Journal of Mining Science, Vol. 45, No. 6, 2009

THE ROCK STRENGTH IN DIFFERENT TENSION CONDITIONS

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The paper discusses the test data obtained in the three-point and four-point bending under uniaxial tension and in the Brazilian test of some rock types.

Strength, tension, bending, Brazilian tension test

A study into the properties of a rock mass begins with a laboratory investigation of the mechanical characteristics of rocks. Though the well-developed methods and standards existent in the field of strength properties of solids allow determining many parameters used then in modeling, calculations, etc., their general focus is structural materials. Rocks are a special group of solid media not always suited to conventional test schemes. It is difficult to sample rocks, especially very hard rocks, when it is only possible to get samples with the extremely simplified geometry. Direct tension tests require either specific rock samples or the specific clamping of them. Such tests are of low use to rocks due to the too much time-consuming preparation. Moreover, the mechanical strength of rocks ranges widely, thus, a test series to be representative should involve not a few samples. The direct rupture tests are replaced now by the indirect experiments (core splitting along generatrix, nonaxial compression and other methods).

In the study below, the uniaxial tension tests included only marble samples with the aim to compare the values of strength under the uniaxial tension and the Brazilian tension test. The simplest manufactured samples shaped as flat beams and disks, usually recommended for express testing, exhibit the uniaxial tensile strength unlike the rupture strength. Though samples fracture under the tensile stresses in this case, the fields largely differ from the uniaxial tension and, as a consequence, the values of strength can vary greatly. The problem although not recent [1 – 4], some experimental evidences have no sound explanations yet. This study objective is to estimate the uniaxial tensile strength of rocks after the bending tests and the Brazilian tests.

THE TESTING PROCEDURE

The tests involved marble, three kinds of granite, gabbro-diorite, gabbride and dolerite. Each test series comprised 7 to 10 samples made of the same slab (in the same direction), fractured over a specified plane. The Brazilian test disks 37.8 mm in diameter were made of slabs 19 to 23 mm thick. For gabbride, dolerite and two kinds of granite, the said thickness disks 26.5 mm in diameter were manufactured to analyze the dimension effect on the sample fracture. The small diameter disks passed series of 3 – 5 tests with each of the rock materials. The beams for the bending testing were 20×20×120 mm. Fracture start side was thoroughly polished. The analysis of the thickness effect consisted of testing other dimension granite and gabbride beams. Some mechanical properties obtained in the tests are given in Table 1: E is the static

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Young modulus; ρ is the density; σ_c is the time compressive strength; σ_t is the time Brazilian tensile strength; K_{1c} is the critical strength intensity factor; C_p is the P-wave rate. The K_{1c} to be determined passed an additional test series with samples with a cut. The beams 20×20×120 mm had in the middle a 5 to 1-mm-long cut made by a 1-mm-thick diamond disk. The result processing procedure is described in [5]. A fracture surface energy γ was calculated by the values of K_{1c}, E and Poisson's ratio $\nu = 0.2$.

The load leveller moved at 0.5 mm/min, which fitted the loading rate 0.3 to 2 MPa/s. In the Brazilian tests, the steel polished dies and the samples were separated by 20-μm-thick fluoroplastic cushions to uniform the load application along the generatrix. The bending test samples were supported by 7-mm-diameter rollers. To escape from slanting, the force was applied to the samples through a ball seat, including the direct tension tests.

THE TEST RESULTS

The average strength measurements are compiled in Table 2, where σ_t is the Brazilian test tensile strength calculated as:

$$\sigma_t = \frac{F}{\pi R t}, \tag{1}$$

σ_b^1, σ_b^2 are, respectively, the one-point and four-point bending strength determined from the assumption of elastic epures:

$$\sigma_b = \frac{6M}{Bt^2} = \frac{3FL}{Bt^2}, \tag{2}$$

where F is the force; M is the force moment; R is the disk sample radius; t is its thickness; B is the sample width; $L/2$ is the arm of force.

TABLE 1. Mechanical Properties of Rocks

Rock	E, GPa	ρ, kg/m^3	σ_{comp}, MPa	σ_{tens}, MPa	K_{1c}, MPa·m$^{1/2}$	γ, J/m^2	C_p, m/s
Ufalei marble	18	2700	80	6.9	0.8	18	6000
Lightcolored granite	70	2620	129.9	10.6	0.8	4.6	5550
Granite	63	2600	168	11.2	1.1	8.7	4880
Biotite granite	40	2600	176.6	10.4	0.7	6.1	3800
Gabbro-diorite	62	2700	189.5	13.4	1.0	8.1	4820
Gabbride	80	3000	290	20.4	2.0	25	5530
Dolerite	112	3000	379	25	1.9	16	6040

TABLE 2. Experimental Strengths of Rocks

Rock	σ_t, MPa	σ_b^1, MPa	σ_b^2, MPa	σ_b^1/σ_b^2	σ_b^2/σ_t	δ, cm
Ufalei marble	6.9	18	15.7	1.15	2.26	0.85
Lightcolored granite	10.6	14.8	13.4	1.10	1.25	0.36
Granite	11.2	19.4	16.6	1.17	1.5	0.61
Biotite granite	10.4	10.9	11.1	0.982	1.06	0.29
Gabbro-diorite	13.4	20.8	19.4	1.07	1.45	0.41
Gabbride	20.4	37	33.6	1.10	1.65	0.61
Dolerite	25	38	33.4	1.14	1.34	0.38

The calculated length parameter in the rightmost column in Table 2 is found as [3]:

$$\delta = \frac{2K_{1c}^2}{\pi \sigma_t^2} \tag{3}$$

and is subsequently used as a structural characteristic of a medium.

The Brazilian Testing. Table 2 omits the comparison data for the Brazilian test and uniaxial tension of Ufalei marble samples. Testing of six samples made of the same slab as the dumbbell shape cores with a length of nearly 50 mm and cross-section 20×20 mm resulted in the following: $\bar{\sigma} = 5.9$ MPa with a 45 % scatter while the average strength of the cores was 6.9 MPa with an 18 % scatter. In this case, the Brazilian test yielded a 15 % overestimate strength as against the uniaxial tension. Some references inform on the cases when the uniaxial tensile strength exceeded the Brazilian tensile strength for graphite [6]. The authors of the works [1, 2] believe that the Brazilian tests biaxiality should decrease a material strength as against the uniaxial tension as the compression in perpendicular to the tension adds extension. With the maximum extension criterion [1], or the maximal tension stress work [2] used, these authors have come to a conclusion on a decreased strength of cores when split along generatrix as compared with the uniaxial rupture. Along with this, there are data, e.g., for concrete [7], showing the higher strength of cores in splitting than in the uniaxial tension. According to Brace [8], it can be expected in the biaxial tests of granite and dolomite that the limits of the uniaxial tension and the Brazilian tension will be equal. Based on the comparative experiment, the authors of [9] drew a deduction on the aboutness of the rock tensile strengths obtained in the both testing modes. To explain this controversy, the work [2] offered a two-parameter model accounting for a medium structure. It follows from the model that given a small ratio of a structural parameter to the core radius, the uniaxial tensile strength will be higher than the Brazilian test strength, and vice versa. The model gives no clear definition of the structural parameter of a medium and only says that this parameter is proportional to the medium grain size. What is important is that the model correctly describes extreme situations: fine-grained graphite has the higher uniaxial tensile strength than the Brazilian tensile strength [6], while coarse-grained rough-structure concrete with $\Delta = 20$ mm filler exhibits the lower value of the former characteristic. The structural parameter-to-radius ratio of a sample determines a zone with equal strength limits both in uniaxial tension and splitting of the core along its generatrix depending on Poisson's ratio. The strength values obtained by the two methods discussed, according to the model from [2], at $\nu = 0.2$ may reach 30 % for a very fine structure material and 45 % for a rough structure material ($\Delta / R \approx 0.5$). Rocks feature a far smaller range of grain sizes and, thus, the less scattered strength values. The Table 2 rightmost column data allow finding the structural parameter/radius of a core in a range from 0.1 to 0.2. In this range, the rupture-to-splitting strength ratio can be both above and below 1 but no more than by 20 % in any case. All the three kinds of granite and gabbride possess large grains, but the structural parameter suggests an inflated value only of strength obtained by splitting along the core generatrix. For gabbro-diorite and dolerite (fine- and mid-grained, small structural parameter), the Brazilian tension and uniaxial tensile strength limits will probably be equal. For biotite and lightcolored granites having large grains but small structural parameter, the tensile strengths in the both methods will probably be the same. This supposition is favored to by the independence of the splitting strength values of the core diameter (37.8 and 26.5 mm). Hence it can be said that the Brazilian test tensile strength not always coincides with the uniaxial tension results but is within the strength range limits for a given rock.

Bending Test. Table 2 data show the higher tension stress limits in three-point bending than in the four-point bending test. This fact was pointed out to by many researchers [3]. The finite element method calculations and the beam loading experiment results yield the 15 % higher extension strain in the four-point bending versus the three-point test, which produces the upward bias of the three-point bending strength. The biotite granite samples are only out of the trend: its three- and four-point bending strengths are equal. Hereinafter, when speaking on bending, we will mean the four-bending strength values. The tensile strength in bending of rocks except for biotite granite largely exceeds the Brazilian tests tensile strength, which may be due to a number of factors. Stress epure is inelastic and the calculation by (2) yields an overestimate, especially if a material is plastic. In a brittle material, the effect of a sample volume and stress gradient shows itself.

The idea on strong rocks to be always brittle is invalid a little. Figure 1 shows loading curves for dolerite and gabbride, the strongest rocks in the discussed testing series. The strain of the end face of the beam, measured by a strain gauge glued at the failure point is given in standard units. The dolerite beam fracture as a main crack onset recorded by an acoustic emission sensor took place at the end of the plastic branch of the loading curve (point *A* in Fig. 1). In this case, it is more correct to calculate the beam stresses by the limit state. The beam fracture moment is set by the following relation [10]:

$$M = \sigma_t \frac{Bt^2}{6}\left[1 + \frac{2\delta_p}{t}\left(1 - \frac{\delta_p}{t}\right)\right], \tag{4}$$

where δ_p is the plastic zone size. For the dolerite beam 20 mm thick, the plastic zone length is 4.3 mm according to (4). The curves analogous to Fig. 1 have been recorded in all the tested rocks. It is interesting that all these curves possess more or less pronounced nonlinearity, though the plastic branch is present in the strain curves not of all the rocks. The maximum stress at the fracture moment in a material that disobeys the Hooke law can be found by using a procedure from [10]. The stresses of a beam in the extreme case of $\delta_p = t/2$, that are calculated by the limit loads (2) and the limit case (4) differ by a factor of 1.5. It is impossible to explain the higher ratio of the bending strength to the uniaxial tensile strength by replacing the elastic moment by the plastic moment. This situation fits the case with Ufalei marble. Besides, for rocks having no clear yield point and pronounced nonlinearity, i.e. for brittle rocks, accounting of nonlinear stress distribution yields no satisfactory results; in our case, these are gabbroid and granite.

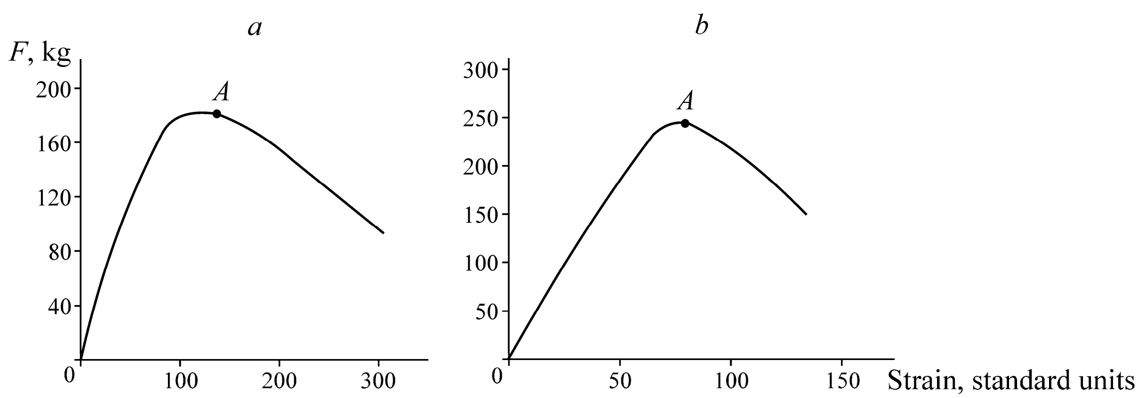

Fig. 1. The beam loading curves: *a* — dolerite, *b* — gabbride

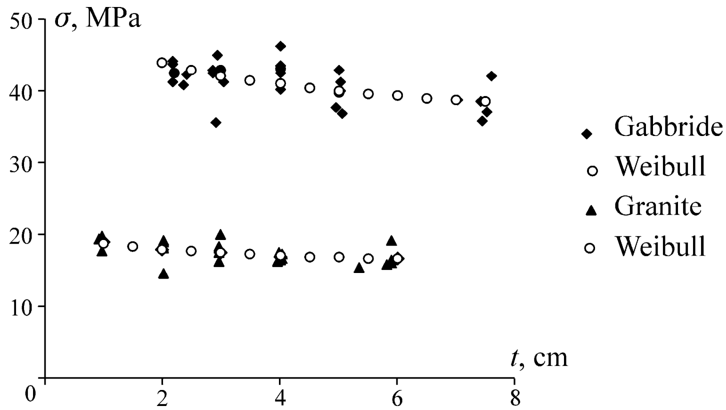

Fig. 2. Strength-to-thickness relationship for granite and gabbride beams

In a brittle medium, the dependence of a sample's strength on its volume and stress state is described by the statistic strength models. The geometrical effect, either volume or surface, is the highest when a concept of the weakest chain is acceptable, which is known as the Weibull theory. Figure 2 illustrates the strength to thickness relationships for granite and gabbroid beams (this is the same case as with the beam volume, the other dimensions unchanged). By sampling on average strength values, on the assumption of the Weibull distribution:

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{V_2}{V_1}\right)^{\frac{1}{m}},$$

we determined a damage density index m: 10 in gabbride and 14 in granite. Such a high value characterizes materials with a small dispersion and uniform distribution of damages, which corresponds, according to Fig. 2, to a slight decrease in a sample's strength with its larger volume. On the other hand, the statistic approach involves the effect of a stress state. In particular, the ratio of the bending tensile strength σ_b and the tensile strength σ_t is [11]:

$$\frac{\sigma_b}{\sigma_t} = \left[2(m+1)\left(\frac{V_t}{V_b}\right)\right]^{\frac{1}{m}},$$

where V_t is the uniaxially tensed sample volume; V_b is bended sample volume. Assuming the equality of the tensile strength and the Brazilian test tensile strength and taking into account the sample dimension, find the desired m: approximately 5 in granite and 4.5 in gabbride. So, the statistical approach explains the situation quite badly for the granite and gabbride samples. This controversy in marble gets even worse. The single-type tests of marble show low dependence between the sample's strength and volume. This fact means low dispersion of damages, thus, the model gives very high values of m. At the same time, given a high bending to tensile strength ratio, m = 2; that is, a model should show the high dispersion damages in a medium. This contradiction indicates the inapplicability of the weakest chain concept.

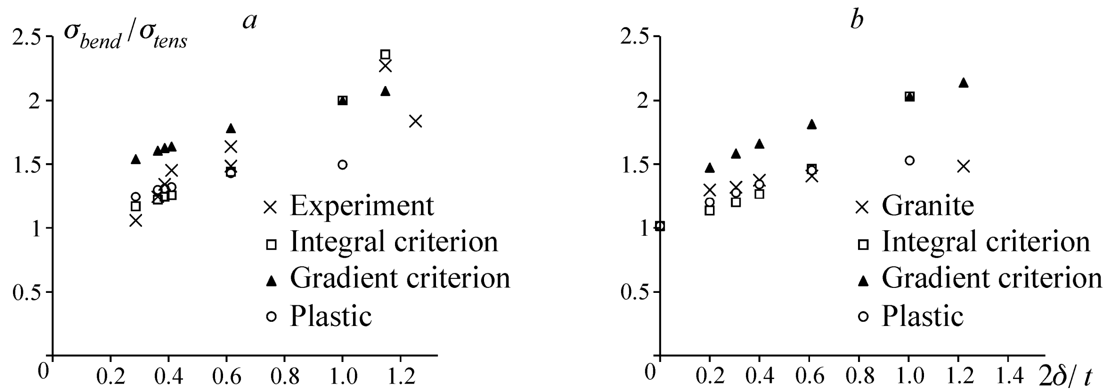

Fig. 3. Bending strength versus tensile strength

Another approach to the problem is accounting for the discontinuity fracture criterion in the stress field under bending. The known results of using the nonlocal strength criteria are presented in Fig. 3, where the crucifixes indicate average values of the ratios between the bending strength and the Brazilian test strength. Each X mark in Fig. 3*a* shows a definite material (test), $2\delta/t$ alters due to a structural parameter, the sample beams have the same thickness of 20 mm, while they are of different thickness in the tests illustrated in Fig. 3*b*. The strength gradient criterion is calculated according to [12, 13]. The structural parameter is chosen as δ. This model calculation data are denoted by the triangles. The integral criterion by Novozhilov [14] agrees better with the experimental data when $2\delta/t < 0.6$ as compared with the gradient criterion. The integral criterion calculations are displayed by the squares. The circles show the values obtained by the model where the structural element δ undergoes the constant σ_t, and the further stress distribution is linear. The determination of the inner forces in this model resembles the limit state calculation; therefore, the legend in Fig. 3 puts these points as "Plastic." A satisfactory agreement between the calculations and tests is observed in the range $2\delta/t < 0.6$ when we apply the integral fracture criterion and the model where the structural element δ feels constant stresses equal to the tensile strength.

CONCLUSION

The calculation of a bending strength for rock beams with the assumed elastic stress epures mostly yields the overestimate tensile strengths. The rock curves $\sigma-\varepsilon$ in these tests are nonlinear. It is recommended to process the experimental data by an elastic-plastic model if the tested material has a yield point, and by the nonlocal fracture criteria for the more brittle materials. In addition, for the results to be reliable, it is advised to obey the condition that $2\delta/t < 0.5$ that sets the minimum thickness of a tested beam sample.

The study was conducted with financial support from V. N. Oparin Scientific School, Grant NSH-3803.20085.

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