J Low Temp Phys (2013) 170:60–67
DOI 10.1007/s10909-012-0658-8

The Temperature Effects on the Parabolic Quantum Dot Qubit in the Electric Field

Ying-Jie Chen · Jing-Lin Xiao

Received: 20 April 2012 / Accepted: 10 July 2012 / Published online: 20 July 2012

Abstract The temperature effects on the parabolic quantum dot qubit in the electric field have been studied under the condition of electric-LO-phonon strong coupling using the variational method of Pekar type. The numerical results lead us to formulate the derivative relationships of the oscillation period of the electron in the superposition state of the ground state and the first-excited state with the electric field, the electron-LO-phonon coupling constant and the confinement length at different temperatures, respectively.

Keywords Quantum optics · Temperature effects · Variational method of Pekar type · Quantum dot · Qubit

1 Introduction

Size reduction in microelectronics and integrated circuits, with the corresponding increase of speed, is approaching a physical limit where quantum effects begin to appear. Since the statement of different quantum algorithms was put forward, it has been demonstrated that quantum mechanics offers unexpected possibilities in information transmission and processing, indicating the potential capabilities of quantum computers for solving intractable problems in a classical computer [1, 2], then the new information and communication protocols are developing. A quantum computer's elementary unit of quantum information is the qubit and two-level systems with long-lived

Y.-J. Chen (✉)
College of Physics and Engineering, Qufu Normal University, Qufu 273165, China
e-mail: sdchenyingjie@126.com

J.-L. Xiao
College of Physics and Electronic Information, Inner Mongolia National University, Tongliao 028043, China

quantum coherence are candidate qubits. A quantum entangled state is considered as a universal resource for quantum information processing because of its quantum state superposition principle and quantum nonlocality. Solid-state implementation for both qubits and entangled states is very promising because of the inherent scalability for realistic application of quantum computers [1, 2] as well as the relative ease of integration within current technologies [1–6]. Among such systems, the use of semiconductor quantum dots (QDs) [1, 2, 7–9] has received increasing attention due to the knowledge of their theoretical and experimental properties and the existence of an industrial base for semiconductor processing. However, there is no unique system of avoiding the ubiquitous problem of decoherence in real quantum objects during an effective implementation of quantum algorithms [10]. Quantum decoherence is inherent quality which results from the imperfect isolation of the quantum system from its environment and is essential in understanding how a quantum system becomes effectively classical [11]. Then how an entangled system undergoes decoherence or how the entanglement changes as a result of interaction with the environment is an important issue. The environment decoherence leads to deterioration of the performance of quantum logic operations and also strongly influences entanglement between qubits [12] that is necessary for quantum gate operation. Therefore, the investigation of decoherence dynamics due to the entanglement of the qubit system with its surrounding environment has become the central issue in the study of quantum information processing [13–18].

In practice, the work of the experiment about quantum qubits is performed at finite temperature. However, quantum systems are very frail and the temperature destroys the quantum coherence of the stored information [19, 20], a process called decoherence. Therefore, the temperature effects of the quantum dot qubit should be studied. The two-level quantum system can be employed as a single qubit in a quantum dot. For this single electron QD qubit, Li et al. presented a kind of parameter-phase diagram schemes and defined the parameters region for the use of an InAs/GaAs as a two-level quantum system [21, 22]. We have studied the temperature dependences of the parabolic linear bound potential quantum dot qubit in our earlier papers [23, 24]. In Ref. [23], we have investigated the dependences of the probability density of electron, which is in the parabolic linear bound potential quantum dot qubit, on the temperature. And in Ref. [24], the lows, that the probability density of electron and the period of oscillation change with the temperature's changing in the parabolic linear bound potential and Coulomb bound potential quantum dot qubit, have been dealt with. In the present paper, we treat the temperature effects on the parabolic quantum dot qubit in the electric field under the condition of electric-LO-phonon strong coupling using the variational method of Pekar type. The paper is organized as follows: in Sect. 2 we present the general Hamiltonian of an electron-phonon system in presence of an electric field F and obtain the relations of the oscillation period of the electron in the superposition state of the ground state and the first-excited state to the temperature, the electric field, the electron-LO-phonon coupling constant and the confinement length. In Sect. 3 we present and discuss the numerical results. Finally, Sect. 4 presents our conclusions. The parabolic confining potential in our proposed structure, that is Gauss potential, is the most likely to the real potential of electron in QD. Consequently our results are more accurate. In the meantime, our results should be meaningful for designing the solid-state implementation of quantum computing both theoretically and experimentally.

2 Theoretical Model

We consider the system where the electrons are bounded by the parabolic potential and the electric field. The electrons are much more confined in one direction (taken as the Z direction) than in other two directions. Therefore, we shall only take the effect of electron and LO-phonon into account assuming that the electrons move on the X–Y plane. The Hamiltonian of an electron-phonon system in presence of an electric field F along the x axis is given by

$$H = -\left(\hbar^2/2m^*\right)\nabla_\rho^2 + m^*\omega_0^2\rho^2/2 + \sum_q \hbar\omega_{LO} b_q^+ b_q + \sum_q \left(V_q e^{i\boldsymbol{q}\cdot\boldsymbol{r}} + h.c\right) - e^* F x, \tag{1}$$

where m^* is the band mass of electron. ρ is the two-dimensional coordinate vector and ω_0 is the confinement constant. $m^*\omega_0^2\rho^2/2$ is the parabolic confining potential in a single QD. $b_q^+(b_q)$ is the creation (annihilation) operator of bulk LO-phonon with the wave vector $\boldsymbol{q}(\boldsymbol{q}_{//}, q_\perp)$. $\boldsymbol{r} = (\boldsymbol{\rho}, z)$ is the coordinate of the electron. e^* is the electron charge, and

$$V_q = (i\hbar\omega_{LO}/q)\left(\hbar/2m^*\omega_{LO}\right)^{1/4}(4\pi\alpha/V)^{1/2}, \tag{2}$$

$$\alpha = \left(e^2/2\hbar\omega_{LO}\right)\left(2m^*\omega_{LO}/\hbar\right)^{1/2}(1/\varepsilon_\infty - 1/\varepsilon_0). \tag{3}$$

Using the Fourier expansion and the LLP transformation and choosing the polaron unit ($\hbar = 2m^* = \omega_{LO} = 1$), we have

$$H' = U^{-1} H U. \tag{4}$$

We choose the trial wave-functions of the electron-phonon system in the ground and the first-excited state [25] as

$$|\phi_{e-p}\rangle = \left[\lambda \exp\left(-\lambda^2\rho^2/2\right)/\sqrt{\pi}\right]\left|\xi(z)\right\rangle|0_{ph}\rangle, \tag{5}$$

$$|\phi_{e-p}\rangle' = \left[\lambda^2\rho \exp\left(-\lambda^2\rho^2/2\right)\exp(\pm i\varphi)/\sqrt{\pi}\right]\left|\xi(z)\right\rangle|0_{ph}\rangle. \tag{6}$$

Then we find the ground state energy and the first-excited state energy of electron as the following forms

$$E_0 = \lambda_0^2 + 1/\lambda_0^2 l_0^4 - \alpha\lambda_0(2\pi)^{1/2}/2 - F/2\sqrt{\pi}\lambda_0, \tag{7}$$

$$E_1 = 2\lambda_0^2 + 2/\lambda_0^2 l_0^4 - 11\alpha\lambda_0(2\pi)^{1/2}/32 - F/4\lambda_0. \tag{8}$$

Where $l_0 = (\hbar/m^*\omega_0)^{1/2}$ is the confinement length.

We can obtain λ_0 by the variational method. And we can get the eigenlevel and the eigenwave-function. Thus, we obtain the two-level system needed by a single qubit.

It is well known that the time evolution of the quantum state of the electron in this system can be written as

$$\psi_{01} = \left[\phi_0(\rho)\exp(-iE_0t/\hbar)\right]/\sqrt{2} + \left[\phi_1(\rho)\exp(-iE_1t/\hbar)\right]/\sqrt{2}. \tag{9}$$

Based on (7)–(9), we can present the probability density in the following form

$$Q(\rho, t) = \big[\left|\varphi_0(\rho)\right|^2 + \left|\varphi_1(\rho)\right|^2 + \varphi_0^*(\rho)\varphi_1(\rho)\exp(i\omega_{01}t) + \varphi_0(\rho)\varphi_1^*(\rho)\exp(-i\omega_{01}t)\big]/2, \tag{10}$$

where $\phi_0(\rho)$ and $\phi_1(\rho)$ of (10) are the ground state and the first excited state, and

$$\omega_{01} = (E_0 - E_1)/\hbar, \tag{11}$$

then the period of oscillation is

$$T_0 = 2\pi/\omega_{01} = 2\pi/\big[\lambda_0^2 + 1/\lambda_0^2 l_0^4 + 5(2\pi)^{1/2}\alpha\lambda_0/32 - (1/4 - 1/2\sqrt{\pi})F/\lambda_0\big]. \tag{12}$$

The mean number of optical phonons of the superposition state around the electron in parabolic quantum dot is

$$N_q = \langle\psi_{01}|U^{-1}\sum_q b_q^+ b_q U|\psi_{01}\rangle = 27(2\pi)^{1/2}\alpha\lambda_0/64. \tag{13}$$

At a finite temperature, the electron-phonon system is no longer in the ground state entirely. The lattice vibrations excite not only the real phonon but also the electron in a parabolic potential well. The properties of the polaron are a statistical average of the electron-phonon system in various states. According to the quantum statistics, the statistical average number of optical phonons is

$$\bar{N}_q = \big[\exp(\hbar\omega_{LO}/k_B T) - 1\big]^{-1}, \tag{14}$$

where k_B is the Boltzmann constant.

With the consideration mentioned above, the value of λ_0 determined by (13) relates to the value of N_q and to the value of $\bar{N}_q$, which should be self-consistent with (14). Therefore, we can obtain the relation of T_0 to N_q, T.

3 Results and Discussions

The effects of the temperature on the period of oscillation, which are extracted from a numerical evaluation, are shown in Figs. 1–3.

Figure 1(a) shows the period of oscillation as a function of the temperature and the electric field when the electron is in the superposition state of ψ_{01} for the electron-LO-phonon coupling constant $\alpha = 6$ and the confinement length $l_0 = 0.5$. It is shown that the period of oscillation increases with the temperature's increasing. The reason is that, the increase of temperature makes the speed of thermal motion of the electron and the phonon rise so that the electron will interact with more phonons. However, the contribution, which the increment of speed of the electron causes the probability of the electron in the superposition state to increase, is relatively strong. And the contribution, which the electron interacts with more phonons to destroy the superposition state, is relatively weak. Therefore, the electron lifetime on the superposition state is prolonged and the period of oscillation increases with the temperature's increasing. Figure 1(a) also shows that the period of oscillation slowly increases with

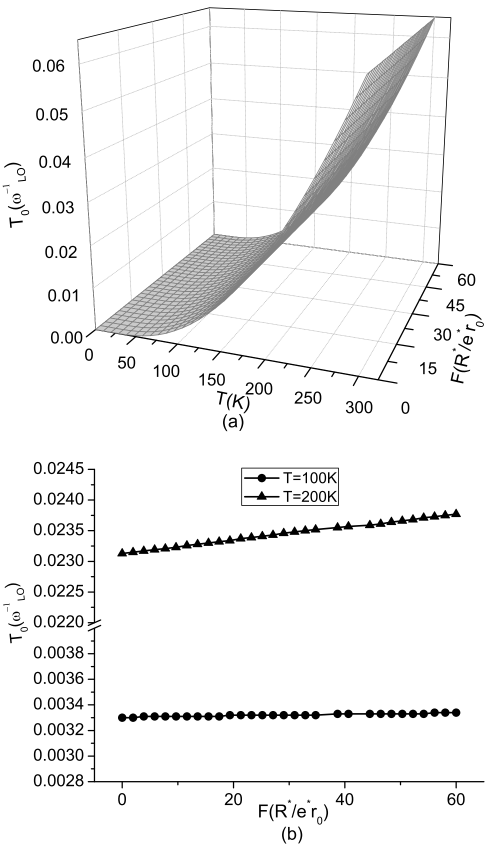

Fig. 1 (**a**) The period of oscillation as a function of the temperature and the electric field. (**b**) The period of oscillation as a function of the electric field at different temperatures

the electric field at any temperatures (Fig. 1(b) is clearer). Due to existence of the electric field, the ground and the first-excited state energies are reduced and the influence on the first-excited state energy is greater than the ground state energy. For this reason, the energy spacing between the ground and the first-excited states decreases and the period of oscillation increases [26]. As is shown in Fig. 1(a), at a finite temperature, the electric field does not impact the dependence of the oscillation period on the temperature.

Figure 2 depicts the period of oscillation as a function of the temperature and the electron-LO-phonon coupling constant for the confinement length $l_0 = 0.5$ and different electric fields $F = 20, 50$. It is shown, when the electric field F is 20 or 50, that the period of oscillation increases with the temperature's increasing. The reason is the same as above discussions. The Fig. 2 also depicts, when the electric field F is 20 or 50, the period of oscillation decreases with the electron-LO-phonon coupling constant's increasing. This is because that the coupling constant of the electron-phonon interaction is weaker in the first-excited state than that in the ground state, the energy spacing increases with the increasing coupling constant. The increase in energy spacing causes a reduction of the period of oscillation [24, 26]. Furthermore, Fig. 2 reveals that the period of oscillation increases with the electric field's increasing as in

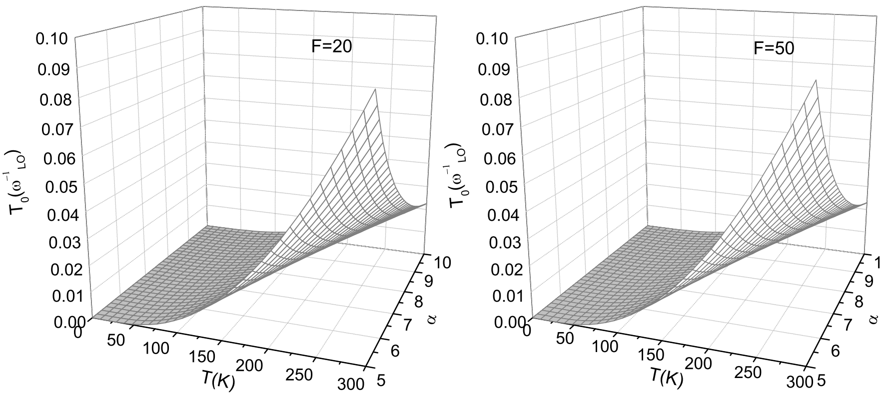

Fig. 2 The period of oscillation as a function of the temperature and the electron-LO-phonon coupling constant for different electric fields

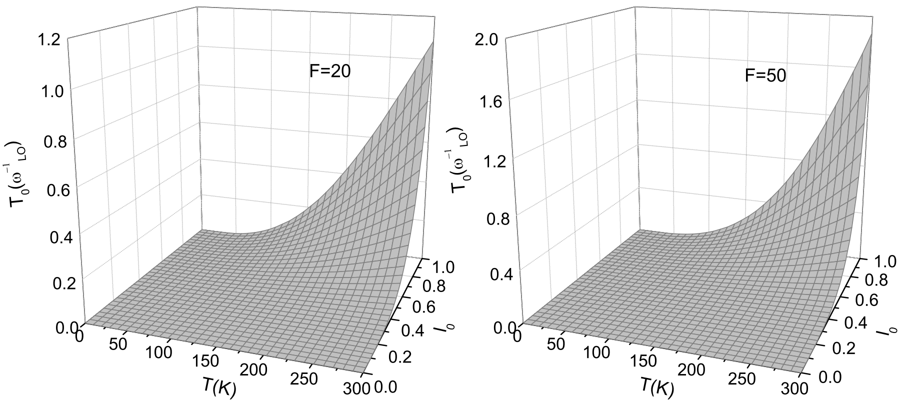

Fig. 3 The period of oscillation as a function of the temperature and the confinement length for different electric fields

Fig. 1. Besides, Fig. 2 also means that the electric field doesn't change the relations of the period of oscillation with the temperature and electron-LO-phonon coupling constant at finite temperature (compare Fig. 2 with Fig. 2 in Ref. [24]).

Figure 3 describes the period of oscillation as a function of the temperature and the confinement length for the electron-LO-phonon coupling constant $\alpha = 6$ and different electric fields $F = 20$, 50. As is seen in the Fig. 3, at different electric fields, the period of oscillation increases with the confinement length's increasing. For the same reason as in the Fig. 2, the period of oscillation fluctuates because of the temperature's increasing and the period of oscillation increases with the electric field's increasing as in Fig. 1. At the same time, Fig. 3 also shows that the electric field doesn't make the relations of the period of oscillation with the temperature and the confinement length change (compare Fig. 3 with Fig. 4 in Ref. [24]).

The above discussions indicate that the period of oscillation increases in superposition state of electron in a QD with the increment of temperature due to the existence the electric field. And our earlier studies show the probability density of electron and the period of oscillation are all effected by the temperature due to the existence of the parabolic linear bound potential or the Coulomb bound potential [23, 24]. However, a qubit cannot be independent of environment and must interact with the heat bath. As a result, the interaction destroys the superposition state of a qubit, which is decoherence [13–18]. The period of oscillation increases, in other words, the lifetime of a qubit increases. So the process of decoherence is slower. It is very useful to store information where the QD is made as its elementary unit. This is in agreement with discussion in Ref. [26].

4 Conclusions

In this paper, we have investigated that the effects of the temperature on the parabolic quantum dot qubit in the electric field using the Pekar variational method. From a numerical evaluation, we have shown that the period of oscillation increases with the electric field's increasing at any temperature. Meanwhile, at lower electric field or higher electric field, our results show: (1) the period of oscillation increases with the increasing temperature; (2) the period of oscillation decreases with the electron-LO-phonon coupling constant's increasing when the temperature is lower or higher; (3) the period of oscillation increases with the confinement length's increasing when the temperature is lower or higher. In addition, our work shows the advantage of the method that we used in this paper is the simplicity of the design of the computation process.

At present, other theoretical and experimental research of temperature effects on the quantum dot qubit is very scare besides our works [23, 24]. So a comparison between our results and experimental observations will be made in the future. But we still hope that our work anticipates some interesting aspects of the temperature dependences on quantum dot qubit.

Acknowledgements This work was supported by the research fund from Qufu Normal University (Grand No. XJZ200839).

References

1. G. Burkard, D. Loss, D.P. DiVincenzo, Phys. Rev. B **59**, 2070 (1999)
2. D. Loss, D.P. DiVincenzo, Phys. Rev. A **57**, 120 (1998)
3. A. Ekert, R. Jozsa, Rev. Mod. Phys. **68**, 733 (1996)
4. A. Imamoglu, D.D. Awschalom, G. Burkard, D.P. DiVincenzo, D. Loss, M. Sherwin, A. Small, Phys. Rev. Lett. **83**, 4204 (1999)
5. Z.J. Wu, K.D. Zhu, X.Z. Yuan, Y.W. Jiang, H. Zheng, Phys. Rev. B **71**, 205323 (2005)
6. S. Vorojtsov, E.R. Mucciolo, H.U. Baranger, Phys. Rev. B **71**, 205322 (2005)
7. J.R. Petta, A.C. Johnson, J.M. Taylor, E.A. Laird, A. Yacoby, M.D. Lukin, C.M. Marcus, M.P. Hanson, A.C. Gossard, Science **309**, 2180 (2005)
8. P. Zhang, Q.K. Xue, X.G. Zhao, X.C. Xie, Phys. Rev. A **66**, 022117 (2002)
9. S. Weiss, M. Thorwart, R. Egger, Europhys. Lett. **76**, 905 (2006)
10. M.A. Nielsen, I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000)
11. W.H. Zurek, Rev. Mod. Phys. **75**, 715 (2003)

12. M. Thorwart, P. Hünggi, Phys. Rev. A **65**, 012309 (2001)
13. A. Hichri, S. Jaziri, R. Ferreira, Physica E **24**, 234 (2004)
14. W.A. Coish, E.A. Yuzbashyan, B.L. Altshuler, D. Loss, J. Appl. Phys. **101**, 081715 (2007)
15. W. Ben Chouikha, S. Jaziri, R. Bennaceur, Physica E **39**, 15 (2007)
16. F. Buscemi, P. Bordone, A. Bertoni, New J. Phys. **13**, 013023 (2011)
17. S. Filippov, V. Vyurkov, L. Fedichkin, Physica E **44**, 501 (2011)
18. Y. Sun, Z.H. Ding, J.L. Xiao, J. Low Temp. Phys. **166**, 268 (2012)
19. C.Z. Li, M.Q. Huang, P.X. Chen, *Quantum Communication and Quantum Computing* (National University of Defence Technology Press, Changsha, 2000)
20. S.H. Xiang, K.H. Song, Acta Phys. Sin. **55**, 529 (2006)
21. S.S. Li, J.B. Xia, F.H. Yang, Z.C. Niu, S.L. Feng, H.Z. Zheng, J. Appl. Phys. **90**, 6151 (2001)
22. S.S. Li, G.L. Long, F.S. Bai, S.L. Feng, H.Z. Zhang, Proc. Natl. Acad. Sci. USA **98**, 11847 (2001)
23. Y.J. Chen, J.L. Xiao, Acta Phys. Sin. **57**, 6758 (2008)
24. Y.J. Chen, J.L. Xiao, Commun. Theor. Phys. **52**, 601 (2009)
25. Z.W. Wang, J.L. Xiao, Acta Phys. Sin. **56**, 678 (2007)
26. J.W. Yin, J.L. Xiao, Y.F. Yu, Z.W. Wang, Chin. Phys. B **18**, 0446 (2009)