

Decision Making Based on Fuzzy Aggregation Operators for Medical Diagnosis from Dental X-ray images

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Abstract Medical diagnosis is considered as an important step in dentistry treatment which assists clinicians to give their decision about diseases of a patient. It has been affirmed that the accuracy of medical diagnosis, which is much influenced by the clinicians' experience and knowledge, plays an important role to effective treatment therapies. In this paper, we propose a novel decision making method based on fuzzy aggregation operators for medical diagnosis from dental X-Ray images. It firstly divides a dental X-Ray image into some segments and identified equivalent diseases by a classification method called Affinity Propagation Clustering (APC+). Lastly, the most potential disease is found using fuzzy aggregation operators. The experimental validation on real dental datasets of Hanoi Medical University Hospital, Vietnam

showed the superiority of the proposed method against the relevant ones in terms of accuracy.

Keywords Decision making · Dental X-Ray image · Dental diagnosis · Fuzzy operators · Medical diagnosis

Introduction

Medical diagnosis is considered as an important step in dentistry treatment which assists clinicians to give their decision about diseases of a patient by enumerating a list of possible diseases accompanied with their possibility degrees. It has been affirmed that the accuracy of medical diagnosis, which is much influenced by the clinicians' experience and knowledge, plays an important role to effective treatment therapies. Many methods were presented to enhance the *accuracy of diagnosis*. In medical diagnosis from dental X-Ray images, this matter relates to the dental segmentation, classification and decision making.

Dental segmentation is defined as the process of dividing a dental X-ray image into isolated parts according to different objectives and purposes [13]. Dental X-ray images can be used as input of some dental related problems such as human identification, health-care support systems, teeth numbering, automated dental identification system, dental age estimation and so on. These images are stored in a computer in the form of either an optical or a digital image. Different X-ray images have different resolutions, orientations and luminance content, depending of the X-ray machine and the dentist who took it [15]. Dental image segmentation is often used as the first step to find hidden dental structures, malignant or benign masses, bone loss, and cavities. There have been some works for dental segmentation such as SSFC-FS in [17, 19].

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After segments are found, possible diseases associated with them are then identified by a classification method. Herein, a number of methods such as Fuzzy Inference System (FIS) [9, 18], artificial neural network [2], Support Vector Machine [12], Bayesian classifier [6] and Virtual Doctor System (VDS) [10] were used accordingly. Tuan and Son [20] compared several classification methods namely Affinity Propagation Clustering (APC), Prim and Kruskal minimum spanning tree methods. The authors conducted that APC achieved better accuracy than other relevant methods.

The last step – decision making evaluates and determine the final disease from a list of possible ones found by the classification step. This is quite important as it supports clinicians to issue correct disease and treatments. Fuzzy aggregation operators are utilized in many articles for decision making problems. Bedregal et al. [5] used weighted average operators based on n-dimensional overlaps to support investors for investment evaluation in multi-attribute group decision making (MAGDM). Merigo' [19] used ordered weighted average operators (OWA) for multi-expert decision-making in production management. Some typical fuzzy aggregation operators are weighted average (WA) operators based on n-dimensional overlaps [5], the ordered weighted average operators (OWA) [14], the triangular intuitionistic fuzzy aggregation operators, generalized ordered weighted averaging operators of triangular intuitionistic fuzzy numbers (TIFNs) [4], interval-valued intuitionistic fuzzy weighted arithmetic average operator [1], triangular intuitionistic fuzzy aggregation operators, generalized ordered weighted averaging operators of TIFNs [22]. Hossain et al. [11] presented fourteen aggregation operators experimented in medical decision support systems.

It has been shown in the relevant works for medical diagnosis from dental X-Ray images [18, 20] that the decision making process was oversimplified or ignored. Moreover, the fuzzy aggregation operators reviewed above were not applied to the dental diagnosis problem. In this paper, we indeed focus on the decision making step in medical diagnosis from dental X-Ray images. We utilize the existing methods for dental segmentation (SSFC-FS [19]) and classification (APC [20]) to extract a list of diseases for decision. Then, a novel decision making method using fuzzy aggregation operators is presented. By using the fuzzy aggregation operators, the accuracy of dental diagnosis system will be enhanced. The whole process will be empirically validated on real datasets. The rest of the paper is organized as follows. Second section summarizes the fuzzy aggregation operators. Third section describes the proposed approach. "Experiments" section validates the proposed method on the real datasets of Hanoi Medical University Hospital, Vietnam. Finally, conclusions and further works are covered in the last section.

Fuzzy aggregation operators

In this section, we recall some aggregation operators used in recent researches. Fuzzy aggregation operations on fuzzy sets are operations by which several fuzzy sets are combined in a desirable way to produce a single fuzzy set [8, 21].

Definition 1 [5]. For each n-dimension vector $w = (w_1, w_2, \dots, w_n)$, a WA operator is:

$$WA(a_1, a_2, \dots, a_n) = \frac{\sum_{j=1}^n w_j a_j}{\sum_{i=1}^n w_j} \quad (1)$$

In the case of vector w satisfying the conditions:

$$0 \leq w_j \leq 1, \forall j = 1, \dots, n, \quad \sum_{j=1}^n w_j = 1 \quad (2)$$

Then, WA has a simple form as,

$$WA(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j a_j \quad (3)$$

Definition 2 [7]. For every n-dimension vector $w = (w_1, w_2, \dots, w_n)$ satisfying the condition (2) and b_j is the j^{th} largest element of group $\{a_1, a_2, \dots, a_n\}$, an OWA operator of n-dimensions symmetric is:

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j \quad (4)$$

Definition 3 [22]: A GOWA operator is a mapping:

$$GOWA(a_1, a_2, \dots, a_n) = \left(\sum_{j=1}^n w_j b_j^\lambda \right)^{\frac{1}{\lambda}}, w_j \in [0, 1], \sum_{j=1}^n w_j = 1, \lambda \in (-\infty, +\infty) \quad (5)$$

By using different values of parameter λ , we get various averaging operators such as the OWA operator, the ordered weighted quadratic averaging (OWQA) operator, the ordered weighted harmonic averaging (OWHA) operator, the ordered weighted geometric averaging (OWGA).

Definition 4 [16]. A triangular fuzzy number (TFN) is a fuzzy number represented with a set of three points $\tilde{A} = (a_1, a_2, a_3)$ interpreted as a form of membership function $\mu_A(x)$ that satisfies the following conditions:

- i) This function is increasing from a_1 to a_2
- ii) This function is decreasing from a_1 to a_2
- iii) $a_1 \leq a_2 \leq a_3$

Definition 5 [16]. An fuzzy generalized ordered weighted averaging distance (FGOWAD) operators of dimension n is a mapping $FGOWAD : \Psi^n \times \Psi^n \rightarrow R$ that has an associated weighting w satisfying (2) such that:

$$FGOWAD(\langle \hat{a}_1, \hat{b}_1 \rangle, \langle \hat{a}_2, \hat{b}_2 \rangle, \dots, \langle \hat{a}_n, \hat{b}_n \rangle) = \left(\sum_{j=1}^n w_j d_j^\lambda \right)^{\frac{1}{\lambda}} \tag{6}$$

where d_j is the j^{th} -largest of the $d(\hat{a}_i, \hat{b}_i)$, $i = 1, \dots, n$ and λ is a parameter, $d(\hat{a}_i, \hat{b}_i)$ is the distance between two TFNs $\hat{a}_i = (a_1^i, a_2^i, a_3^i)$, $\hat{b}_i = (b_1^i, b_2^i, b_3^i)$.

$$d(\hat{a}_i, \hat{b}_i) = \frac{1}{3} (|a_1^i - b_1^i| + |a_2^i - b_2^i| + |a_3^i - b_3^i|) \tag{7}$$

Definition 6 [3]. An intuitionistic fuzzy set (IFS) is the universe X is:

$$A = \{ \langle x, t_A(x), f_A(x) \rangle | x \in X \} \tag{8}$$

where $t_A(x)$ is membership function and $f_A(x)$ is non-membership function satisfying $t_A(x) + f_A(x) \leq 1, \forall x \in X$. For convenience, an IFS A in X can be rewritten as

$$A = \{ \langle x, [t_A(x), 1 - f_A(x)] \rangle | x \in X \} \tag{9}$$

Definition 7 [16]. Let $\tilde{a}_j = [t_{\tilde{a}_j}, 1 - f_{\tilde{a}_j}]$, $j = 1, 2, \dots, n$ be a collection of intuitionistic fuzzy values on IFS A, IFWA and IFOWA operators are:

$$IFWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = w_1 \tilde{a}_1 \oplus w_2 \tilde{a}_2 \oplus \dots \oplus w_n \tilde{a}_n \tag{10}$$

$$IFOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = w_1 \tilde{a}_{\sigma(1)} \oplus w_2 \tilde{a}_{\sigma(2)} \oplus \dots \oplus w_n \tilde{a}_{\sigma(n)} \tag{11}$$

where w is weighting vector and $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(1, 2, \dots, n)$.

Definition 8 [16]. A fuzzy generalized ordered weighted averaging distance (FGOWAD) operator of dimension n is:

$$WHFGOWAD(\langle \hat{a}_1, \hat{b}_1 \rangle, \langle \hat{a}_2, \hat{b}_2 \rangle, \dots, \langle \hat{a}_n, \hat{b}_n \rangle) = \left(\sum_{j=1}^n w_j \tilde{d}_j^\lambda \right)^{\frac{1}{\lambda}} \tag{12}$$

where \tilde{d}_j is the j^{th} -largest of the $\tilde{d}(\hat{a}_i, \hat{b}_i)$, $i = 1, \dots, n$ and λ is a parameter, $\tilde{d}(\hat{a}_i, \hat{b}_i)$ is the weighted Hamming distance between two TFNs $\hat{a}_i = (a_1^i, a_2^i, a_3^i)$, $\hat{b}_i = (b_1^i, b_2^i, b_3^i)$.

$$d(\hat{a}_i, \hat{b}_i) = w'_1 |a_1^i - b_1^i| + w'_2 |a_2^i - b_2^i| + w'_3 |a_3^i - b_3^i|, \quad w'_i \in [0, 1], i = 1, 2, 3; \sum_{i=1}^3 w'_i = 1 \tag{13}$$

The proposed method

In this section, we introduce a novel decision making method called Operator Disease Diagnosis System (ODSS) for medical diagnosis as in Fig. 1. Firstly, from an inputted dental X-ray image, the dental features are extracted and segmented into C* segments by the mean of a semi-supervised fuzzy clustering method called SSFC-FS [19]. For each segment, APC [20] is used to determine the most appropriate disease patterns corresponding to the segment based on a graph representation of dental features. A disease decision table is then generated as in Table 1 where \bar{d}_{ij} ($i = 1..n, j = 1..m$) is the probabilities of disease patterns into each segment. Herein, we should apply fuzzy aggregation operators to get the final diagnostic results (See the end of Fig. 1 for details).

Denote

$$y_j = OWA(\bar{d}_{1j}, \bar{d}_{2j}, \dots, \bar{d}_{nj}), j = 1, \dots, m \tag{14}$$

$$y'_j = FGOWAD(\bar{d}_{1j}, \bar{d}_{2j}, \dots, \bar{d}_{nj}), j = 1, \dots, m \tag{15}$$

Based on the discrete symptoms on each segment of the image, the model gives the diagnosis for each segment with equivalent diseases. A disease with highest probability is selected from the segments as the final result. It is obvious that probability values are separated and uncertain, we should use the combination of aggregation operators in order to get the best synthetic results. In this paper, the combination of OWA (Def. 2) and FGOWAD (Def. 8) is chosen to specify the most appropriate disease pattern from the disease decision table. A schema using fuzzy aggregation operators is proposed in Table 2. In this schema, steps 1–2 are demonstrated in Eqs. (14–16) respectively.

$$y''_j = (OWA(\bar{d}_{1j}, \bar{d}_{2j}, \dots, \bar{d}_{nj}) + FGOWAD(\bar{d}_{1j}, \bar{d}_{2j}, \dots, \bar{d}_{nj})) / 2, j = 1, \dots, m \tag{16}$$

The effectiveness of this combination is shown by experiments. Lastly, the synthetic results are shown in

Fig. 1 Operator Disease Diagnosis System (ODSS)

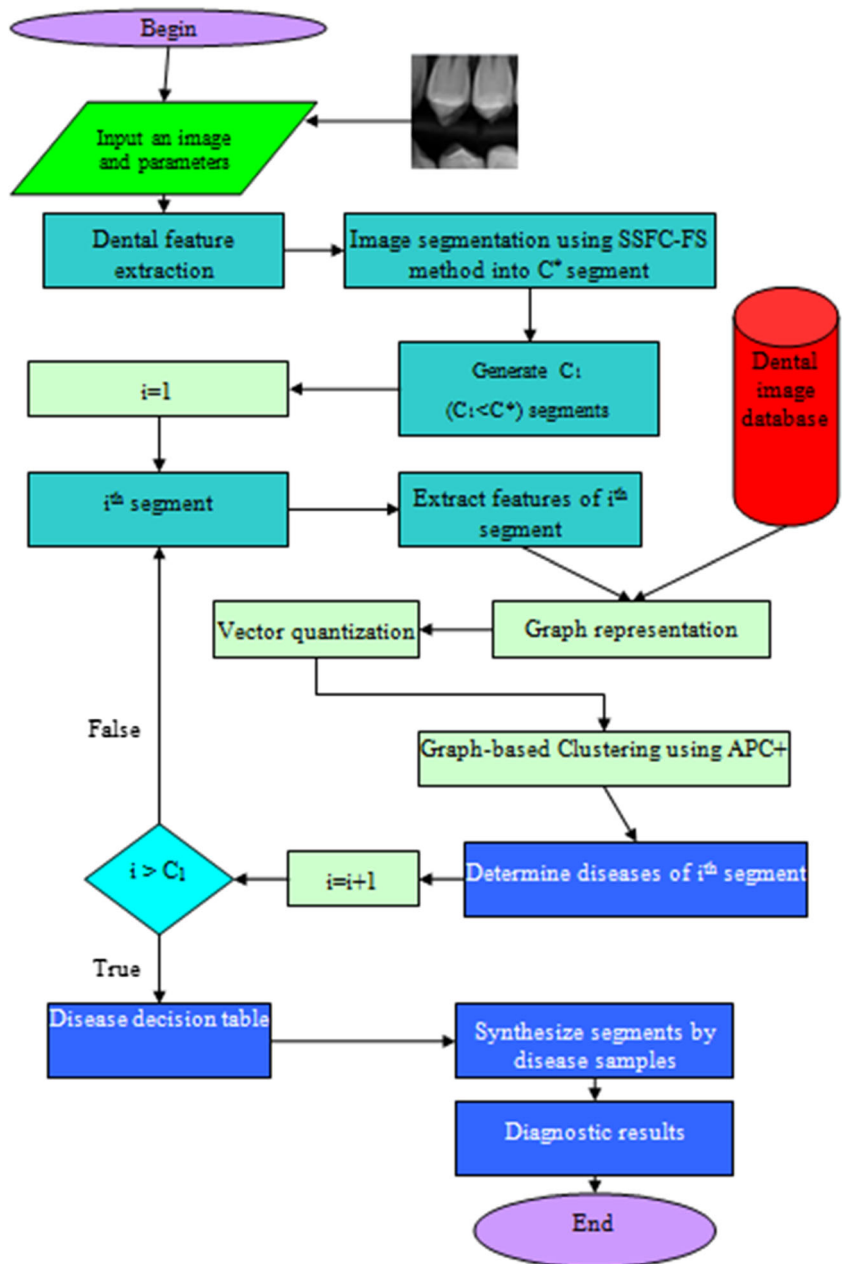


Table 3 with final disease being determined by maximal operator:

$$y_{\max} = \max(y_1, y_2, \dots, y_m); \tag{17}$$

Table 1 Disease decision table

	Disease pattern 1	Disease pattern 2	Disease pattern m
Segment 1	\bar{d}_{11}	\bar{d}_{12}		\bar{d}_{1m}
Segment 2				
.....				
Segment n	\bar{d}_{n1}	\bar{d}_{n2}		\bar{d}_{nm}

Example 1. Consider an input 3×3 image and its matrix form as in Fig. 2. We aim to validate whether this image belongs to either one of the following five diseases: cracked dental root, include teeth, decay, hypodontia and resorption of periodontal bone.

SSFC-FS [19] is applied to divide the image into 3 segments, thus the cluster centers and membership matrix are obtained and presented as in Tables 4 and 5 where P_i ($i = 1..9$) denote the i th pixel in the image.

Table 2 Disease specification

<i>Input</i>	Information about the probabilities of disease patterns into each segment ($\bar{d}_{ij}, i = 1, \dots, n; j = 1, \dots, m.$)
<i>Output</i>	Give the diagnosis about the most likely disease
1	Compute the synthetic results $y_i (i = 1, \dots, m)$ using OWA and FGOWAD operators
2	Aggregate these results
3	Give the diagnosis

Table 3 Synthetic results

Image	Disease pattern 1	Disease pattern 2	Disease pattern m
OWA	y_1	y_2	y_m
FGOWAD	y'_1	y'_2	y'_m
OWA-FGOWAD	y''_1	y''_2	y''_m

Based on the segmentation results using SSFC-FS, segment 1 consists of pixels 2, 3, 7, 9; segment 2 consists of pixels 4, 5 and segment 3 consists of the remaining pixels. The weight vector of similarity in each segment is $W = (0.54, 0.98, 0.75)$. By applying the APC algorithm [20] and diagnosis procedure, the disease decision table for input image is addressed as in Table 6.

In order to decide the most appropriate disease for a segment, we try to build a synthetic table for all combinations of fuzzy aggregation operators described in “Fuzzy aggregation operators” section. The synthetic results are presented in Table 7. It is clear that all combinations give the final diagnosis as Disease 1 (cracked dental root) with the highest synthetic value in a row. However, OWA-FGOWAD has higher value than others. From the example, we recognize that the proposed method can recommend the possible disease for an image using the combination of segmentation, classification, and fuzzy operator.

Remark 1. The proposed method (ODSS) has some advantages:

- 1) This method is a hybrid model between image segmentation and decision making with aggregation operators. Image segmentation helps the decision making procedure concentrate into the important regions in the image.
- 2) ODDS is easy to implement and straightforward.

Experiments

Database, tools and evaluation Based on the real dataset including 66 dental X-ray images from Hanoi Medical University Hospital, we validate the proposed method by validity indices (MAE, MSE and Accuracy presented in details below). In order to evaluate the performance of new algorithm, some other related methods are implemented on the same dataset. These methods are fuzzy inference system (FIS) [18] and affinity propagation clustering (APC) [20]. All these methods have been implemented using Matlab 2014.

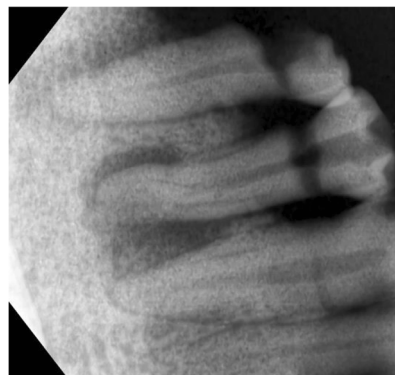
Dental feature extraction There are five dental features that are concerned in this paper, such as: Entropy, edge-value and intensity feature (EEI), Local Binary Patterns (LBP), Red-Green-Blue (RGB), Gradient feature (GRA), Patch Level Feature (PAT) [15].

Disease patterns Corresponding to five common dental diseases (cracked dental root, include teeth, decay, hypodontia, and resorption of periodontal bone). These diseases are labeled as 1, 2, 3, 4 and 5 respectively.

Validity indices MSE (Mean Squared Error), MAE (Mean Absolute Error) and Accuracy.

Experimental results Based on the algorithm described above, the ODDS is illustrated in the real dataset. The first experiment is to find out the best value of parameter λ of

Fig. 2 A 3×3 dental image (left) and a part of the matrix representation



59	125	97
72	142	114
71	146	116
49	132	102
41	130	98
48	145	112
47	148	114
33	139	103
34	142	106

Table 4 Cluster centers of the segmented image using SSFC-FS (bold values imply pixels of the image that are likely to acquire diseases)

	P1	P2	P3	P4	P5	P6	P7	P8	P9
Segment 1	0.15	0.78	0.87	0.23	0.28	0.12	0.44	0.17	0.56
Segment 2	0.34	0.12	0.07	0.45	0.42	0.21	0.15	0.25	0.21
Segment 3	0.51	0.1	0.06	0.32	0.3	0.67	0.41	0.58	0.23

Table 5 The membership matrix of the segmented image using SSFC-FS

Segment 1	71.28	150.76	115.32
Segment 2	38.67	146.37	102.45
Segment 3	45.38	127.34	102.34

Table 6 Decision table

	Disease 1	Disease 2	Disease 3	Disease 4	Disease 5
Segment 1	0.70	0.70	0.00	0.40	0.23
Segment 2	0.40	0.30	0.00	0.30	0.34
Segment 3	0.50	0.50	0.87	0.50	0.50

FGOWAD (Def. 8). By changing values of parameter λ when applying ODSS method for the dataset, values of validity indices are shown in Table 8.

From the results in this table, with two validity indices MSE and MAE, ODDS method gets the best performance when $\lambda = 3$. In Accuracy index, it is the best in the case of $\lambda = 2$. Thus, $\lambda = 3$ is selected for the next experiment. When applying these instances into the dental X-ray image dataset, the numerical results are shown as in Table 9. The experimental results show that ODDS is the best method in terms of validity indices. In both MSE and MAE, ODDS has the much less than those values of APC and FIS methods. The Accuracy value of ODDS is a bit higher than this of two other methods.

In the view of medical diagnosis, ODDS gives the most possible dental disease presented via symptoms on X-ray images. The model concentrates into the segments with

Table 7 Synthetic table using various combinations (bold values means the best)

	Disease 1	Disease 2	Disease 3	Disease 4	Disease 5
WA-FGOWAD	0.64	0.51	0.34	0.46	0.41
WA-IFWA	0.62	0.50	0.30	0.42	0.34
WA-IFOWA	0.57	0.49	0.30	0.40	0.40
OWA-FGOWAD	0.65	0.52	0.30	0.46	0.40
OWA-IFWA	0.62	0.51	0.26	0.43	0.33
OWA-IFOWA	0.57	0.50	0.26	0.40	0.39

Table 8 Experimental results of ODDS method with various values of λ (bold values imply the best)

	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$
MSE	0.1655	0.0981	0.0881	0.0897
MAE	0.1655	0.0981	0.0881	0.0897
Accuracy	93.08	93.12	93.02	92.98

high ability of being affected by diseases. A part from that, in this model, the combination of OWA and FGOWAD is used to deal with the vague and uncertain information. Thus, the final diagnosis has a higher accuracy than other methods. The obtained results from this model have important significance in supporting disease diagnosis. Using this model, the disease analysis is performed in small segments which sometimes cannot be discovered. This is good in the case of there are many diseases affected. Moreover, the diagnosis result is not only considered to isolated segments but also is synthesized for the final decision.

Conclusions

In this paper, we concentrated on the medical diagnosis from dental X-ray image. The main contribution of this research is a new diagnosis scheme using fuzzy aggregation operators. In this model, dental diagnosis was combined with image segmentation to deliver the diseases in each segment. OWA and FGOWAD were used to aggregate the highest ability of disease pattern in whole input image. The synthetic disease is given by a decision making procedure using the weights of segments. Experimental results were performed in a real database of dental X-ray images classified by five common dental diseases. From this illustration and three validity indices, it has been validated and showed better performance than other related methods. For the future research, our method could be improved by: i) to enhance the diagnosis results in clinical diagnosis; ii) to apply the ODDS into a big database including the noise; iii) to increase the processing rate of support system (decrease the time remaining).

Table 9 Evaluation performance of all methods (bold values imply the best)

	APC	FIS	OWA	FGOWAD	ODDS
MSE	0.821	0.2445	0.0902	0.0954	0.0881
MAE	0.701	0.1264	0.0902	0.0954	0.0881
Accuracy (%)	89.10	90.29	92.34	91.86	93.02

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Human and animal rights and informed consent This article does not contain any studies with human participants or animals performed by any of the authors.

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