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Effect of Anti-dots on the Magnetic Susceptibility in a Superconducting Long Prism

C. A. Aguirre[1] · **Miryam R. Joya[1]** ·
J. Barba-Ortega[1]

Abstract The magnetic susceptibility of a long mesoscopic superconducting square prism containing one/two (dot) anti-dots is calculated in the framework of the Ginzburg–Landau theoretical model. This magnetic susceptibility shows jumps at each of the vortex transition fields. We studied the influence of the number, size and geometry of the anti-dots on the magnetic susceptibility in a superconducting sample. We found interesting physical behavior when several kinds of materials filled into the anti-dot are considered.

Keywords deGennes parameter · Magnetic susceptibility · Mesoscopics · Anti-dot

1 Introduction

The magnetic property of a superconducting material at low temperature in contact with different kinds of materials is one of the aspects of the physics of condensed matter (superconductivity) that has been studied more in recent years. Current applications of high-temperature superconductivity include magnetic devices that protect medical imaging systems, SQUIDs, infrared sensors, accelerators of particles, magnetic levitation transport vehicles, etc. It is well known that when a superconductor is in contact with other materials such as metal, ferroelectric, superconductor at higher critical temperature, the proximity effects can induce surface domains, where nucleation of the superconductivity can be decreased/increased [1–5]. Heat capacity in mesoscopic system studies contributes significantly to technological innovation by improving the capabilities of electronic devices [6–8]. The heat capacity $C_{P,V}$ was measured in

✉ J. Barba-Ortega
jjbarbao@unal.edu.co

[1]   Departamento de Física, Universidad Nacional de Colombia, Bogotá, Colombia

the phase transitions between vortex states in mesoscopic superconductors, and the experiments were carried out on arrays of mesoscopic disks and rings to maximize the output signal and found that the information process could be easily manipulated in nano-structured devices [6,9,10]. In a recent work, a Ginzburg–Landau approach was applied to calculate the magnetic susceptibility (χ_m) for two- and three-dimensional superconducting samples with different boundary conditions and sizes, and the authors found a strong dependence of the boundary conditions, dimensionality and size of the sample on the signature in the magnetic susceptibility of a superconducting sample [11]. A link between deGennes microscopic boundary condition and the Ginzburg–Landau approach, and a discussion of some relevant experiments for slabs, cylinders, spheres, hypercubes were found by Fink [12], and in this work the authors studied the effect upon the transition temperature T_c of a superconductor via the deGennes parameter b. A complete theoretical analysis of the specific heat of the mixed state in superconductors was first discussed by deGennes [13] and Fetter [14], and in this analysis, the authors described the specific heat change between two different vortex states, which is proportional to the difference obtained in χ_m [15] leading to the prediction of specific heat jumps at magnetic fields for which a vortex enters or leaves the sample [16]. Other attempts have used the BCS theory or the Eilenberg equations to describe calorimetric properties of superconductors [17]. In the present paper, we report changes in the numerical calculus of magnetic susceptibility due to the choice of different values of (dot) anti-dot size, internal boundary conditions and geometry of the defect. We simulated (via the deGennes parameter) an anti-dot defect in the sample filled by a dielectric material $b \rightarrow \infty$, a metal $b > 0$, or a higher critical temperature superconductor $b < 0$. As is well known, the order parameter or superconductivity is suppressed (enhanced) in the anti-dot surfaces when $b > 0$ ($b < 0$) is used [11].

2 Theoretical Formalism

We considered a long superconducting mesoscopic prism (either of circular or of rectangular cross section) surrounded by a uniform magnetic field $\mathbf{H_0}$, with one/two anti-dots filled by different kinds of materials (see Fig. 1). The derivation of the G–L equations takes the dimensionless units as follows: $|\psi|$ is the order parameter in units of $\psi_\infty(0) = \sqrt{-\alpha(0)/\beta}$, lengths in units of the coherence length $\xi(0)$; $\mathbf{A}$ in units of $H_{c2}(0)\xi(0)$, where $H_{c2}(0)$ is the second critical field, temperature in units of the critical temperature T_c [13]. For the borders of the anti-dots (internal boundary conditions), we assume that the perpendicular component of the superconducting current is equal to zero at the surface ($\mathbf{J} \cdot \mathbf{n} = 0$), where the suffix n denotes the direction normal to the surface (i.e., $(\nabla - i\mathbf{A}) \cdot \mathbf{n} = a\psi_s/b$). a is the unit cell and b the deGennes parameter. For a better analysis, we choose a new parameter $\gamma = 1 - a/b$. In $\mathbf{n}$ direction, we introduce the superconductor–metal ($b > a$, $0 < \gamma < 1$), the superconductor–superconductor ($b < 0$, $\gamma > 1$) and superconductor–vacuum (or dielectric) ($b \rightarrow \infty$, $\gamma = 1$) boundary conditions [18,19]. Since the external magnetic field is always parallel to the z axis, we take $\mathbf{H_0} \equiv H_0\hat{\mathbf{z}}$. As a model system, we considered a square sample of area L^2

(A) Case A (B) Case B (C) Case C

(a)
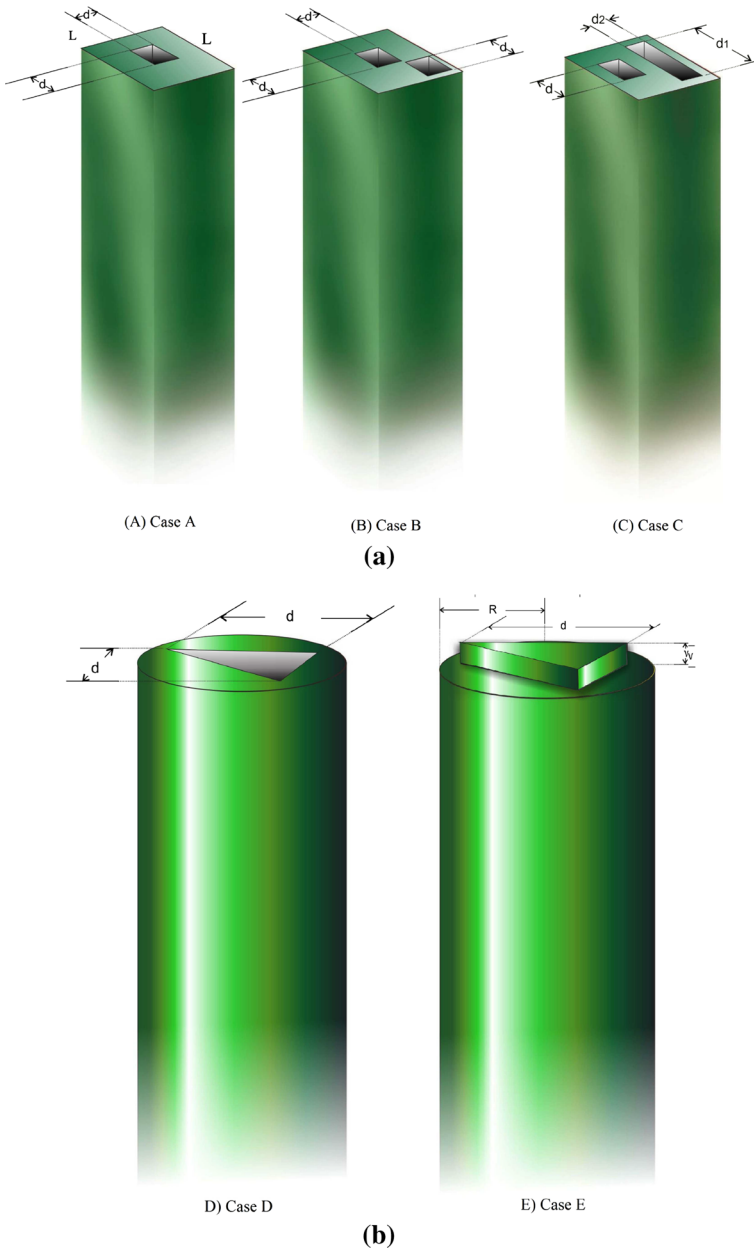
D) Case D E) Case E

(b)

Fig. 1 (Color online) Layout of the studied samples with (dot) anti-dots: **a** Case A: a long cylinder of rectangular cross section with one square anti-dot of size $d \times d$, *b* Case B: a long cylinder of rectangular cross section with two asymmetrical squares anti-dots of size $d \times d$, *c* Case C: a long cylinder of rectangular cross section with one square anti-dot of size $d \times d$ and one rectangular anti-dot of size $d_1 \times d_2$. **b** *d*) Case D: a long cylinder of circular cross section of radius R with one central isosceles triangle anti-dot of edge d, *e* Case E: a long cylinder of circular cross section with a central isosceles triangle dot of edge d and weight w

with an anti-dot of size $d_1 \times d_2$, respectively, with unit cell $a = 0.125$. The Ginzburg–Landau parameter $\kappa = \lambda/\xi = 1.0$ which is close to the experimental value for Al, Nb, or Pb mesoscopic samples. We solved the Ginzburg–Landau (G–L) equations by considering the magnetic induction and density of the superconducting electrons invariant on the z axis, in this scenario the demagnetization effects are not present, and we have a bi-dimensional problem, that is, the local magnetic field outside the sample is the external applied magnetic field [18–24].

$$\frac{\partial \psi}{\partial t} = -(-i\mathbf{\nabla} - \mathbf{A})^2 \psi + (1 - T)\psi(1 - |\psi|^2) \tag{1}$$

$$\frac{\partial \mathbf{A}}{\partial t} = (1 - T)\mathrm{Re}\left[\bar{\psi}(-i\mathbf{\nabla} - \mathbf{A})\psi\right] - \kappa^2 \mathbf{\nabla} \times \mathbf{\nabla} \times \mathbf{A} \tag{2}$$

Notice that Eqs. 1 and 2 are gauge invariant since they do not change under the transformation $\psi^{'} = \psi exp(i\chi)$, $\mathbf{A}^{'} = \mathbf{A} + \nabla\chi$ and $\phi^{'} = \phi - \partial\chi/\partial t$. We choose the zero-scalar-potential gauge, that is, $\phi^{'} = 0$ at all times. The presence of a dot in the Ginzburg–Landau model is considered by the w function, so Eq. 1 is:

$$\frac{\partial \psi}{\partial t} = -(-i\mathbf{\nabla} - \mathbf{A})^2 \psi + (1 - T)\psi(1 - |\psi|^2) - i(i\mathbf{\nabla} + \mathbf{A}) \cdot \frac{\nabla w}{w} \tag{3}$$

where $0 < w \leq \xi$ is a function that describes the effect of the variation of the sample thickness on the superconducting condensate. In this case, we can assume that the superconducting condensate is homogeneous along the z-direction and consequently we may take for the volume of the sample $dV = wdS$, where dS is the surface differential perpendicular to axe z. $w = 1$ everywhere, except in the dot region where is $w = 1.12$ (for more details of this calculus, see Ref. [21]). Equation 2 remains unchanged. Equations 1 and 2 are solved using the iterative procedure as outlined in Ref. [22]. To calculate the magnetic susceptibility, we used:

$$\chi_m = \frac{\partial M}{\partial H_0} \tag{4}$$

$$4\pi M = \langle H \rangle - H_0 \tag{5}$$

where $M(H_0)$ is the magnetization as a function of the external magnetic field H_0. The magnetic susceptibility is numerically calculated using the Runge–Kutta method of fifth order [24].

3 Results and Discussion

The magnetic susceptibility χ_m as a magnetic field function at $T = 0$ for case A with $L = 8\xi$, considering a superconducting–vacuum interface $\gamma = 1.0$ for (a) $d = 0\xi$, $H_1 = 1.05H_{c2}$, (b) $d = 2\xi$, $H_1 = 0.8H_{c2}$, (c) $d = 3\xi$, $H_1 = 0.75H_{c2}$ and (d) $d = 4\xi$, $H_1 = 0.75H_{c2}$, is shown in Fig. 2. As the magnetic field is increased, at a
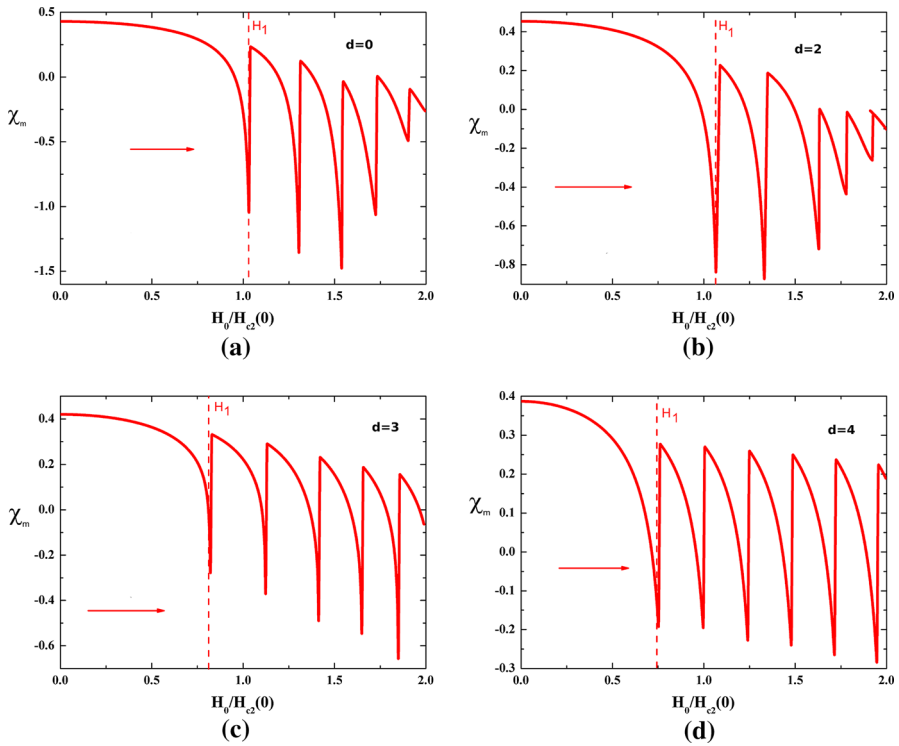
Fig. 2 Magnetic susceptibility for case A with $L = 8\xi$, considering a superconducting–vacuum interface $\gamma = 1.0$ for **a** $d = 0\xi$, $H_1 = 1.05H_{c2}$, **b** $d = 2\xi$, $H_1 = 0.8H_{c2}$, **c** $d = 3\xi$, $H_1 = 0.75H_{c2}$. H_1 and **d** $d = 4\xi$, $H_1 = 0.75H_{c2}$. H_1 is the critical field in which the first vortex entry occurs and it decreases as anti-dot size d increases (except for $d = 0$ case) (Color figure online)

certain point H_1 a Meissner–Abrikosov state transition occurs (in the magnetization $-4\pi M$ curve as a function of the applied magnetic field, the first vortex entry magnetic field H_1 is easily identified). At this point, the susceptibility jumps abruptly, then as the field continues increasing, the magnetic susceptibility decreases until another vortex (or chain of vortices) penetrates when it jumps occurs. This continues until the superconducting–normal state transitions. The effect of the size of the defect d is large; we estimate that the first jump occurs in $H_1/H_{c2} = 1.1, 1.05, 0.80, 0.75, 0.38$ for $d/\xi = 0, 2, 3, 4, 5$, respectively. We can see that the height of the first peak changes. All the samples show approximately equal diamagnetism. As we know once the vortices penetrate into the sample, they move from the screening currents in its boundary, but the vortices cannot reach the exact middle of the sample due to screening currents around the defect, as the size of the defect is bigger, the variation in the number of jumps in the magnetization and flux into the defect is major. The magnetic susceptibility χ_m as a magnetic field function for case A, with $L = 8\xi$, $d = 2\xi$ considering the internal boundary conditions: (A) $\gamma = 0.98$, $H_1 \approx 0.98H_{c2}$, (B) $\gamma = 1.05$, $H_1 \approx 0.99H_{c2}$, (C) $\gamma = 1.01$, $H_1 \approx 1.0H_{c2}$ (D) $\gamma = 1.2$ $H_1 \approx 1.0H_{c2}$ is shown in Fig. 3. When the border of the anti-dot is in contact with a thin layer of another
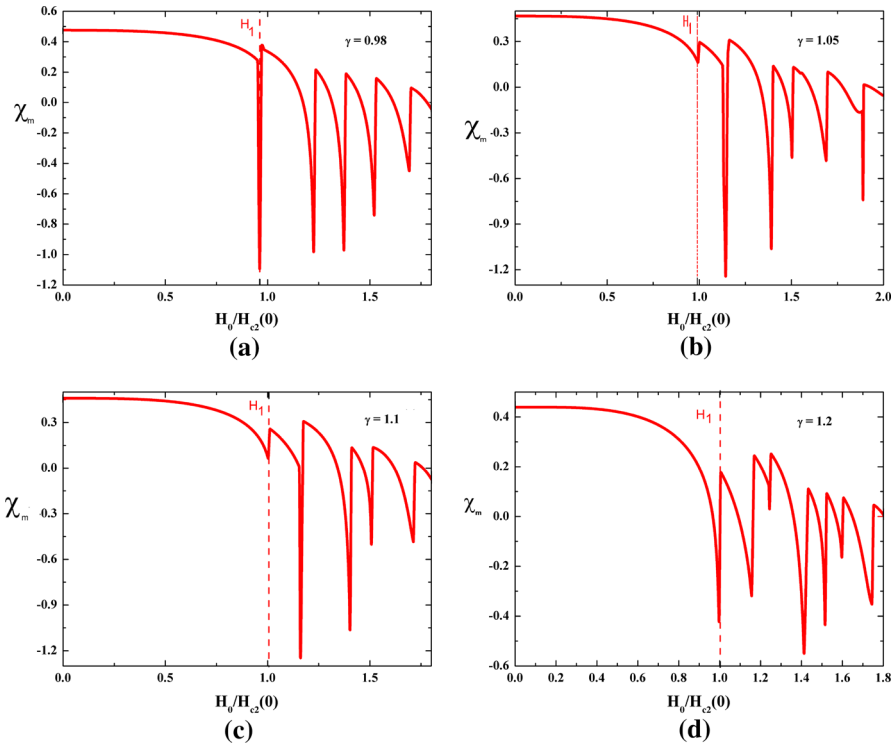
Fig. 3 Magnetic susceptibility for case A with $L = 8\xi$, $d = 2\xi$ considering the following internal boundary conditions: **a** $\gamma = 0.98$, $H_1 \approx 0.98 H_{c2}$, **b** $\gamma = 1.05$, $H_1 \approx 0.99 H_{c2}$, **c** $\gamma = 1.01$, $H_1 \approx 1.0 H_{c2}$ **d** $\gamma = 1.2$ $H_1 \approx 1.0 H_{c2}$. H_1 increase as γ increases

material (different values of deGennes parameter b related to γ), the susceptibility exhibits several peaks. We can appreciate that the difference of peaks in χ_m between adjacent vortex states is higher for a higher number of magnetic fluxoids Φ_0 in the sample, while the amplitude of the oscillation is smaller for a lower number of Φ_0. For $\gamma = 0.98, 1.05, 1.10, 1.25$, we found $H_1/H_{c2} = 0.98, 0.985, 0.990, 1.010$. H_1 increase as γ increase. In Figs. 4 and 5, we studied the magnetic susceptibility for case B with $L = 8\xi$, $d = 2\xi$, considering the internal boundary conditions: (a) $\gamma = 0.90$, $H_1 \approx 0.55 H_{c2}$, (B) $\gamma = 0.98$, $H_1 \approx 0.71 H_{c2}$ (C) $\gamma = 1.10$, $H_1 \approx 1.18 H_{c2}$ and (a) $\gamma = 0.90$, $H_1 \approx 0.81 H_{c2}$, (b) $\gamma = 0.98$, $H_1 \approx 0.71 H_{c2}$, respectively. When the superconducting–metal ($0 < \gamma < 1$) and superconducting–superconducting at higher critical temperature ($\gamma > 1$) internal boundary conditions are used in the anti-dot, we found that the highest values of χ_m are reached with the highest values of γ, so the diamagnetism can remain in the sample and thus is important when used in technological devices. In Fig. 6a, b, we studied the magnetic susceptibility for cases D and E, with $R = 8\xi$, $d = 2\xi$, considering the boundary conditions $\gamma = 1.0$, $H_1 \approx 1.05 H_{c2}$ for both studied cases. The curves show that the highest (and lowest) values of magnetic susceptibility are independent of the presence of triangular dot or anti-dot for this studied case.
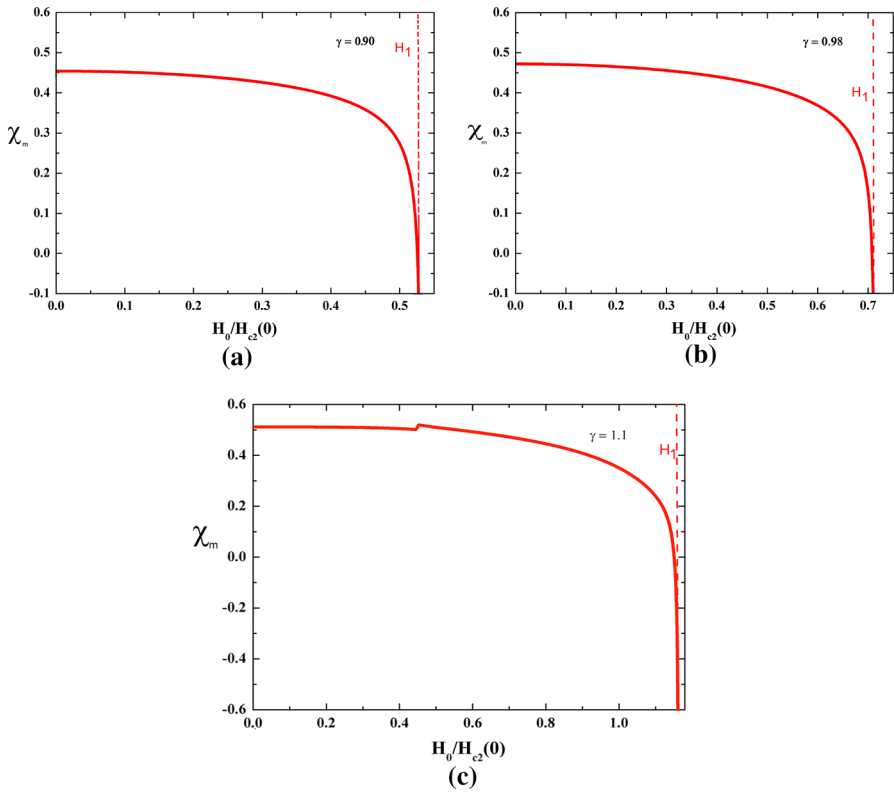
Fig. 4 Magnetic susceptibility for case B with $L = 8\xi$, $d = 2\xi$, considering the following internal boundary conditions: **a** $\gamma = 0.90$, $H_1 \approx 0.55H_{c2}$, **b** $\gamma = 0.98$, $H_1 \approx 0.71H_{c2}$ **c** $\gamma = 1.10$, $H_1 \approx 1.18H_{c2}$
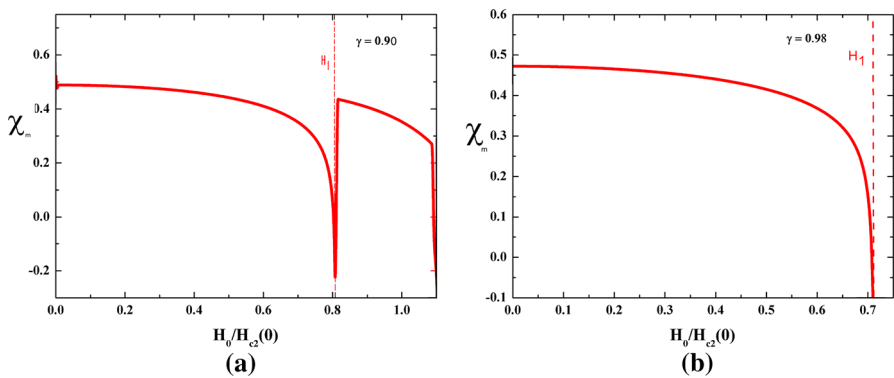

Fig. 5 Magnetic susceptibility for case C with $L = 8\xi$, $d = 2\xi$, $d_1 = 4\xi$, $d_2 = 1\xi$, considering the following internal boundary conditions: **a** $\gamma = 0.90$, $H_1 \approx 0.81H_{c2}$, **b** $\gamma = 0.98$, $H_1 \approx 0.71H_{c2}$
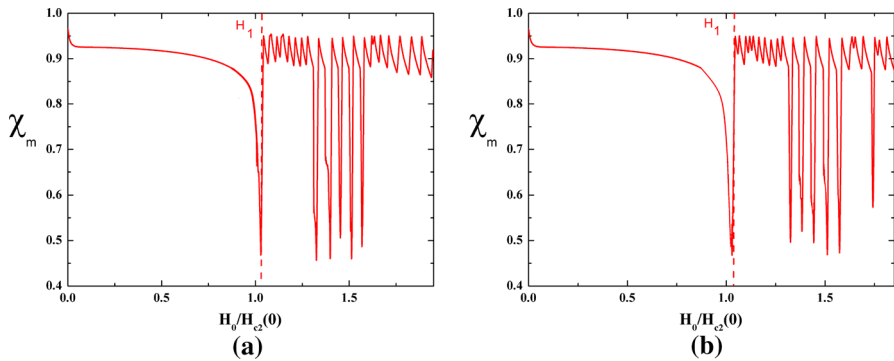
Fig. 6 Magnetic susceptibility for cases (D, E) with $R = 8\xi$, $d = 2\xi$, $\gamma = 1.0$ and $w = 1.12$ for the dot (case D), $H_1 \approx 1.05 H_{c2}$ for both cases

4 Conclusions

We calculate the magnetic susceptibility $\chi_m = \partial M / \partial H_0$ within the Ginzburg–Landau theory for superconducting samples containing anti-dots. We studied the effect of the anti-dot size and the internal boundary conditions on the magnetic susceptibility $\chi_m(H)$ in a long square prism. We found that the variations will produce different signatures in the magnetic susceptibility. Lastly, we also found that H_1 increase as γ increase into the anti-dots. For the triangular dot case, the presence of the dot does not show relevant changes of the signature of the magnetic susceptibility.

References

1. W.A. Little, R.D. Parks, Phys. Rev. Lett. **9**, 9 (1962)
2. D.Y. Sharvin, Y.V. Sharvin, JETP Lett. **34**, 272 (1981)
3. R.A. Webb, S. Washburn, C.P. Umbach, R.B. Laibowitz, Phys. Rev. Lett. **54**, 2696 (1985)
4. L.P. Levy, G. Dolan, J. Dunsmuir, H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990)
5. B. Bhushan, *Springer Handbook of Nanotechnology* (Springer, Berlin, 2004)
6. O. Bourgeois, S.E. Skipetrov, F. Ong, J. Chaussy, Phys. Rev. Lett. **94**, 057007 (2005)
7. P. Singha Deo, J.P. Pekola, M. Manninen, Europhys. Lett. **50**, 649 (2000)
8. W.C. Fon, K.C. Schwab, J.M. Worlock, M.L. Roukes, Nano Lett. **5**, 1968 (2005)
9. F.R. Ong, O. Bourgeois, Eur. Phys. Lett. **79**, 67003 (2007)
10. M. Berciu, T.G. Rappoport, B. Janko, Nature **435**, 71 (2005)
11. C. Aguirre, J. Gonzalez, J. Barba-Ortega, J. Low Temp. Phys. **182**, 51 (2016)
12. H.J. Finky, S.B. Haleyy, C.V. Giuraniucz, V.F. Kozhevnikovz, J.O. Indekeu, Mol. Phys. **103**(21), 2969 (2005)
13. P.G. deGennes, *Superconductivity of Metals and Alloys* (Addison-Wesley, New York, 1994)
14. A.L. Fetter, P.C. Hohenberg, *Superconductivity* (Marcel Dekker, New York, 1969)
15. J.D. Patterson, B.C. Bailey, *Solid-State Physics* (Springer, Berlin, 2007)
16. B. Xu, M.V. Milosevic, F.M. Peeters, Phys. Rev. B **81**, 064501 (2010)
17. K. Watanabe, T. Kita, M. Arai, Phys. Rev. B **71**, 144515 (2005)
18. M.M. Doria, A.R. Romaguera, F.M. Peeters, Phys. Rev. B **75**, 064505 (2007)
19. J. Barba-Ortega, E. Sardella, J. Albino Aguiar, Supercond. Sci. Technol. **24**, 015001 (2011)
20. J. Barba-Ortega, J.D. González, E. Sardella, J. Low Temp. Phys. **177**, 193 (2014)

21. G.R. Berdiyorov, M.V. Milosevic, B. Baelus, F.M. Peeters, Phys. Rev. B **70**, 024508 (2004)
22. V.A. Schweigert, F.M. Peeters, P.S. Deo, Phys. Rev. B **57**, 13817 (1998)
23. M. Tinkham, *Introduction to Superconductivity*, 2nd edn. (Dover Publication, New York, 1975)
24. M. Rainville, *Ecuaciones Diferenciales*, 8th edn. (Prenctice Hall Hispanoamerica, Ciudad de Mexico, 1997)