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A Review of the Metal Magnetic Memory Method

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Abstract

Metal magnetic memory (MMM) method is a non-destructive testing (NDT) technology which has potentials to detect early damage. A review is presented in this paper about the development of this method, including the theoretical studies of the magnetic/stress coupling effect, factors influencing the detection signals, the criteria for judging the damage state and defect identification. Directions of future research are also discussed.

Keywords Metal magnetic memory · Magnetic/stress coupling effect · Influencing factors · Stress concentration · Defect identification

1 Introduction

Russian expert Dubov proposed the metal magnetic memory (MMM) method, and then introduced its principles and applications in boiler, pressure vessel, pipelines, etc. to China [1–3], which has caused a strong reaction in the NDT field. Non-destructive methods such as visual examination (VE), ultrasonic testing (UT), radiography testing (RT), magnetic particle testing (MPT), and eddy current testing (ECT) [4–8], are mainly aimed at detection, interpretation, and measurement of already developed macro-defects in the ferromagnetic components. However, the MMM technique takes advantages of detecting micro-defects caused by stress concentration, which has significant meaning for preventing the early failure of the ferromagnetic components.

However, this technique is still limited in its mechanism research and quantitative detection due to its short history and various disturbance factors in testing. This paper summarizes the research progress of the MMM technique in the past decades. Some key points are discussed, including the physical fundamentals of this technique, factors influencing the detection signals, the criteria to judge the stress concentration, and defect identification. Finally, the future development trends about the MMM technique are discussed.

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2 Physical Fundamentals of the MMM Technique

The physical essence of the MMM technique is the magnetic/stress coupling effect, which is also called the magnetomechanical effect. The magnetic/stress coupling effect is a phenomenon that the stress-induced permeability variation will lead to the variation of the surface magnetic intensity of ferromagnetic materials. In fact, the conceptual description of the interaction between the stress and magnetism may date back to more than half a century ago. In 1945, Bozorth [9] reported that the length of ferromagnetic materials will change as they are magnetized, which is called the magnetostriction effect. And the changes of magnetization occur in ferromagnetic materials when small stresses are applied or removed, which is called the magnetomechanical effect. Later on, according to Le Chatelier's principle, Cullity [10] proposed that the rate of change of magnetostriction with the magnetic field at constant stress is equal to the rate of change of magnetic induction with the stress at constant field. However, as Jiles [11] pointed out, this equation is quite misleading as a description of the magnetomechanical effect in ferromagnetic materials, because the magnetization process is hysteretic and therefore inherently irreversible in nature. Brown [12] and Burgel [13] proposed a theory that the change in magnetization due to domain wall motion should obey Rayleigh's law at low magnetizations. In fact, even at low fields, the data of Lliboutry [14] was not consistent with this theory, because the magnetization caused by tensile and compressive stress is asymmetric, which should be taken into consideration. Craik and Wood [15] concluded with the statement that the results caused by stress cannot be reconciled with any theory based simply on the movement of existing domain walls. The theory of magnetization under stress must take the discontinuous changes in domain structure into account. Birss [16] also found that the changes in the domain wall pinning energies and irreversible changes in domain structure cannot be described by Brown's theory.

More systematic researches of the magnetomechanical effect were presented by the research group of Jiles. In 1984, Jiles and Atherton [17] proposed the concept of "law of approach", which states that the change in flux density is proportional to the difference between the anhysteretic value of flux density and the initial value, and the application of stress no matter compressive or tensile will cause the magnetization to approach the anhysteretic curve. Later on, the group of Jiles [11, 18] calculated the reversible component and the irreversible component of magnetization respectively, and then presented the Jiles-Atherton model of ferromagnetic materials at uniaxial stress based on the law of approach. This model has been proved by Pitman [19], Maylin [20] and Squire [21] successively, and is widely used for describing the changes in magnetization of ferromagnetic materials when subjected to an applied uniaxial stress. In subsequent studies, Jiles and Li proposed an improved model which considers the effect of different kinds of crystal anisotropy in materials [22–24], and a new approach to investigate the magnetomechanical effect with the application of the Rayleigh law [25–27]. Sablik [28–32] modified the Jiles-Atherton model by elucidating the variation of magnetic properties with the grain size and the dislocation density, and established the Jiles-Atherton-Sablik(J-A-S) model for modeling plastic deformation effects in steel. In addition, other models [33–38] (e.g. the homogenized energy model) were established to characterize the stress-induced magnetic behavior of ferromagnetic materials.

The classical Jiles-Atherton-Sablik model has limitations in describing the asymmetry in magnetic behavior under tensile and compressive stress. The group of Xu [39, 40] modified the model by the incorporation of a stress demagnetization term, a stress-dependent domain coupling coefficient, a variable pinning coefficient, and stress-dependent saturation magnetostriction. The modified model provides a much better description of asymmetrical magnetic properties under tension and compression. Li [41], Wang [42] and Shi [43] also proposed modified models to predict the magnetic behavior for the plastic deformation case and the compression case. However, the group of Liu [44, 45] compared the existing magneto-plastic models established by different researchers and found certain mistakes, such as the mixing of anhysteresis magnetization and magnetization, considering the irreversible magnetization energy as actual total magnetization energy, etc. Figure 1 shows the comparison of the different results obtained from the models and the experiments. By comparing Fig. 1a, b, one may observe that the classical J-A model [11] is not consistent with the experimental results of Craik and Wood [15], especially in the compression-release process. Both of the modified models revised the classical J-A model in the compressive stage. Compared to the model of Xu [39] (Fig. 1c), the model of Liu [45] (Fig. 1d) describes the experimental phenomenon more accurately.

In a word, the essence of the magnetomechanical effect is the magnetic/stress coupling effect under various external fields. The weak magnetic testing method such as the MMM technique mainly focuses on the research of the magnetic/stress coupling effect under the ambient geomagnetic field. Therefore, the research of traditional magnetomechanical effect is the theoretical basis of the research of weak magnetic testing methods.

3 Recent Research of Weak Magnetic Testing Methods

Figure 2 shows the testing principle of the weak magnetic testing [46]. One may observe that the weak magnetic testing is consisted of microscopic structural changes and macroscopic external detection. Under the effect of the geomagnetic field and the mechanical load, magnetic signals which can be measured change due to the movement of the domain. The distribution of the magnetic signals is related to the stress state and the damage degree of ferromagnetic materials. Therefore, the stress state and the damage degree can be evaluated by measuring the magnetic signals on the surface of the materials. Generally, the magnetic signals in the defect area are shown in Fig. 3. The magnetic signals present a linear distribution along the longitudinal axis of ferromagnetic components when there is no defect [47–49]. However, the magnetic signals change abruptly in the stress concentration zones caused by the defect, where the tangential signal H_t reaches the maximum and the normal signal H_n changes polarity and has a zero value [50–52]. Many studies have been carried out to investigate the microscopic structural changes induced by the applied stress and to explain the distribution of the magnetic signals.

3.1 Microscopic Structural Changes

According to ferromagnetic physics, the magnetic changes of ferromagnetic materials can be attributed to the change of magnetic domain structure. The applied stress will affect the direction and structure of domains and generate net magnetic moment on the surface [53]. Most of the domains are lamellar before loading, and the labyrinth domains are appearing and the number of the labyrinth domains increases as

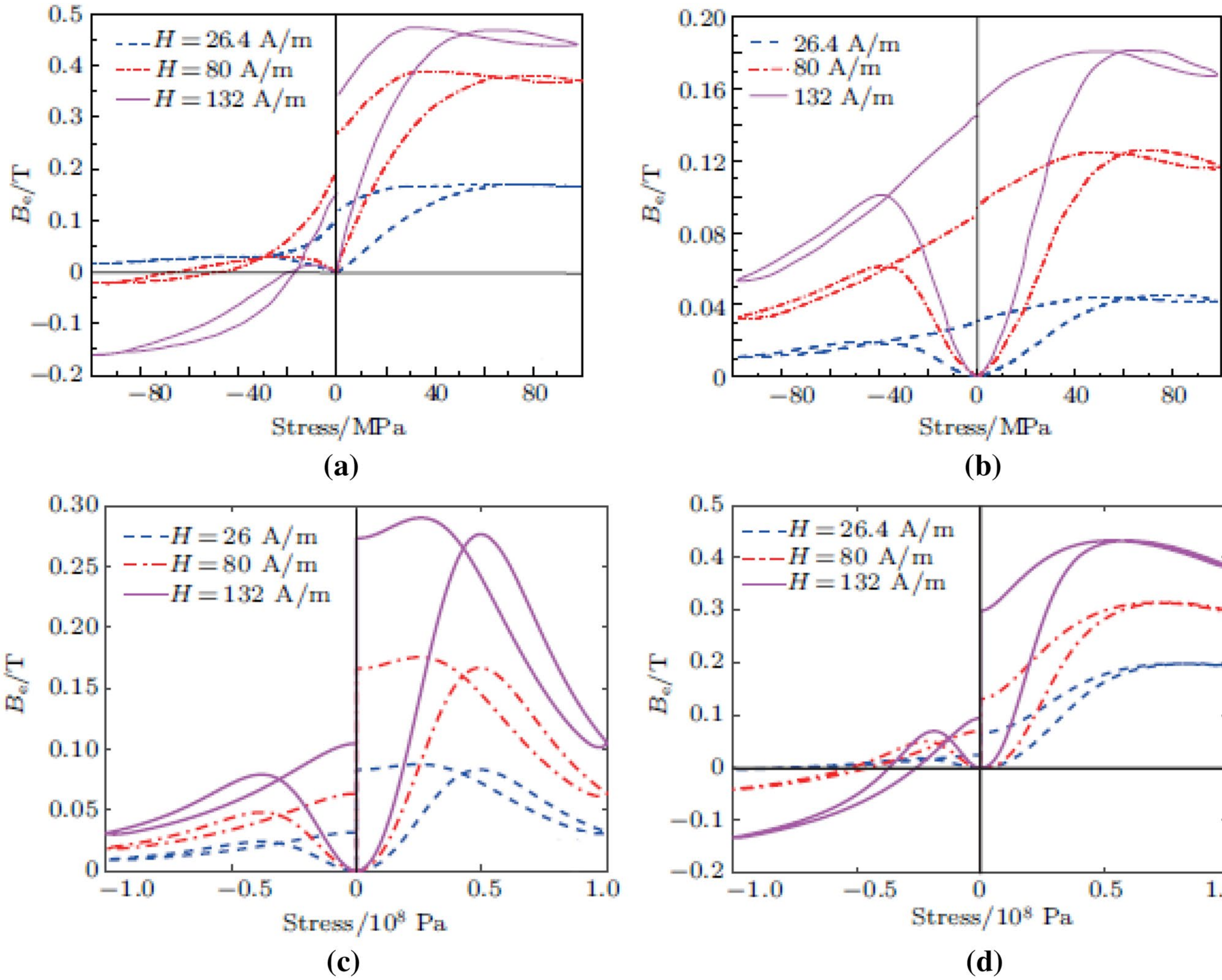

Fig. 1 Comparison of the different results obtained from the models and the experiments: **a** experimental results of Craik and Wood [15]; **b** Jiles-Atherton model [11]; **c** modified J-A model of Xu [39]; **d** modified J-A model of Liu [45]

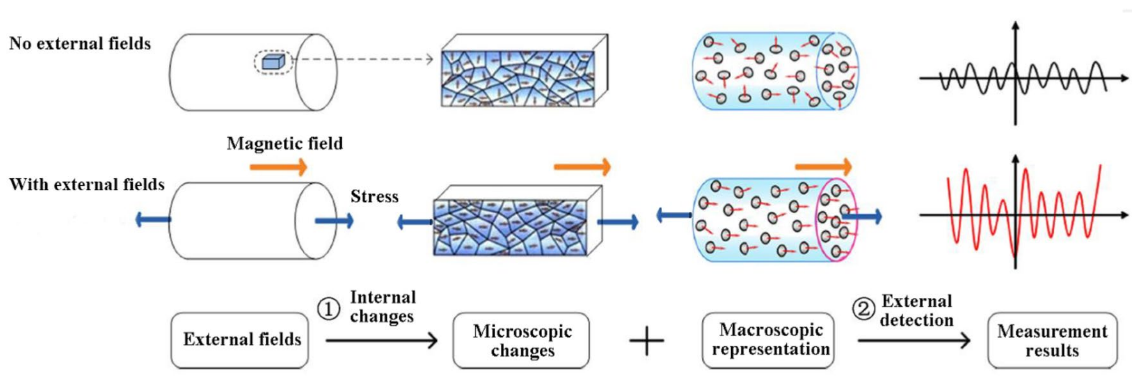

Fig. 2 Testing principle of the weak magnetic testing [46]

the load increases [54]. The magnetic signals change with the movement of magnetic domains. Song [55–57] further investigated the mechanism of domain motions by fracture mechanism and dislocation theory and proved that the weak

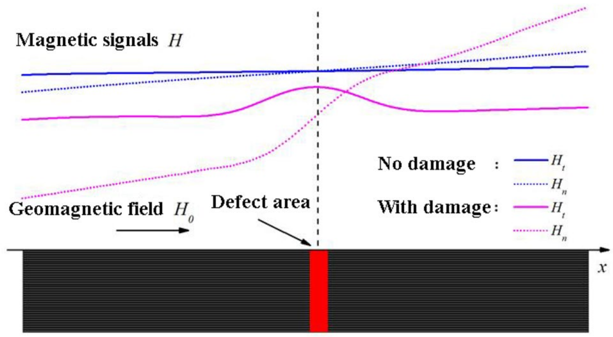

Fig. 3 Weak magnetic signals in the defect area

magnetic testing method can be used to detect the early damage.

Many studies have been carried out to investigate the relationship between the magnetic/stress coupling effect and the microscopic structure of ferromagnetic materials based on different theories, such as the principle of equivalent stress magnetic field, the law of electromagnetic induction, the law of conservation of energy, etc [58–63]. Different theoretical models were established, such as the linear magnetic-charge model, stress-magnetization coupled model, etc., to predict the basic characteristics of magnetic signals in the defect zone [64–68]. The analytical expressions of the magnetic signals for different defects obtained by Shi [69, 70] provide a possibility for quantitative inspection of the defect. However, a lot of researches only target the surface defects, further work is needed to investigate the relationship between the magnetic signals and the buried defects [71].

3.2 Distribution of the Magnetic Signals

Zhou [72] investigated the effective field theory and gave an explanation for the phenomenon that at the stress concentration zone, the tangential magnetic field has a maximum value and the normal magnetic field acquires zero value. The slope of the normal magnetic field increases continuously in the elastic stage, but decreases in the plastic stage. This may be induced by the dislocation of magnetic domains and the residual stress [73–75]. In the fatigue test, the shapes of magnetic curves will gradually reach a stable stage as the number of cycles increases, which is consistent with the J-A model [76, 77].

Additionally, Li [78, 79] and Zheng [80] established magnetomechanical models to simulate the spatial magnetic field distributions around the defect. The comparison for the theoretical results from different constitutive relations and experimental data [51] is given in Fig. 4 [80]. Compared to other models, the model of Zheng is more consistent with the variations of normal magnetic signals. As a result, the theoretical analysis for the stress concentration proposed a possibility for the early diagnosis of ferromagnetic components using the magnetic memory method.

4 Factors Influencing the Magnetic Signals

The MMM technique is a weak magnetic testing method, many factors influencing the variations of the magnetic signals have been investigated.

4.1 External Fields

The geomagnetic field is the main external field in the MMM technique. There are two views on the effect of the geomagnetic field on magnetic signals. Li [81] held the view that the geomagnetic field has little effect on the formation of magnetic abnormalities, whereas Ren [61] and Zhong [82] found that the geomagnetic field is a driving source in the process of generating weak magnetic signals.

Recently, a more unified view on the effects of the geomagnetic field has been proposed. Li [83] and Yu [84, 85] has proved by experiments that the geomagnetic field influences the magnetic intensities rather than the magnetic distributions. The numerical results of Yao [86] further verified this conclusion. He suggested that one may eliminate the effect of the geomagnetic field using the RMF gradient parameters.

Huang [87, 88] found that other external fields such as the artificial exciting magnetic field only influence the magnetic intensities. Figure 5 [87] shows the variations of the normal components of the magnetic signals excited by the applied magnetic fields with different intensities. One may observe that the values of $H_p(y)$ increase as the magnetic field intensity increases, and the variations of $H_p(y)$ are similar when the magnetic field intensities are different. Therefore, a certain applied magnetic field can help to strengthen the magnetic signals by highlighting its characteristic values and improve the accuracy of weak magnetic detection [89–92]. Additionally, the external field and the applied stress will both influence the magnetic distributions. The final distributions are determined by the stronger one [93, 94].

4.2 Loading Type

The loading types include static tension loads, fatigue loads, compression loads, etc. The group of Xu [95–97] found that under static tension loads, the magnetic field curves demonstrate different behaviors in the elastic stage as well as in the plastic stage. However, the magnetic field curves of different cycles are similar under fatigue loads. The effect of compression loads on magnetization is much smaller than that of tension loads [98], which is the reason that few studies were carried out to investigate the

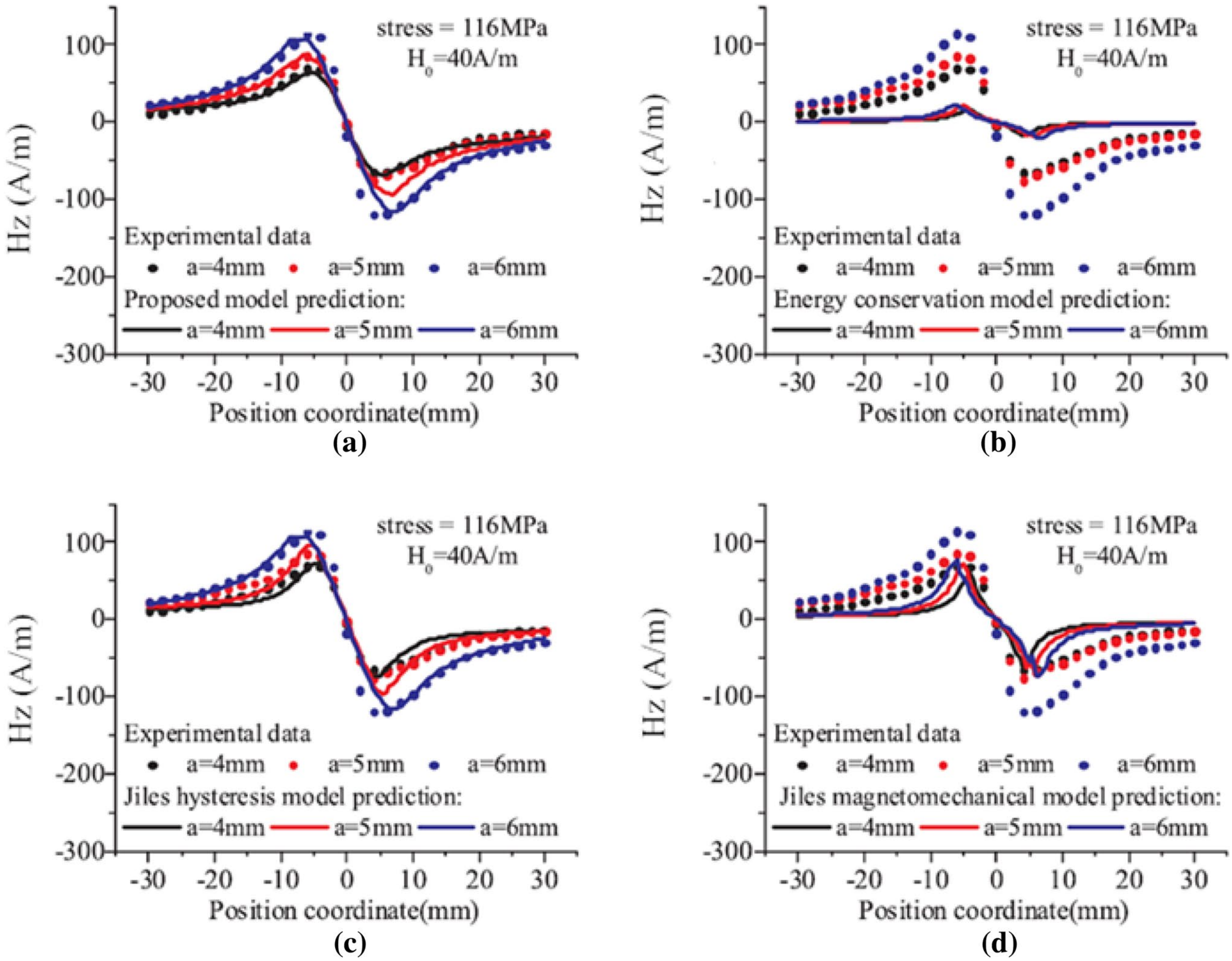

Fig. 4 Comparison of different results obtained from models and experiments: **a** the proposed model by Xiaojing Zheng [80]; **b** the energy conservation model [63]; **c** the Jiles hysteresis model [11, 17, 18]; **d** the Jiles magnetomechanical model [52, 71]

metal magnetic memory effect under compression loads. The group of Xu [99–101] did a series of experiments to investigate the variations of magnetic fields induced by different loading types, e.g. static tension, rotary bending fatigue tests and three-point bending tests, and found that magnetic signals are different under these loads. e.g. under static tension loads, magnetic signals increase linearly and then vary in small ranges, which means the specimen is near the yield limit. Sun's three-dimensional stress-induced magnetic anisotropic constitutive model display the same feature and trend [102]. The types of loads influence the magnetic parameters in different directions, such as magnetic susceptibility or permeability, which causes the couple of the components of magnetization and magnetic field in different directions.

4.3 Chemical Compositions of Ferromagnetic Materials

Bao [49] studied the relationships between the stress and the magnetic signal in three different material steels, Q235, 45# and Q345, subjected to tensile stress. The results showed that different materials demonstrate different characteristics. The experiment results of Dong [103] also proved this viewpoint. Zhang [104] proposed that, in the yielding and necking stages, the specimen materials may only influence the magnetic intensities, but cannot change the shape and the distribution of magnetic field curves.

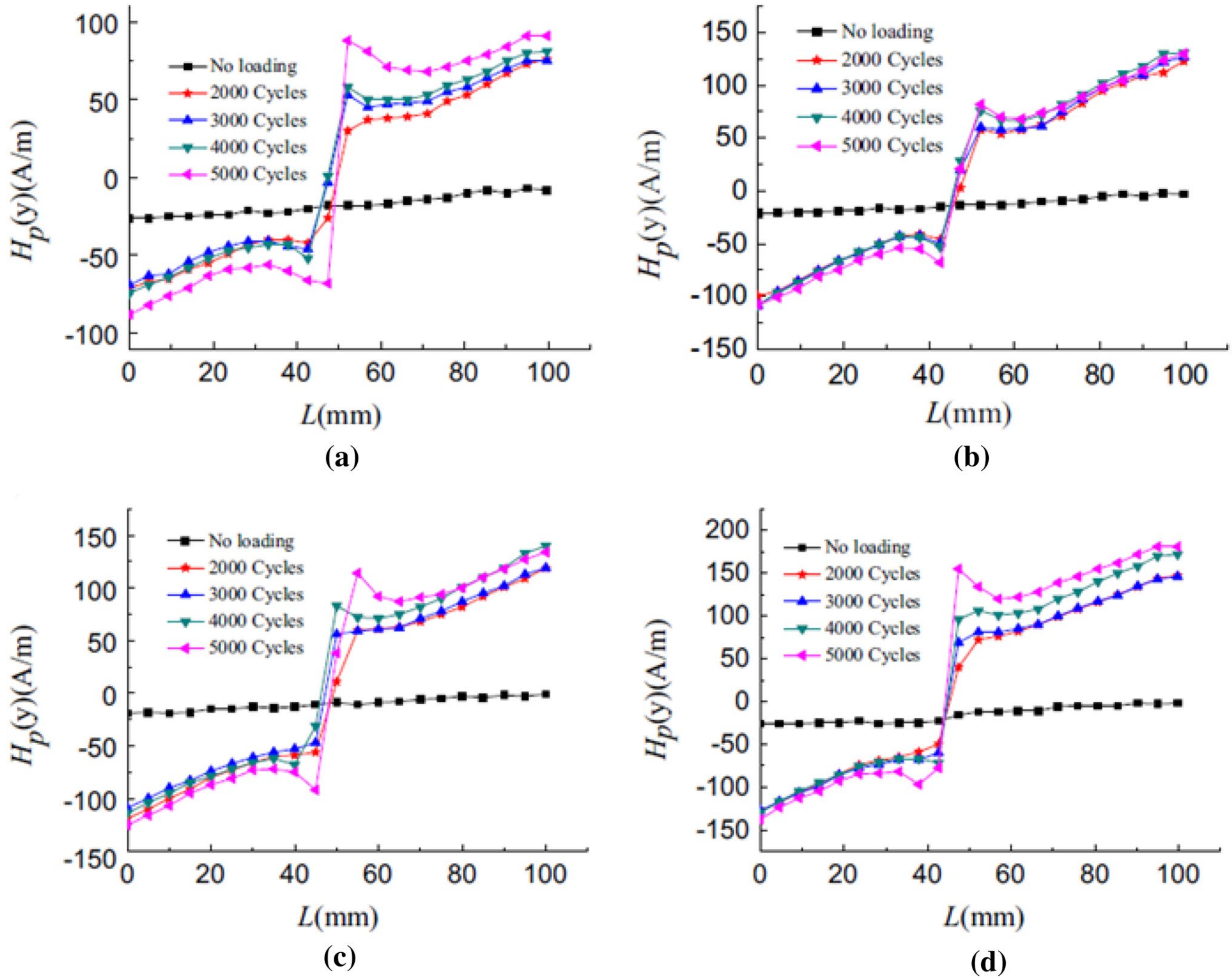

Fig. 5 Variation of the normal component of the magnetic signals excited by the applied magnetic field with different magnetic field intensities: **a** H_B; **b** 1.5 H_B; **c** 2 H_B; **d** 2.5 H_B (the applied magnetic field $H_B = 200$ A/m) [87]

4.4 Geometry and Dimensions of the Defect

The group of Zheng [71, 80] and the group of Wang [64, 65, 86] presented numerical studies to investigate the influences of the defect length, width and depth on the surface magnetic signals. They found that the amplitude of magnetic signals increases with an increase in the defect length, width or depth. Ding [105] established a model of the stress field around cracks and the variation of the magnetic field, and presented the rules of the variation of magnetic signals with different crack depths, widths, trends, etc. He found that the geometric features of the defect only influence the magnitude of the magnetic field.

4.5 Initial Remanent States

Leng [106] measured the normal magnetic field intensities on the undemagnetized and the demagnetized specimen surfaces. The results are shown in Fig. 6 [106]. As can be seen in Fig. 6a, the H_y component of the sample with no demagnetization tended to remain stable and regular as tension increased. In contrast, the H_y component of the demagnetized specimen became spread at higher stress level in the plastic stage, as shown in Fig. 6b. The initial remanent states have a great effect on magnetic field variations induced by the stress. And the initial remanent states may eliminate the essential characteristics of the magnetic signals produced by the early damage. Gorkunov [107] also pointed out that

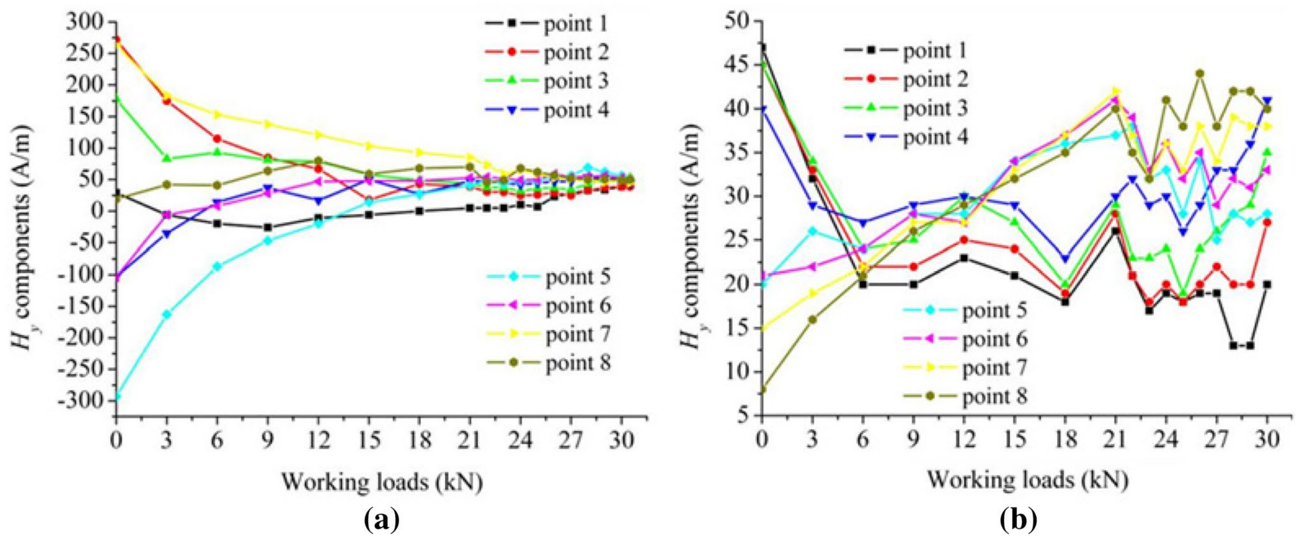

Fig. 6 Effect of initial remanent states on the weak magnetic signals: **a** the specimen with no demagnetization; **b** the demagnetized specimen [106]

the stability of the testing results is significantly governed by the initial remanence state(magnetic prehistory) using the MMM technique.

4.6 On-Line or Off-Line Testing

A series of tensile experiments, including on-line testing and off-line testing, were carried out by Jian [108]. The results showed that there is no obvious correlation between the magnetic gradient and the tensile stress in on-line testing. However, if the specimen was taken off from the testing machine, the measured magnetic gradient varied linearly with the prior maximum stress. Xu [109] and Ren [110] also thought the testing results are better in off-line testing than in on-line testing. However, a more direct and accurate relationship between the stress and the magnetic field can be investigated using on-line testing, if the influence of testing machine grips can be eliminated.

4.7 Loading Speed

Bao [111, 112] investigated the effect of the loading speed on the stress-induced magnetic behavior of a ferromagnetic steel. The results are shown in Fig. 7. One may observe from Fig. 7a that the loading speed imposes strong impact on the variation of the magnetic field signals. The most visible

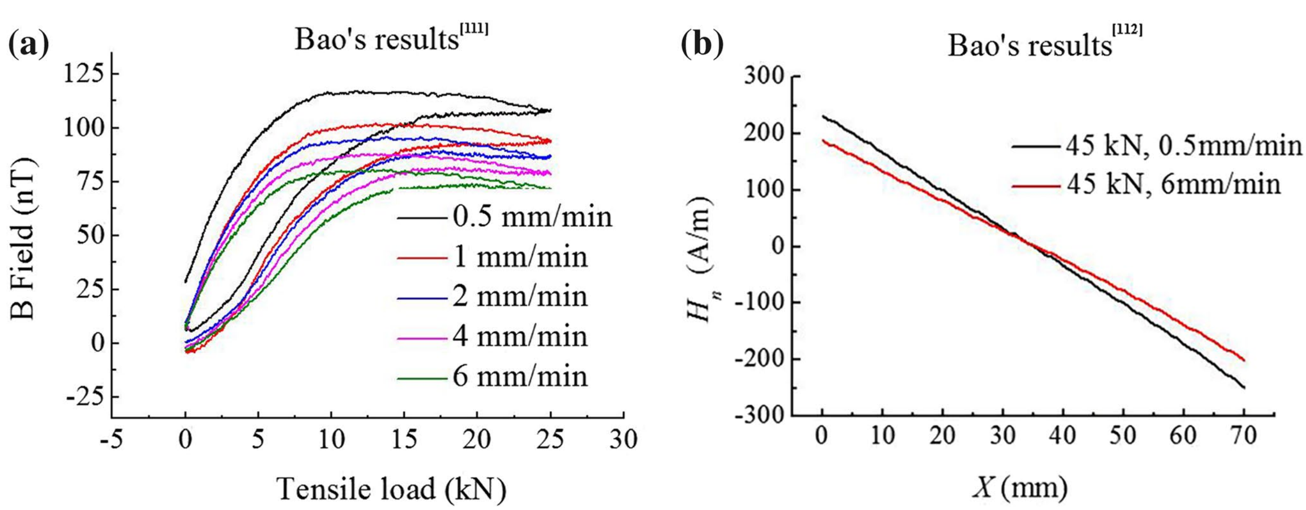

Fig. 7 Effect of loading speed on the weak magnetic signals

feature is that the amplitude of the magnetic field decreases as the loading speed increases from 0.5 mm/min to 6 mm/min. In Fig. 7b, when the tensile load is 45 kN, the normal magnetic curves rotate anticlockwise around the centre of the specimen as the loading speed increases from 0.5 mm/min to 6 mm/min. These results demonstrate that it is meaningful to study the effect of loading speed on the induced magnetic field variation of steel. Therefore, the classical J-A model theory of the magnetomechanical effect should be amended by considering the effect of loading speed.

4.8 Lift-Off Value

Yu [84] found that the lift-off values of magnetic sensors affect both the magnetic intensities and their gradients, but the positions of the peaks of the magnetic intensity curves do not change. The numerical results of Shi [80] and Yao [86] also proved this viewpoint. The lift-off value has a significant influence on the intensity of RMF signals. And if the lift-off value is greater than 5 mm, the change of lift-off values has almost no effect on magnetic memory signals. However, the influence of the lift-off values is depended on the defect size.

4.9 Temperature and Manufacturing

Huang [113] proposed a quantitative relationship among temperature, applied stresses, and spontaneous magnetic signals in ferromagnetic steels, and found that the mean value of magnetic signals decreases with the increase in temperature. Dong [114] investigated the variations of metal magnetic memory signals of 18CrNiWA steel after the processes of forging, milling, grinding and heat treatment. The results showed that the magnetic signals will be influenced by the different external loads induced by the machining process.

5 Criteria for Judging the Damage State

5.1 Variations of Magnetic Fields Under Different Stress States

Current research has found that the magnetic fields can be used to characterize stress states. The variations of magnetic signals change accordingly with the changes of stress states. Yu [115] and Dong [116] found that when the specimen is not treated by annealing or demagnetization, the magnetic intensities decrease gradually in the elastic stage and then stay stable in the plastic stage, finally changing rapidly after fracture. Leng [117] further investigated the variation regularities of the magnetic field intensities and pointed out a correlation coefficient to predict whether the specimen is in a certain critical or limit condition. It can be concluded that there is a qualitative relationship between the magnetic field and the stress. However, it is difficult to establish a quantitative relationship of them because of the numerous interference factors in weak magnetic field excitation [118, 119]. When the smooth specimen is treated by annealing or demagnetization, the normal magnetic signals on the surface can be considered as a linear distribution along the measuring line, and the changes of its slope can be used to characterize the changes of the stress state. Figure 8 shows the relationship between the slope of normal magnetic signals and the applied stress. One may observe that the relations between the slope of normal magnetic signals and the applied stress are quite different in the elastic stage and in the plastic stage. The experimental results of Dong [75, 120], Guo [48] and Bao [49, 121, 122] showed the slope of normal magnetic field curve increases linearly as the applied stress increases in the elastic stage, and reaches a local maximum near the yield limit, then decreases in the plastic stage. However, the experimental results of Shi [123] and Liu [124] showed that with the increase of tensile stress, the slope of normal magnetic field curve first increases slowly and then increases rapidly in the elastic stage. The results of Dong [95] showed the slope of the normal magnetic field curve continues to increase in the plastic stage, which is different from the results of Bao [49]. The slope of the normal magnetic field curve is effective in differentiating the elastic and plastic stages. Additionally, in the elastic stage, it seems that the applied stress is highly related to the slope coefficient which provides a promising tool to analyze the stress in ferromagnetic materials. However, in the plastic stage, the results from these four figures are quite different. More work is needed to investigate the relationship between magnetic signals and stresses in the plastic stage.

The above researches focus on the normal component of magnetic signals. There is no clear conclusion about which component of magnetic signals is more related to the applied stress. Roskosz [125] measured three components of RMF signals on the specimen surface including tangential component perpendicular to the load direction $H_{s,x}$, tangential component parallel to the load direction $H_{s,y}$ and normal component $H_{n,z}$. And he found that the tangential component parallel to the load direction is best correlated with the stress level, which is consistent with the results of Wilson [73]. Additionally, Roskosz [126–128] also proposed that the values and the distributions of the RMF gradients show a good correlation with the values and distributions of residual stress, both qualitatively and quantitatively.

5.2 Stress Concentration

The metal magnetic memory method is mainly used for the detection of stress concentration. Dubov [1, 2, 129] pointed out that in the stress concentration zones, the tangential

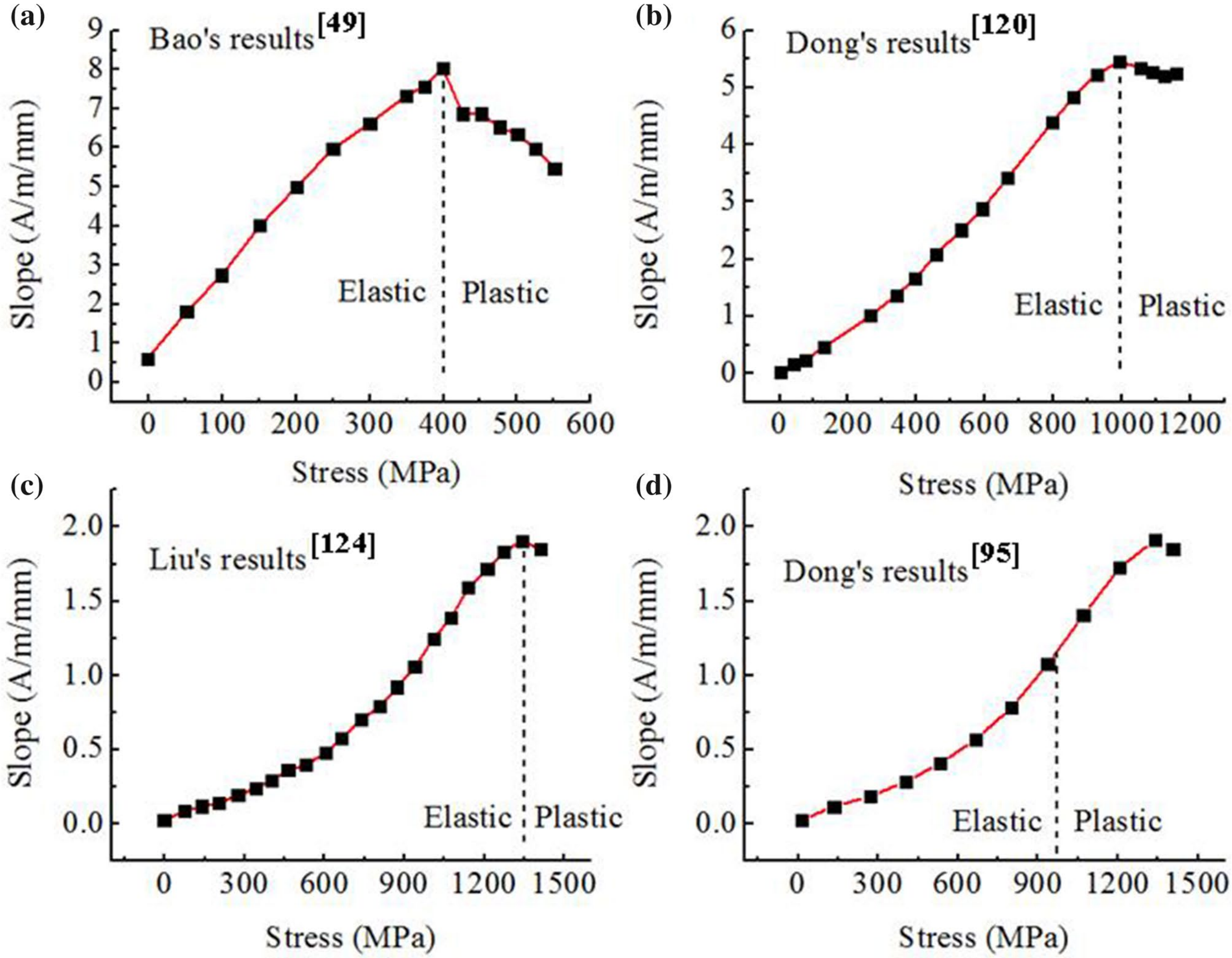

Fig. 8 Relationship between the slope of normal magnetic signals and the applied stress

magnetic signal reaches a maximum value, and the normal magnetic signal has a zero value. The experimental results of Zhang [130], Ren [131] and Dai [132] indicated that the MMM signal has an apparent relationship with the damage of components, which can be used to detect the hidden damage in the ferromagnetic components. However, the normal magnetic signal may not necessarily have a zero value in the stress concentration zones. Therefore, the diagnosis of stress concentration positions may be incorrect if only considering the zero value of the normal magnetic signals [133, 134], especially during the on-line testing [109]. Many researchers have proposed new methods to detect stress concentration zones, as follows:

- Using the zero point of the curve obtained by the normal magnetic signal under loads minus the one under no load [124].
- Using the position of the maximum slope of the normal magnetic field curve [124].
- Combining the peak-peak and gradients of the magnetic signals [135].
- The location of the close area in a Lessajou figure obtained by the magnetic signals is the place of the stress concentration zone [136].
- Analyzing the distribution characteristics of the magnetic gradient tensor modulus and the gradient local wave number [137].

The above studies are mainly used to find the locations of the stress concentrations. The possibilities for evaluating the stress concentration degree on the basis of the magnetic signals have also been widely investigated. Hu [138] and Dong [139] found that the magnetic gradient K increases as the stress concentration factor increases, which can be used to characterize the stress concentration degree of ferromagnetic materials. Huang [68] proposed that the ratio of the maximum normal magnetic gradient K_{max} to the average normal magnetic gradient K_{std} may be used to describe the stress concentration

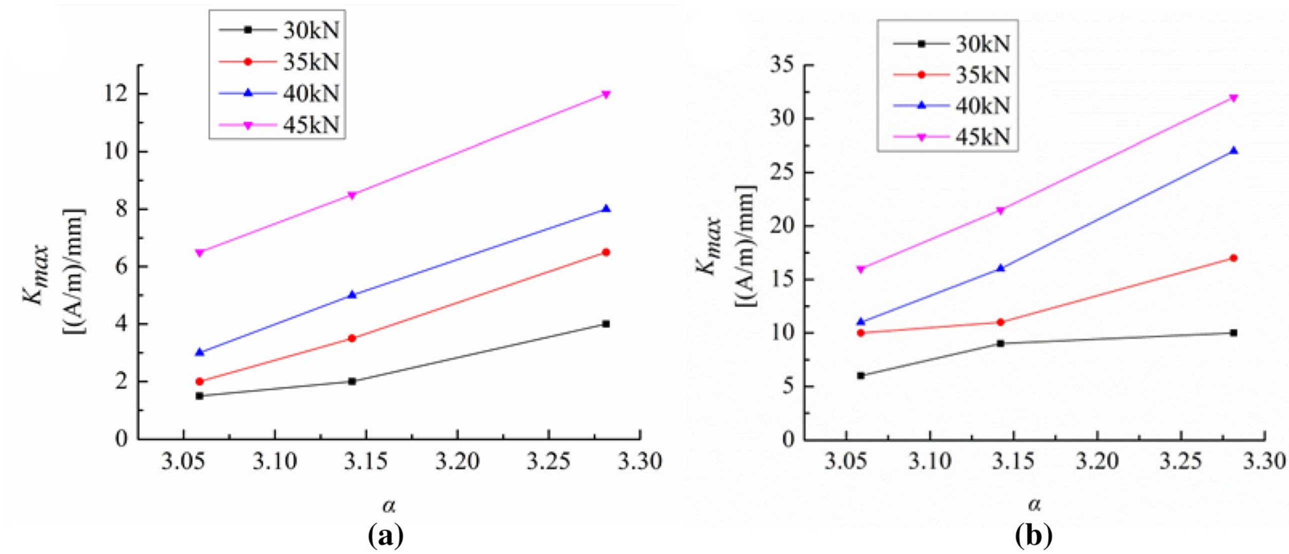

Fig. 9 Relationship between the maximum gradient of the weak magnetic signals and the stress concentration factor: **a** tangential gradient component; **b** normal gradient component

degree quantitatively. The group of Bao have carried out a lot of studies to investigate the quantitative inspection of the stress concentration in ferromagnetic steels using the RMF measurements. Figure 9 shows the relationship between the maximum gradient of the weak magnetic signals and the stress concentration factor [140]. One may observe that the maximum gradients of tangential and normal components both increase as the stress concentration factor increases. Interestingly, the maximum tangential magnetic gradient is entirely proportional to the stress concentration factor as the tensile load is above 35 kN, and the maximum normal magnetic gradient is almost proportional to the stress concentration factor when the tensile load is above 40 kN. It can be assumed that there exists a linear relationship between the maximum gradient and the stress concentration factor, which is considered to be tenable only when the applied tensile load exceeds a certain value. Additionally, according to the definition of the stress concentration factor, Bao [141] proposed a magnetic concentration factor α_m, which is the ratio of the abnormal gradient of the normal magnetic field near the defect to the magnetic gradient of the normal magnetic field away from the defect. α_m has good numerical stability and is related to the stress concentration factor, which can be used to characterize the stress concentration degree without identifying its specific stress state.

6 Defect Identification

The MMM technology can be used for early diagnosis of the microscopic damage and the stress concentration, but it is difficult to reconstruct the defect profile. The defects in engineering are usually irregular and it is difficult to characterize the defects by a single parameter. The length, width, depth and location of defects influence the magnetic signals simultaneously, which is the main obstacle in the defect identification. Chen [142] and Bao [143] defined RMF characteristic parameters to capture the location and the shape of defects. Yao [144] presented a finite element analysis about the RMF signals under different plastic-zone sizes, lift-off values and testing directions. Some RMF parameters, e.g. the tangential gradient component and the normal gradient component, were defined to image the shape of the plastic zone, as shown in Fig. 10. One may observe that the RMF parameters of the tangential component and the normal component can both capture the shape of the plastic zone, especially when the lift-off value is small enough. By comparing Fig. 10a, b, it can be seen that the tangential gradient component images the plastic zone more accurately than the normal gradient component. This suggests an effective defect identification method with the MMM technique. More systematic experimental studies and numerical simulations should be performed to uncouple the relationship between the defects and the magnetic parameters.

7 Conclusions

This paper outlines the research progress of the MMM technique in the past decades, including the theoretical studies of the magnetic/stress coupling effect, factors which can influence the detection signals, the criteria for judging the damage state and defect identification. Some attractive advantages and key problems of the MMM technique are summarized. However, the theoretical studies of the

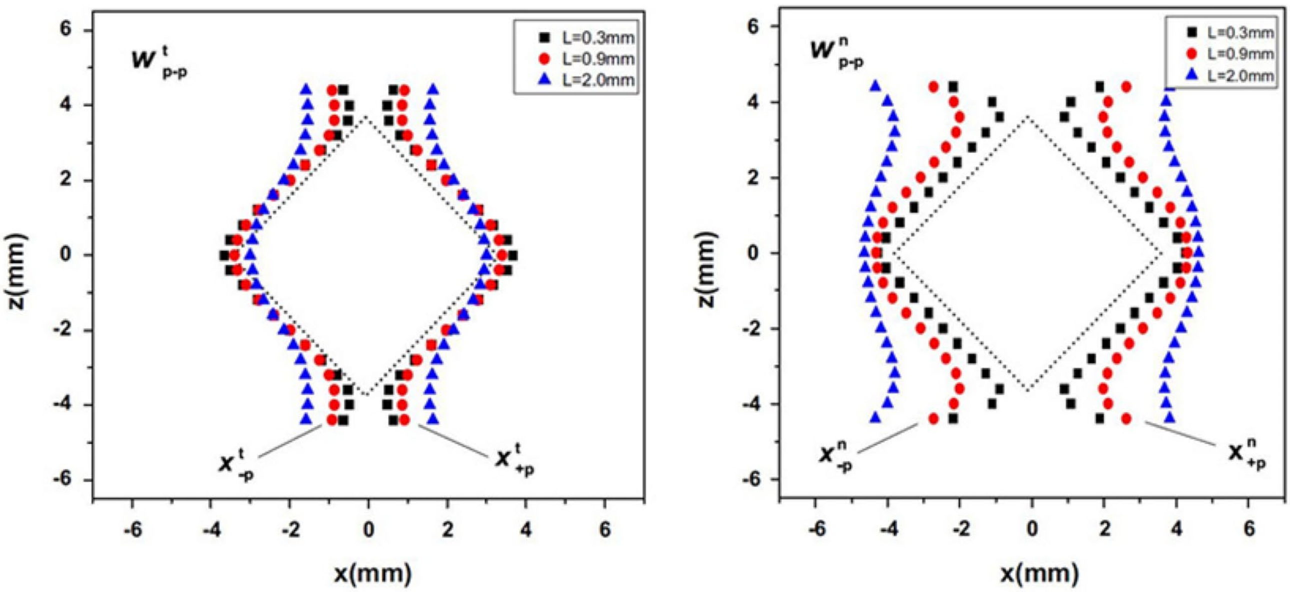

Fig. 10 Surface plot of the diamond plastic zone based on the weak magnetic gradient signals: **a** tangential gradient component; **b** normal gradient component [144]

magnetic/stress coupling effect are incomplete and more work is needed to improve the magnetic/stress coupling model. In the current research, few models could describe the different magnetization features in tension-release and compression-release processes accurately. Further work should be aimed at optimizing the related parameters to make the model more accurate. The influence of temperature could be added to improve the model. And more basic parameters (e.g. magnetic permeability) may be considered to characterize the stress-strain states of ferromagnetic materials quantitatively. Many studies have been carried out to investigate the relationship between the magnetic field and the stress in the stress concentration area. However, there are few studies on defect identification based on the metal magnetic memory method, for the magnetic signals are influenced by the coupling of stress states, defect shapes, defect depths, material types, etc. Defect identification may be a focus in future research.

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