loser has obtained the zero utility by submitting the true budget of 5. It cannot be a winner by sending the smaller budget. So, we suppose that it submits the fake budget higher than 7.5. Then, the SU can be a winner, but its utility decreases because the evaluation of the SU is not large enough to be a winner. In our DEAL, no participant, either winner or loser, can improve its utility by the fake bid.

We also choose one winner and one loser arbitrarily in the Global Auction and examine how their utilities change when submitting fake bids in Fig. 13. We need to mention that the winner has the true bid of 78 and the loser has the true bid of 73, respectively. We can see that the winner obtains the same utility for any bid above 60. However, if the SB submits the fake bid smaller than 60, it fails to purchase a channel and has zero utility. The loser has zero utility with the true bid since it does not qualify the reserve price. When the loser submits a fake bid less than 85, it still does not win the contention and obtains zero utility. The SB can be a winner with the fake bid larger than 85, but the utility decreases further since the cost is larger than the value of the purchased channel. This verifies that no SB can increase its utility by the fake bid in the Global Auction.

### 6.3 Key Results and Insights

From the simulation results, we can observe the followings. First, SEAL has better utility and social welfare than TASG, our main competitor. Both schemes have the same Local Auction algorithm. So the performance gain of SEAL comes from the proposed Global Auction algorithm. Second, DEAL outperforms SEAL significantly. We note that DEAL and SEAL share the same auction algorithms in both Local Auction and Global Auction but, differently from SEAL, DEAL adapts the parameters in the algorithms. We hence attribute the performance gap to the

parameter adaption of DEAL. Third, our parameter adaption method works well even in the dynamic environment with time-varying number of users. Fourth, DEAL has the desirable economic properties, as proved in Sect. 5. Every entity has non-negative utility, which shows the budget-balanced. An SU cannot increase its utility by the fake bid, which implies the truthfulness.

Considering all these, we propose that DEAL is an attractive solution to judiciously value and efficiently distribute the precious resources such as radio spectrum.

## 7 Conclusion

We have proposed an auction-based spectrum leasing framework, called DEAL, for the spectrum market of CR networks. Our proposed framework consists of two nested auction algorithms, where the outer auction between each SB and its serving SUs is called the Local Auction and the inner auction between the SA and SBs is called the Global Auction, respectively. Through the multi-stage auction design, our DEAL supports the group-buying so that the SUs with small budgets can purchase the (expensive) spectrum channels with the help of SBs. Our auction framework is budget-balanced and individually rational while ensuring the non-negative utility to all the sellers and buyers involved. Furthermore, it is truthful since each buyer cannot increase its utility by submitting a fake bid. The simulation results show that, in addition to these properties, our DEAL framework provides substantially higher utility (or revenue) to the SA and SUs than the previous TASG framework, due to the enhanced Global Auction algorithm and the dynamic adaption of parameters.

We have some remaining issues on the DEAL framework, which will be our future work. First, we have not considered the co-channel interference in the system model. When adjacent SBs are allocated the same spectrum channel, the channel quality degrades significantly due to the mutual interference. However, each SU cannot tell the auction result in advance such that its valuation on the channels may not be accurate. Second, DEAL is not truthful if SUs are allowed to collude in the auction. For example, suppose that all the SUs submit half of the true budget for channels. The SUs then enjoy larger utility since the channel cost goes down. However, this is not likely to happen practically since SUs are competitors to each other. Third, our parameter adaption method is not very scalable since it requires more memory and computational power in proportion to the number of emulated systems.

**Acknowledgements** This work was supported by the 2015 Yeungnam University Research Grant.

## References

1. Report of the spectrum efficiency working group: FCC Spectrum Policy Task Force (2002). <http://www.fcc.gov/sptf/reports.html>
2. Mitola, J., Maguire, G.Q.: Cognitive radios: making software radios more personal. *IEEE Pers. Commun.* **6**(4), 13–18 (1999)

3. Huang, J., Berry, R., Honig, M.L.: Auction-based spectrum sharing. *ACM/Springer Mob. Netw. Apps.* **11**(3), 405–418 (2006)
4. Gandhi, S., Buragohain, C., Cao, L., Zheng, H., and Suri, S.: A general framework for wireless spectrum auctions. In: *Proceedings of the IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'07)*, pp. 22–33 (2007)
5. Ji, Z., and Liu, K. J. R.: Belief-assisted pricing for dynamic spectrum allocation in wireless networks with selfish users. In: *Proceedings of IEEE Int'l Conference on Sensor, Mesh, and Ad Hoc Communications and Networks (SECON)*, pp. 119–127 (2006)
6. Wang, B., Ji, Z., Liu, K.J.R.: Primary-prioritized Markov approach for dynamic spectrum access. In: *Proceedings of the IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'07)*, pp. 507–515 (2007)
7. Zheng, H., Peng, C.: Collaboration and fairness in opportunistic spectrum access. In: *Proceedings of IEEE International Conference on Communications (ICC'05)*, pp. 3132–3136 (2005)
8. Keshavamurthy, S., Chandra, K.: Multiplexing analysis for spectrum sharing. In: *Proceedings of IEEE MILCOMM*, pp. 1–7 (2006)
9. Akyildiz, I.F., Lee, W.-Y., Chowdhury, K.R.: CRAHNS: Cognitive radio ad hoc networks. *Elsevier Ad Hoc Netw. J.* **7**(5), 810–836 (2009)
10. Liang, Y.-C., Zeng, Y., Peh, E., Hoang, A.T.: Sensing-throughput tradeoff for cognitive radio networks. *IEEE Trans. Wirel. Commun.* **7**(4), 1326–1337 (2008)
11. Yucek, T., Arslan, H.: A survey of spectrum sensing algorithms for cognitive radio applications. *IEEE Commun. Surv. Tutor.* **11**(1), 116–130 (2009)
12. Sahai, A., Hoven, N., Tandra, R.: Some fundamental limits in cognitive radio. In: *Proceedings of Allerton Conference on Communication Control and Computing*, pp. 1662–1671 (2004)
13. Ha, S., Sen, S., Wong, C. J., Im, Y., Chiang, M.: Tube: Time-dependent pricing for mobile data. In: *Proceedings of ACM SIGCOMM*, pp. 247–258 (2012)
14. Zhang, L., Weijie, W., Wang, D.: Time dependent pricing in wireless data networks: Flat-rate vs. usage-based schemes. In: *Proceedings of IEEE INFOCOM*, pp. 700–708 (2014)
15. Gerpott, T., Jakopin, N.: Firm and target country characteristics as factors explaining wealth creation from international expansion moves of mobile network operators. *Telecommun. Policy* **31**, 72–92 (2007)
16. Zhu, Y., Li, B., Li, Z.: Truthful spectrum auction design for secondary networks. In: *Proceedings of the IEEE INFOCOM*, pp. 873–881 (2012)
17. Zhou, X., Zheng, H.: TRUST: A general framework for truthful double spectrum auctions. In: *Proceedings of the IEEE INFOCOM*, pp. 999–1007 (2009)
18. Khaledi, M., Abouzeid, A.: Auction-based spectrum sharing in cognitive radio networks with heterogeneous channels. In: *Proceedings of IEEE Information Theory and Applications Workshop* (2013)
19. Myerson, R.B., Satterthwaite, M.A.: Efficient mechanisms for bilateral trading. *J. Econ. Theory* **29**(2), 65–281 (1983)
20. Zhou, X., Gandhi, S., Suri, S., Zheng, H.: eBay in the sky: strategy-proof wireless spectrum auctions. In: *Proceedings of the MobiCom* (2008)
21. Yang, D., Fang, X., Xue, G.: Truthful auction for cooperative communications. In: *Proceedings of the Twelfth ACM International Symposium on Mobile Ad Hoc Networking and Computing*, 9 (2011)
22. Wu, F., Vaidya, N.: SMALL: A strategy-proof mechanism for radio spectrum allocation. In: *Proceedings of the IEEE INFOCOM*, pp. 81–85 (2011)
23. Kasbekar, G.S., Sarkar, S.: Spectrum auction framework for access allocation in cognitive radio networks. *IEEE/ACM Trans. Netw.* **18**(6), 1841–1854 (2010)
24. Wang, X., Li, Z., Xu, P., Xu, Y., Gao, X., Chen, H.H.: Spectrum sharing in cognitive radio networks—An auction-based approach. *IEEE Trans. Syst. Man Cybern. B Cybern.* **40**(3), 587–596 (2010)
25. Rawat, D.B., Shetty, S., Xin, C.: Stackelberg-game-based dynamic spectrum access in heterogeneous wireless systems. *IEEE Syst. J.* **10**(4), 1494–1504 (2016)
26. Roy, A., Midya, S., Majumder, K., Phadikar, S., Dasgupta, A.: Optimized secondary user selection for quality of service enhancement of two-tier multi-user cognitive radio network: a game theoretic approach. *Comput. Netw.* **123**, 1–18 (2017)
27. Zheng, Z., Chen, G.: A strategy-proof combinatorial heterogeneous channel auction framework in noncooperative wireless networks. *IEEE Trans. Mob. Comput.* **14**(6), 1123–1137 (2015)

28. Feng, X., Chen, Y., Zhang, J., Zhang Q., Li, B.: TAHES: Truthful double auction for heterogeneous spectrums. In: Proceedings of 31st Annual IEEE International Conference Computer Communucation pp. 3076–3080 (2012)
29. Chen, Y., Zhang, J., Wu, K., Zhang, Q.: TAMES: a truthful double auction for multi-demand heterogeneous spectrums. *IEEE Trans. Parallel Distrib. Syst.* **25**(11), 3012–3024 (2014)
30. Wang, S., Derong, L.: A truthful multi-channel double auction mechanism for heterogeneous spectrums. *Wirel. Pers. Commun.* **77**(3), 1677–1697 (2014)
31. Zhao, F., Ji, S., Chen, H.: A spectrum auction algorithm for cognitive distributed antenna systems. *Ad Hoc Netw.* **58**, 269–277 (2017)
32. Pandit, S., Singh, G.: An overview of spectrum sharing techniques in cognitive radio communication system. *Wirel. Netw.* **23**(2), 497–518 (2017)
33. Lin, P., Feng, X., Zhang, Q., Hamdi, M.: Groupon in the air: a three-stage auction framework for spectrum group-buying. In: Proceedings of the IEEE INFOCOM (2013)
34. FCC Public notice News Media Information 202 / 418-0500. Advanced Wireless Services (AWS-3) auction summary (2015). [http://wireless.fcc.gov/auctions/default.htm?job=auction\\_summary&id=97](http://wireless.fcc.gov/auctions/default.htm?job=auction_summary&id=97)
35. Al-Ayyoub, M., Gupta, H.: Truthful spectrum auctions with approximate revenue. In: Proceedings of the IEEE INFOCOM, pp. 2813–2821 (2011)
36. Kushner, H.J., Yin, G.G.: Stochastic approximation and recursive algorithms and applications. Springer, Berlin (2003)

**Muhammad Shafiq** is a Ph.D. candidate in the Department of Information and Communication Engineering, Yeungnam University, Korea. He received an M.S. degree in Computer Science from the University Institute of Information Technology, Arid Agriculture University, Rawalpindi, Pakistan. He also received a Master's degree in Information Technology (IT) from the University of the Punjab, Gujranwala, Pakistan. He has been working with the Faculty of Computing and IT at the University of Gujrat, Gujrat, Pakistan as a faculty member since 2010, and formerly held the same position with the Department of Computer Science and IT, Federal Urdu University, Islamabad, Pakistan. His research interests include the design of spectrum management, routing, and medium access control protocols for mobile ad hoc networks, IoT, and cognitive radio networks.

**Jin-Ghoo Choi** received his Ph.D. degree from the School of Electrical Engineering and Computer Science, Seoul National University in 2005. From 2006 to 2007, he worked for Samsung Electronics as a senior engineer. In 2009, he was with the Department of Electrical and Computer Engineering in The Ohio State University as a visiting scholar. He joined the Department of Information and Communication Engineering in Yeungnam University as a faculty member in 2010. His research interests include performance analysis of communication networks, resource management in wireless networks, and wireless sensor network.