

Anisotropic Formation of Quantum Turbulence Generated by a Vibrating Wire in Superfluid ^4He

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Abstract To investigate the formation of quantum turbulence in superfluid ^4He , we have studied the emission of vortex rings with a ring size of larger than $38\ \mu\text{m}$ in diameter from turbulence generated by a vibrating wire. The emission rate of vortex rings from a turbulent region remains low until the beginning of high-rate emissions, suggesting that some of the vortex lines produced by the wire combine to form a vortex tangle, until an equilibrium is established between the rate of vortex line combination with the tangle and dissociation. The formation times of equilibrium turbulence are proportional to $\varepsilon^{-1.2}$ and $\varepsilon^{-0.6}$ in the directions perpendicular and parallel to the vibrating direction of the generator, respectively, indicating the anisotropic formation of turbulence. Here, ε is the generation power of the turbulence. This power dependence may be associated with the characteristics of quantum turbulence with a constant energy flux.

Keywords Quantized vortex · Quantum turbulence · Superfluid ^4He

1 Introduction

Quantum turbulence in superfluids has been extensively investigated due to the fact that it has a simpler structure than classical turbulence in viscous fluids [1]. In superfluid ^4He , quantum turbulence is simply a tangle of superfluid vortices. The vortices are stable topological defects carrying the same quantized circulation, and their cores are very thin. For example, a vortex in superfluid ^4He has a core radius of $a_0 \sim 0.1\ \text{nm}$. Therefore, a vortex can be presumed to be identical to a vortex line. Since vortex

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lines cannot end within a superfluid, they form rings or terminate at the boundaries of the superfluid. If a vortex line is attached to an oscillating obstacle, the relative superflow along its surface may extend the vortex line for high velocity oscillations. In experiments, an oscillating object often generates turbulence in superfluid helium even at very low temperatures due to so-called remanent vortices. They nucleate during cooling through the superfluid transition, remaining attached to the walls of immersed objects or the container. If a superflow is applied relative to an obstacle's motion, remanent vortex lines expand unstably, resulting in the formation of turbulence [2] and the emission of vortex rings into the surrounding volume [3]. These phenomena have been observed for both superfluid ^4He [4–6] and $^3\text{He-B}$ [7,8] for various obstacles.

The emission rates of vortex rings vary with the generation power of the turbulence generated by the oscillating obstacle. The emission rate is proportional to the generation power at low powers and is less than proportional to the generation power at high powers [9]. In addition, for turbulence in superfluid $^3\text{He-B}$, the motion of a majority of the vortex lines is observed to become slower at higher powers [10]. Such behavior suggests that vortex lines reconnect with each other, forming a tangle of vortex lines [11,12]. Since turbulence in superfluids is simply a tangle of vortex lines, it is interesting that the supply of vortex lines is a sufficient condition for forming turbulence, which is very different from the case for classical turbulence.

In the present work, to investigate the formation of turbulence, we study vortex emissions from a turbulent region generated by a vibrating wire in superfluid ^4He . Using a set of two vibrating wires and a technique for selecting vortex rings [13], we report times-of-flight of emitted vortex rings of large diameter between the two wires. From the time-of-flight distribution as a function of the generation power of the turbulence, we discuss the formation of a tangle of vortex lines created between the generator wire and the detector wire.

2 Experimental Setup

We measured the times-of-flight of vortices between a turbulence generator and a vortex detector. A vibrating wire is an efficient tool for the continuous generation of turbulence [14]. During wire vibration in a turbulent state, the wire experiences a constant drag force, suggesting that the generation rate of vortices in the path of the wire vibration is balanced against the dissipation or the escape rate of the vortices. We also used a vibrating wire as a vortex detector. The wire driven in metastable laminar regime detects only the first approaching vortex line as the flow around oscillating wire switches to a turbulent state after the capture of the vortex by the wire. The wire can be reset to the vortex-free state after sufficient reduction of its velocity or complete stopping. We have investigated the distribution of vortex emissions from a continuously generated turbulence using this method [9].

To detect vortex flight paths, we mounted three vibrating wires made of NbTi wire with a diameter of $\sim 2\ \mu\text{m}$ in a copper chamber with a pinhole. Two detector wires, detector A and detector B, are located in directions perpendicular and parallel, respectively, to the vibration direction of a generator wire, as shown in Fig. 1.

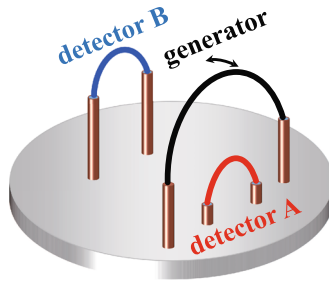


Fig. 1 Schematic illustration of vibrating wires. The detector wires are mounted around the generator wire. Detectors A and B are located in directions perpendicular and parallel, respectively, to the vibration direction of the generator indicated by the arrows. The distances between the generator and detectors A and B are 0.84 mm and 0.83 mm, respectively (Color figure online)

The distance between the apexes of the generator and detector A is set to be 0.84 mm, with the distance between the generator and detector B set to be almost the same at 0.83 mm. The bending shape of detector A is nearly identical to that of detector B. The resonance frequencies in vacuum at 4.2 K are 1145 Hz for the generator, 2906 Hz for detector A and 3282 Hz for detector B.

To study vortex emissions for large vortex rings, we performed time-of-flight measurements in superfluid ⁴He at a temperature of 1.2 K. At a finite temperature, a normal component of superfluid helium provides dissipation and vortex rings shrink during flight and may thereby fade away. The flight distance *l* and the flight time τ until disappearance of a circular vortex ring with an initial radius R_0 are given by $l = R_0/\alpha$ and $\tau = R_0/2\alpha v_0$ [15], where α is the mutual friction coefficient between a normal fluid component and a vortex line, and v_0 is the velocity of a ring with radius R_0 given by

$$v_0 = \frac{\kappa}{4\pi R_0} \left(\ln \frac{8R_0}{a_0} - \frac{1}{2} \right), \tag{1}$$

where κ and a_0 are the circulation quantum and the radius of the vortex core, respectively. In the present setup, the detectors can detect emitted vortex rings with an initial diameter only above 38 μm , because smaller vortex rings may disappear before reaching the detectors. The flight time of the fastest ring is estimated to be 0.07 s under this condition. The setup controlled at a finite temperature enables the study of vortex emissions for large vortex rings.

3 Distribution of Time-of-Flights of Vortex Rings

Time-of-flight measurements were performed with a similar method to those used in previous studies [9, 16]. We measured the period between the start of turbulence generation by the generator and the detection of a vortex ring by the detector. The period, therefore, corresponds to the creation time of a vortex ring plus the flight time from the generator to the detector. We measured periods repeatedly at a temperature of 1.2 K for a power of 540 pW injected in a turbulence region using the generator and detector B, and found that the observed periods were distributed widely from 0.5

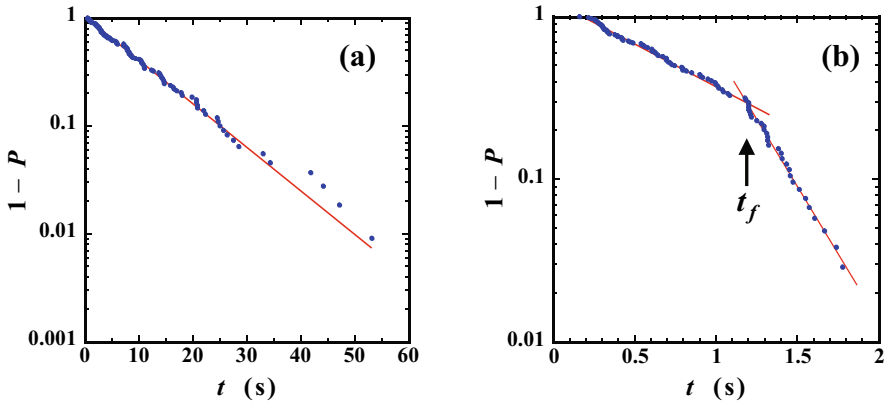


Fig. 2 Distributions of detection periods measured by detector B for generation power of **a** 540 pW and **b** 5000 pW of turbulence generated by the generator. The distributions at low power are exponential. At high power, double exponential behaviors were observed, as shown in **(b)**. The *arrow* in **(b)** indicates the point where the distribution changes (Color figure online)

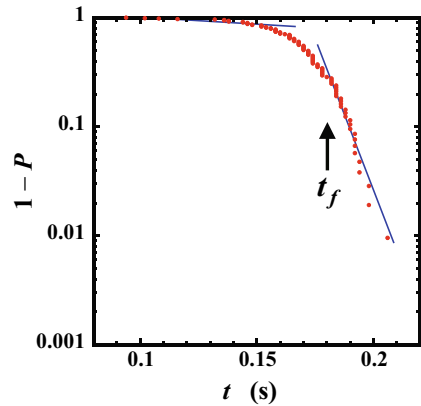
s to 60 s for 100 measurements. The non-detection probability $1 - P$ is plotted as a function of time in Fig. 2a, where P is the detection probability within a time window t . The data of the probability $1 - P$ are fitted to the exponential function

$$1 - P = \exp\left(-\frac{t - t_0}{t_1}\right), \quad (2)$$

shown as the solid line in Fig. 2a, indicating that the vortex detection is a Poisson process. The fitting parameters t_0 and t_1 are estimated to be 0.25 s and 11 s, respectively. The parameter t_0 corresponds to the creation time plus the flight time of the fastest vortex ring [9]. As mentioned in the previous section, the flight time of the fastest vortex ring that can reach the detector is expected to be 0.07 s. The creation time of a vortex ring by the generator is expected to be less than 10 ms [9]. Therefore, the time t_0 is estimated to be almost zero in the present time range. The mean detection period corresponds to the fitting parameter t_1 . Hence, the detector observes vortex rings at irregular intervals with a mean interval time of t_1 . Since the detector is triggered only by a vortex ring that reaches it, the emission of vortex rings is random with respect to the generator's vibration direction. Note that the generator wire is expected to produce a large amount of vortex rings with diameters distributed in the range from below $0.1 \mu\text{m}$ to a size comparable to the amplitude of the wire vibration [9, 13]. In the present setup, however, only the rings larger than $38 \mu\text{m}$ can reach the detector. The distribution shown in Fig. 2a indicates that the emission of vortex rings limited to large sizes is still random as well as the emission of whole rings [9].

Since emitted vortex rings become more dense as the generation power increases, it is expected that a tangle of vortex lines is created by reconnection of emitted vortex rings at high powers, as reported for quantum turbulence in superfluid $^3\text{He-B}$ [10, 11]. To study this behavior, we measured times-of-flight for higher generation powers and found that the distributions indicate a double exponential function, as shown in Fig. 2b.

Fig. 3 Distribution of detection periods measured by detector A for generation power of 490 pW. The distribution indicates a double exponential behavior, even though the power is low. The *arrow* indicates the point after which the distribution obeys an exponential behavior (Color figure online)

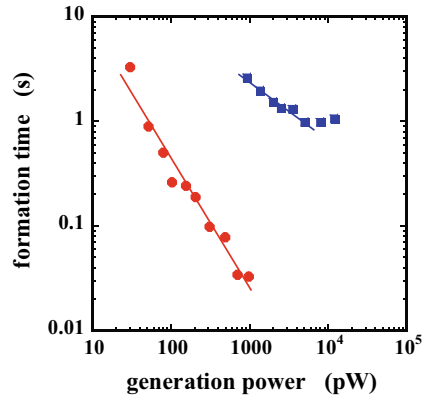


The probability data are well fitted to the single exponential function of Eq. (2) both before and after the time t_f , where the distribution changes, shown by the arrow in Fig. 2b, but for different fitting parameters. The fitting parameters t_0 and t_1 are estimated to be $t_0 = 0.19$ s and $t_1 = 0.83$ s for the early time and $t_1 = 0.28$ s for the late time. In the perpendicular direction, the double exponential distributions were also observed even at low generation powers as shown in Fig. 3, while those in the parallel direction were observed only at powers higher than 1000 pW. Since the slope t_1 of a distribution corresponds to the mean detection period proportional to the inverse of the emission rate, the vortex detection indicates that emission of the vortex rings is suppressed at early times. Nevertheless, during the same time, the generator wire experiences a constant drag force, suggesting a constant emission rate of vortex rings. Hence, a mechanism reducing the number of vortex rings must exist between the generator and the detector. We suggest that a tangle of vortex lines is created between the generator wire and the detector wire for high generation powers.

4 Formation of Vortex Tangle

At the beginning of turbulence generation, the generator produces vortex rings by stretching vortex lines attached to it [17]. The vortex rings move into the surrounding volume, colliding and reconnecting with other rings for a high producing rate of vortex rings, and forming a tangle of vortex lines [11]. The tangle of vortex lines captures more vortex rings, becoming larger. It is considered that during this process, some of the vortex rings escape from the tangle, though the number of escaped rings should remain low. The tangle will grow until the equilibrium between the captured and escaped vortex rings is established. The dissipation rate of vortex lines in the tangle is also linked with equilibrium density, because vortex lines dissipate due to the mutual friction between a vortex and the normal fluid component at a temperature of 1.2 K. In the equilibrium state, the number of escaped vortex rings becomes larger and, therefore, the mean detection period t_1 decreases as the equilibrium distribution is

Fig. 4 Formation time of a vortex tangle in an equilibrium state created between the generator and detector A (red circles) and detector B (blue squares). The formation time is proportional to $\varepsilon^{-1.2}$ in the perpendicular direction (red line) to the vibrating direction of the generator and $\varepsilon^{-0.6}$ in the parallel direction (blue line), where ε is the generation power of the turbulence (Color figure online)



reached after the time t_f shown in Figs. 2b and 3. The equilibrium vortex tangle has also been studied for quantum turbulence in superfluid $^3\text{He-B}$ [18].

Considering the formation process described above, we denote the time from the beginning of detections t_0 to the time t_f as a formation time of the equilibrium tangle of vortex lines. The equilibrium state is influenced by the number of captured vortex rings, which is likely to be proportional to the emission rate and consequently proportional to the generation power required to produce turbulence. Hence, the formation time depends on the generation power. Figure 4 shows the measured formation time of equilibrium tangle as a function of generator wire power. The formation times are plotted for two directions around the generator, indicating the anisotropic behavior with respect to the formation direction of vortex tangle. This behavior is associated with the emission rate of vortex rings generated by the generator [13]. For these experimental conditions, the detectable vortices are limited to sizes larger than a diameter of 38 μm , comparable to the vibrating amplitude of the generator. If such a large vortex is stretched in the wire path by the wire motion, the produced vortex ring will likely move perpendicular to the direction of the wire path. Consequently, the emissions are more frequent in the perpendicular direction than in the parallel direction. The directional dependence appears in the mean detection periods; for instance, $t_1 = 0.007$ s for detector A shown in Fig. 3 and $t_1 = 11$ s for detector B shown in Fig. 2a at a generation power of 500 pW. These results indicate that the large vortex rings observed here are emitted anisotropically against the vibrating direction of the generator.

We note that the vibrating direction of a detector against the approaching direction of vortex rings may influence the detection rate. Detector A is triggered by rings flying from the above and perpendicular to its motion, while detector B detects rings that move from its side and in the same direction as detectors displacement. A vibrating wire can be triggered into a turbulent state by a vortex ring with a velocity equal to or lower than the mean velocity of the wire vibration [9, 17]. The peak velocities of the both detectors were set to be 200 mm/s, corresponding to detected ring diameters above 1 μm . Vortex rings are expected to be small, the order of 1 μm , in the vicinity of the detectors due to the mutual friction between a vortex and the normal fluid component. Hence, the detection rate is considered not to be so influenced by the vibrating direction

of the detector. Therefore, the directional dependence of the detection rates as shown in Figs. 2a and 3 is mostly associated with the anisotropic emission of large vortex rings produced by the generator.

In the perpendicular direction, the formation time is proportional to $\varepsilon^{-1.2}$, where ε is the generation power of the turbulence. Since the product of the generation power and formation time is proportional to the total energy injected into a vortex tangle reaching an equilibrium state, the present result suggests that an almost constant energy is necessary for an equilibrium tangle. However, during the formation of an equilibrium tangle, some vortex rings escape from the tangle or dissipate in the tangle due to the mutual friction. Therefore, the characteristics of an equilibrium tangle such as the size and vortex density cannot be estimated in the present study. In the parallel direction, we find another power dependence $\varepsilon^{-0.6}$, as shown in Fig. 4. The power dependence of $\varepsilon^{-1.2}$ in the perpendicular direction and $\varepsilon^{-0.6}$ in the parallel direction is expected to be related to the statistics of quantum turbulence with a constant energy flux. Further investigations into vortex tangles are necessary to clarify these points.

5 Conclusions

We studied the times-of-flight of vortices emitted from quantum turbulence generated by a vibrating wire in superfluid ^4He at a high temperature of 1.2 K. By controlling the generation power of the turbulence, we observed a double exponential distribution at high powers. The point of change of the distribution corresponds to the formation time of an equilibrium vortex tangle. The formation times are proportional to $\varepsilon^{-1.2}$ and $\varepsilon^{-0.6}$ for directions perpendicular and parallel, respectively, to the vibrating direction of the generator wire, indicating the anisotropic formation of vortex tangle. This power dependence is expected to be related to the statistics of quantum turbulence with a constant energy flux.

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