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Probing the Frequency and Wavevector Dependent Response of ^{3}He Using Patterned Piezoelectric Transducers

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Abstract Techniques for studying the combined temporal and spatial response of ^{3}He are few in number. Here we describe, qualitatively, some strategies to probe single particle and collective responses in normal and superfluid ^{3}He in a frequency and wave-vector specific manner with the goal of stimulating such experiments in the future.

Keywords Wavevector specific coupling · Helium 3 collective modes · Patterned ultrasonic transducers

Ultrasound has proved to be a powerful probe of the physical properties of the helium liquids [1–3]. On the whole such measurements have been carried out using techniques similar to those commonly employed when studying solids. Here we examine a more unconventional approach.

In an earlier paper we studied the coupling to single particle and collective excitations in normal ^{3}He arising from an off-resonant, Rayleigh-like, surface excitation generated by inter-digitated electrodes patterned on the surface of a piezoelectric platelet [4]. To our knowledge such an experiment has not been attempted, in spite of the fact that our model calculations (see below) suggested that it should be possible to make a direct measurement of the Fermi velocity, a quantity currently inferred from thermodynamic measurements of the specific heat. The underlying physical idea is that as one varies the effective phase velocity v_s of the surface excitation, signatures appear when $v_s = v_F$, c_{0l}, and c_{0t} where v_F, c_{0l}, and c_{0t} correspond to the Fermi velocity and the longitudinal and transverse zero sound velocities respectively. Here $v_s = \omega/q$, where $q = 2\pi/d$ and d is the repeat distance of the electrode pattern. The predicted behavior is reproduced in Fig. 1.

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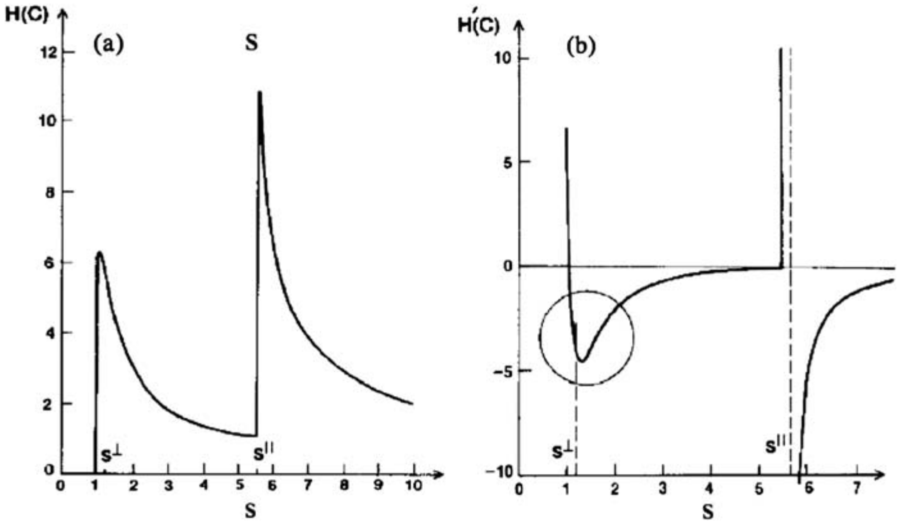

Fig. 1 The quantity $H(c)$, shown in (**a**), is proportional to the energy flux from the transducer into the liquid associated with a sinusoidal oscillatory displacement of the interface with frequency ω and wave vector q; $H'(c)$ shown in (**b**) is the frequency derivative. The *horizontal axis* is an effective surface wave phase velocity, $s = v_s/v_F$, measured in units of the Fermi velocity. Note that $H(c)$ shows strong singularities associated with the Fermi velocity, $s = 1$, and the longitudinal sound velocity, $s^{\parallel}$; $H'(c)$ emphasizes an additional singularity associated with the transverse zero sound velocity, $s^{\perp}$, at the position of the *vertical dashed line* to the *left*. (After Bogacz and Ketterson [4])

The present note identifies a strategy to greatly enhance the coupling to, and sensitivity to absorption by, single particle and collective modes of both the normal and superfluid phases of ^{3}He in a similar wave-vector-specific manner. A drawback of the earlier proposed methodology of generating in-plane periodic displacements of the piezoelectric platelet referred to above is the extremely small amplitudes so produced.[1] Our proposed new strategy is to: (i) generate large displacements by exciting a fundamental or overtone resonance of a conventional longitudinal (e.g., x-cut quartz) or shear (e.g., y or ac cut quartz) transducer that is surrounded by the liquid, (ii) use these transducer displacements to generate periodic in-plane tangential and normal displacements of the liquid by *lithographically patterning an array of parallel grooves* on one of the transducer faces, and (iii) sweep the characteristic fluid velocities by varying the hydrostatic pressure, p, in the liquid so as to achieve the above matching condition.

The proposed transducer configuration with its etched pattern is shown schematically in Fig. 2. The line spacing fixes the q-vector as defined above. The overall perpendicular or tangential motion generated by the transducer at resonance will produce periodic fluid flows with a q-vector $2\pi/d$ together with spatial harmonics. These flows will act like an antenna array and generate an outgoing collective

[1]Note that the standard surface acoustic waves generated by resonantly driving such devices have phase velocities that are greatly mismatched to those characteristic of the liquid by one to two orders of magnitude.

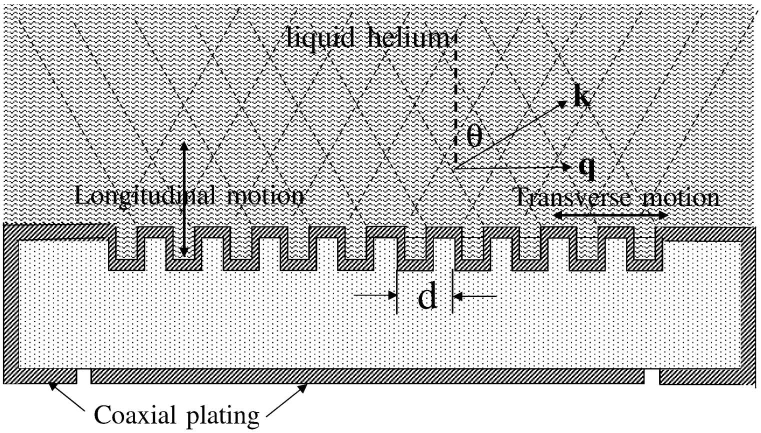

Fig. 2 A piezoelectric transducer produces longitudinal or transverse displacements of a face in contact with the liquid. Patterning the surface with a series of parallel grooves causes the fluid flows so generated to have a periodic in-plane component which acts as a phased antenna array that in turn radiates a collective density/velocity disturbance into the liquid helium at an angle determined by $k \sin\theta = q = 2\pi/d$. Plating with a magnetic or superconducting material in the presence of a static field would generate a periodic magnetic disturbance to launch spin waves

wave (e.g., a sound wave in the normal fluid or, possibly, order-parameter modes in the superfluid), the wave vector k of which satisfies the spatial resonance condition $k \sin\theta = q = 2\pi/d$.

The strongest interaction between the liquid and the grating is expected to occur when the collective wave being launched runs *tangent* to the transducer (and normal to the grating lines); i.e., when $k = q$. In the Appendix we present a simple hydrodynamic model demonstrating this enhanced coupling as contained in the expressions for the associated velocity fields. The effect described is in some sense the inverse of the well known Wood's anomaly [5] in optics (later explained by Rayleigh). There the phenomenon refers to a dip in the intensity of the specularly reflected light from a diffraction grating when the angle or wavelength of the incoming beam is such that the diffracted beam propagates *tangent to the grating,* as shown schematically in Fig. 3 for the case of normal incidence. Such conditions result in an enhanced coupling of the incoming light with the diffracted beam thereby reducing the intensity of the reflected beam. In the present case the role of the "incoming wave" is played by the oscillating transducer surface and at the Wood's anomaly (determined by the wavelength in the liquid) a mode is launched tangent to the grating formed by the array of patterned grooves.

The static wave vector q is fixed by the patterned line spacing while $k = \omega/c_{l,t}$ is determined by the frequency and the mode velocity. Now the whole point of the present approach involves exploiting the large displacements of the transducer which only occur when ω matches the fundamental or an overtone resonance; hence ω is constrained to these frequencies. Fortunately liquid helium is quite compressible implying the various velocities, e.g., $c_{l,t}(p)$, can be swept continuously over some range simply by changing p [6, 7]; for longitudinal hydrodynamic sound in ^{3}He it ranges from 183–406 m/s as the pressure goes from ambient to 30 bar at 150 mk [8]. Hence

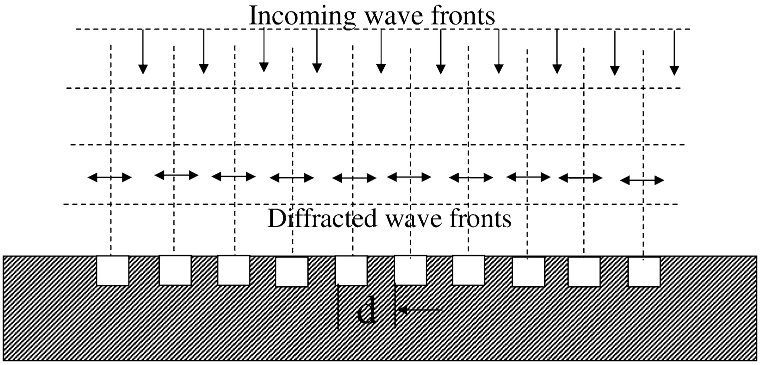

Fig. 3 An incoming wave with wavelength $\lambda = d$ impinging perpendicular to a grating will be diffracted tangent to the grating resulting in a reduction of the intensity of the reflected beam relative to other angles. This is a special case of Wood's phenomena

by a proper choice of the d-spacing one can arrange for the above condition to be satisfied. By incorporating several transducers with different resonant frequencies and/or d-spacings additional velocities can be obtained. Also by exploiting overtones of the fundamental resonance frequency of the transducer, ω_0, which occur at $(2n+1)\omega_0$ where n is an integer, and exploiting *spatial harmonics* of the q-vector of the patterned grating, mq where m is also an integer, one can arrange for additional absorption features to occur when

$$\frac{(2n+1)\omega_0}{mq} = c_{l,t}.$$

Since shear wave transducers are minimally loaded by the fluid (implying a higher quality factor Q) the desired signatures should be most apparent for this polarization.[2] In any case one would design the acoustic cell to minimize reflections associated with the *conventional plane waves* which are launched normal to a transducer surface (e.g. by incorporating a rough or porous "sound spoiler" opposing, but separated from, the transducer). Note conventional ultrasonic velocity measurements performed using a time of flight measurement or an evolving standing wave pattern *require* a reflecting (or receiving) surface, an exception being an acoustic impedance measurement.[3] For longitudinal waves, the effectiveness of the cell with respect to both suppressing conventional standing waves and detecting the launching of wave-vector specific longitudinal sound modes could of course be evaluated with liquid ^{4}He.

[2]Note we can pattern two distinct "polarizations" for which the grating lines run (i) parallel or (ii) perpendicular to the in-plane displacement direction of a shear transducer. At the onset the former couples to transverse zero sound the latter to longitudinal sound.

[3]Acoustic impedance measurements were first applied to study longitudinal zero sound in ^{3}He by [9]. Propagating transverse zero sound was studied by [10].

Now when $k < q$ launching a wave is kinematically forbidden; the disturbance in the fluid corresponds to an evanescent wave in this regime.[4] As the pressure is varied such that we pass through the condition $k = q$, and a wave can be launched, the Q factor associated with the transducer will abruptly drop, since a new channel for dissipation (acoustic radiation in this case) has opened up. From the onset of the Q shift we obtain a direct measurement of the mode velocity.[5] By locking an r.f. generator to a transducer resonance using an f. m. technique [13] and measuring the resonance width by monitoring the second harmonic, it is expected that the pressure at which the onset of propagation in the liquid is reached can be determined relatively accurately. Applying the method to study shear waves, in both the normal and superfluid states, could prove especially interesting. Perhaps the most exciting experiment would be a direct measurement of the Fermi velocity as discussed in [1]. But the ability to directly couple to collective modes in a wave vector specific manner, such as the so called imaginary squashing mode in ^{3}HeB, is also of interest. Since the this mode's frequency scales like the energy gap, one can use both temperature and pressure to adjust the mode frequency.

We now describe how the technique might be adapted to also study spin waves in ^{3}He. It has been shown earlier that one can couple to spin waves in a wave-vector specific manner by applying an r.f. current to a patterned meander line coil to generate a spatially periodic r.f. magnetic field and monitoring the absorption[14]. The dispersion of spin waves is described by $\omega = \gamma H + Dk^2$ where γ is the ^{3}He gyromagnetic ratio and the constant D measures the dispersion (with $D < 0$ in ^{3}He). If the wave vector satisfies the $k = q$ condition we again anticipate maximal coupling. If the patterned acoustic transducer is coated with a ferromagnetic (or possibly superconducting) film, field inhomogeneities in the presence of an external field that are created by the patterning will be periodically displaced by the motion of the transducer, thereby setting up an oscillating, spatially periodic, magnetic field. Although q is again fixed by the patterning, here we have an added flexibility in that by sweeping H we sweep ω.

In summary, we have presented some strategies to couple to a variety of single-particle and collective density and spin degrees of freedom of normal and superfluid ^{3}He in a wave vector specific manner. As noted above, the sensitivity of the technique for coupling to a density wave could be determined in a trial experiment with hydrodynamic (or collisionless below about 1 K) sound in ^{4}He.

Appendix: Liquid velocity profile

Here we obtain the velocity profile in a non viscid (Euler) liquid driven by the periodic displacement of a transducer on which a line grating has been patterned. The analysis

[4] This suggests the possibility of probing the liquid/solid interface in a depth dependent manner. In the driven evanescent regime we would have $\omega^2/c(p)^2 = k^2 - \gamma^2 = q^2 - \gamma^2$ where $\gamma = \gamma(p)$ measures the decay rate of the disturbance in the fluid. With such a probe one may be able to study surface bound quasiparticle states or surface phases. Such states are predicted in high temperature superconductors (e.g., see [11]). For the case of superfluid ^{3}He films a crystalline-like surface phase has been predicted by [12].

[5] Knowing that $k = q$ at the onset, we immediately have the mode velocity, ω/k, unlike with conventional ultrasound measurements where one must determine the time of flight.

is based on a conventional hydrodynamic model for the response of the liquid.[6] In such a fluid the velocity can be obtained from a scalar potential according to

$$\mathbf{v} = \nabla\phi \tag{A.1}$$

where ϕ obeys the wave equation

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0. \tag{A.2}$$

We first treat the case of a grating-patterned longitudinal sound transducer. We anticipate that ϕ will have the form

$$\phi(x, z, t) = \Re e^{-i\omega t}\left(\phi_+ e^{ik_x x + ik_z z} + \phi_- e^{-ik_x x + ik_z z}\right) + \Re\phi_\perp e^{-i\omega t + ikz} \tag{A.3}$$

where $k_x^2 + k_z^2 = k^2 = \omega^2/c^2$; the first two terms represent outgoing "diffracted" waves while the third is the conventional wave launched perpendicular to the transducer surface. The position of the static transducer surface for the case of a grating represented by a single spatial Fourier harmonic can be written as $\zeta(x) = \zeta_0 \cos qx$. When the transducer is excited the interface oscillates vertically and its position then becomes

$$\zeta(x, t) = \zeta_0 \cos qx + \zeta_1 \cos\omega t, \tag{A.4}$$

where ζ_1 is the amplitude of displacement associated with the longitudinal motion of the transducer. The boundary condition for an Euler fluid is that the magnitude of its velocity v_n normal to the surface match that of the surface where

$$v_n(x, z = \zeta, t) = \mathbf{v}(x, z = \zeta, t) \cdot \hat{\mathbf{n}}(x)$$

and $\hat{\mathbf{n}}(x) = \cos\theta\hat{\mathbf{z}} + \sin\theta\hat{\mathbf{x}}$ is the vector normal to the surface with $\tan\theta = \partial\zeta/\partial z$. We will assume $q\zeta_0 \ll 1$ in which case $\cos\theta \cong 1$ and $\sin\theta \cong \partial\zeta/\partial x = -\zeta_0 q \sin x$. Using (A.3) and (A.1) and retaining only terms involving the 0th nd 1st order spatial Fourier harmonic we have

$$v_n(x, z = \zeta, t) \cong \Re e^{-i\omega t}\left[ik_z\left(\phi_+ e^{ik_x x} + \phi_- e^{-ik_x x}\right) + ik\phi_\perp(1 + ik\zeta_0 \cos qx)\right]. \tag{A.5}$$

To the same order the velocity of the surface along its normal is

$$\dot{\zeta}(x, t) = -\omega\zeta_1 \sin\omega t. \tag{A.6}$$

Equating the fluid and surface velocities normal to the surface in the limit $k\zeta_0 \to 0$ yields

$$\phi_\perp = -\frac{\omega}{k}\zeta_1 = -c\zeta_1. \tag{A.7}$$

[6] It could only qualitatively apply in the collisionless regime of ^{3}He.

With $\phi_\perp$ now determined, thereby fixing the amplitude of the outgoing wave along $\hat{\mathbf{z}}$, we then obtain the amplitudes $\phi_\pm$ of the "diffracted" waves by demanding the three terms involving $k\zeta_0$ cancel:

$$ik_z\big(\phi_+ e^{ik_x x} + \phi_- e^{-ik_x x}\big) - k^2\phi_\perp\zeta_0\cos qx = 0 \tag{A.8}$$

which requires setting $q = k_x$ and equating $\phi_+ = \phi_-$ whereupon we obtain

$$\phi_+ = -i\frac{k^2\zeta_0}{2k_z}\phi_\perp = i\frac{ck^2\zeta_0}{2k_z}\zeta_1. \tag{A.9}$$

Using (A.1), (A.3), (A.7) and (A.9) we obtain the velocity in the fluid as

$$\begin{aligned}\mathbf{v}(x,z,t) \cong \Re e^{-i\omega t}\bigg[&-ick^2\zeta_0\frac{k_x}{k_z}e^{ik_z z}\sin k_x x\hat{\mathbf{x}}\\ &-\big(ck^2\zeta_0 e^{ik_z z}\cos k_x x + icke^{ikz}\big)\hat{\mathbf{z}}\bigg]\zeta_1.\end{aligned} \tag{A.10}$$

We see that the x component of the velocity is large for waves propagating nearly parallel to the grating; note (A.9) breaks down as $k_z \to 0$ as it was obtained on the basis of an iteration assuming $\phi_\perp \gg \phi_+$ and ignoring higher order spatial harmonics.

We now consider the case of longitudinal waves diffracted by a grating patterned along the x direction on a shear transducer. For this case the equation for the transducer surface analogous to (A.4) is

$$\zeta(x,t) = \zeta_0\cos q(x + \xi_1\cos\omega t)$$

where ξ_1 is the amplitude of the oscillatory displacement of the transducer surface generated by the shear motion of the transducer. The velocity in the direction perpendicular to the surface is now

$$\dot{\zeta}(x,t) = q\omega\xi_1\zeta_0\sin\big[q(x + \xi_1\cos\omega t)\big]\sin\omega t.$$

Assuming $q\xi_1 \ll 1$ and carrying terms to the same order as for the longitudinal case (which after our approximations involved only the vertical component of the fluid velocity) we set

$$\dot{\zeta}(x,t) = v_z(x, \zeta = z, t)$$

or

$$q\omega\xi_1\zeta_0\sin qx\sin\omega t\Re = \Re e^{-i\omega t}ik_z\big(\phi_+ e^{ik_x x} + \phi_- e^{-ik_x x}\big)$$

which requires setting $\phi_+ = -\phi_- \equiv i\phi_1$ and $q = k_x$ yielding

$$\phi_1 = -\frac{\omega k_x\zeta_0}{2k_z}\xi_1.$$

The resulting velocity in the fluid is then

$$\mathbf{v}(x,z,t) \cong \Re e^{-i\omega t}e^{ik_z z}\bigg[i\frac{\omega k_x^2}{k_z}\sin k_x x\hat{\mathbf{x}} + \omega k_x\cos k_x x\hat{\mathbf{z}}\bigg]\zeta_0\xi_1.$$

We again see that the coupling increases for waves traveling nearly parallel to the surface.

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