

# Transition to Turbulence and Critical Velocity in Superfluid Solution of $^3\text{He}$ in $^4\text{He}$

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**Abstract** By the method of oscillating tuning fork, we carried out researches of the transition to turbulence in superfluid solution of 5%  $^3\text{He}$  in  $^4\text{He}$  at temperatures of 100 mK–300 mK. The critical velocity  $v_c$  of the turbulence appearance is determined through measuring the volt-ampere characteristics. It is established that in the mixture the temperature dependence of the critical velocity is non-monotonous and differs strongly from that in pure  $^4\text{He}$ . Unlike  $^4\text{He}$ , the step-like anomalies on resonance curves were observed which, presumably, is connected with instability of the vortex system under the conditions where the core of the vortex is filled by the atoms of  $^3\text{He}$ . It is shown that such anomalies appear at the temperatures below 0.9 K, at the same time at temperatures below  $\sim 0.5$  K they appear even at  $v < v_c$ .

**Keywords** Turbulence · Superfluid mixture · Vortex · Instability · Tuning fork

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## 1 Introduction

Recently, the origin and development of turbulence in superfluid helium have been investigated by immersing a vibrating body of different shapes (wire, sphere, grid, quartz tuning fork) into a liquid [1–19]. The main results of these experiments are concerned with pure  $^4\text{He}$ . In this context of special interest is the region of low temperatures (below  $\sim 1$  K) where He II actually has no normal component possessing viscosity.

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So far very little is known about the transition from a laminar flow to turbulence in another class of superfluid systems, in particular in superfluid  $^3\text{He}$ – $^4\text{He}$  solutions. At sufficiently low temperatures the density of the normal component of solutions is mainly dependent on the impurity-induced excitations which are responsible for the kinetics and the dissipation processes in the system. Previously, the stall of a laminar flow was usually investigated producing heat flows in the system [20, 21] or on operating dilution refrigerators [22, 23].

This preliminary investigation of the transition to turbulence in a superfluid solution of  $^3\text{He}$  in  $^4\text{He}$  solutions was made using a quartz tuning fork. The technique has been applied recently to pure  $^4\text{He}$  at high [16–18] and low [19] temperatures. The obtained resonance curves and the  $I$ – $V$  characteristics have provided the first information about the critical velocity in dilute  $^3\text{He}$ – $^4\text{He}$  solutions.

## 2 Experimental Technique

The measurements were performed using a quartz tuning fork (the vibrating length of the prong 3.7 mm, and prong cross section  $0.6 \times 0.3$  mm) with the resonance frequency 32 kHz which was submerged into a 5% solution of  $^3\text{He}$  in  $^4\text{He}$ ,  $T = 0.3$ – $2.6$  K. The tuning fork was placed inside a cylindrical copper cell with a volume of  $2.3 \text{ cm}^3$ . The cell was in a thermal contact with the dilution chamber plate. For convenience, a hole was drilled in the factory-made cap of the tuning fork to fill it with the liquid to be investigated.

The tuning fork was excited with a continuous sine signal generator. The current  $I$  owing to the piezoelectric effect is proportional to the fork oscillation amplitude and the velocity of oscillation. Current was measured with a two phase lock-in analyzer (5208) by the resistance  $R = 1 \text{ k}\Omega$  voltage drop. The proportionality coefficient between current and velocity of oscillation for actual fork was determined by normalizing the measured values of  $I$  on resonance frequency at 1.48 K to the current-voltage characteristic obtained in [18], where the value of the velocity were found, for the tuning fork.

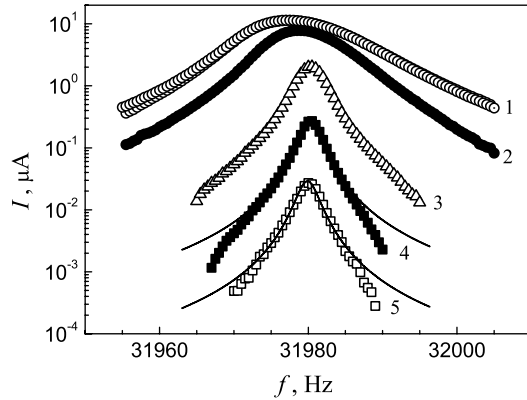
To find out the proper characteristics of the tuning fork, the measurements were first made in vacuum at the cell temperature below 1.5 K. The obtained resonance frequency was  $f_0 = 32715.278 \text{ Hz}$  and the width of the resonance curve was 0.035 Hz. Initially, the measurement was made on He II (like in [19]). Then the measuring cell was warmed to room temperature, evacuated thoroughly, cooled and the measurement was continued on a solution of 5%  $^3\text{He}$  in  $^4\text{He}$ .

The experiment with the  $^3\text{He}$ – $^4\text{He}$  solution proceeded as follows. The amplitude-frequency characteristics of the quartz tuning fork were measured at constant stabilized temperature of the cell in the interval 0.3–2.6 K and at different excitation voltages (1 mV–20 V).

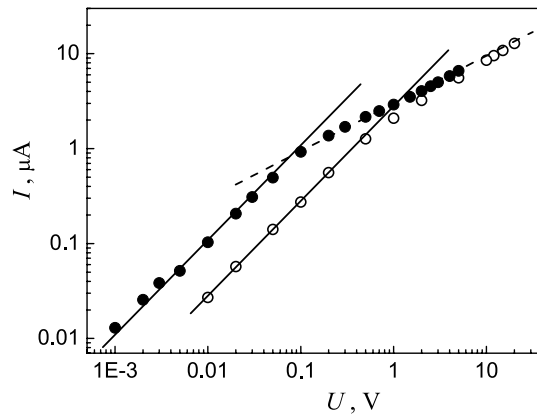
## 3 Analysis of Resonance Curves and $I$ – $V$ Characteristics

The typical amplitude-frequency dependence taken at  $T = 1.03 \text{ K}$  and several excitation voltages is illustrated in Fig. 1. For a convenient comparison of the scanning

**Fig. 1** The amplitude-frequency characteristic of the quartz tuning fork in the solution at different excitation voltages: 1 20 V, 2 10 V, 3 1 V, 4 0.1 V, 5 0.01 V.  $T = 1.03$  K



**Fig. 2**  $I$ – $V$  characteristic of the tuning fork in a 5%  $^3\text{He}$ – $^4\text{He}$  mixtures,  $T = 1.03$  K ( $\circ$ ),  $^4\text{He}$ ,  $T = 1.16$  K ( $\bullet$ ). The solid and broken lines are explained in the text



results and different excitation voltages, the experimental results are presented on semi logarithmic scale. It is seen that the resonance curve width and the resonance frequency are invariable at low excitation voltages (Fig. 1, curves 4, 5). As the exciting voltage grows, the resonance curve width increases and the resonance frequency decreases (curves 1–3).

Figure 2 shows the  $I$ – $V$  characteristic of the quartz tuning fork immersed into the investigated solution at  $T = 1.03$  K. The  $I$ – $V$  characteristic was plotted using the data of Fig. 1. For comparison, Fig. 2 contains the  $I$ – $V$  characteristic of  $^4\text{He}$  at a close temperature ( $T = 1.16$  K). At low excitation voltage the signal amplitude is proportional to the excitation voltage (solid line) in both  $^4\text{He}$  and 5%  $^3\text{He}$ – $^4\text{He}$  solution, that is typical for the laminar flow. As the excitation voltage increases, the laminar flow changes to turbulence.

In this case the squared amplitude of the signal is proportional to the excitation voltage (Fig. 2, broken line). The intersection of the solid and dashed lines is the critical signal amplitude corresponding to the change in the flow conditions. It is seen in Fig. 2 that turbulence in the mixture and in He II is characterized practically by the same dependence of the signal amplitude upon the excitation voltage. However,

the change of the laminar flow into turbulence with an increasing excitation voltage is described by smooth curve while the dependence for He II has a kink.

#### 4 Evolution of Resonance Curves at Low Temperatures

The amplitude-frequency dependences change appreciably when the temperature of the mixtures decreases. For example, these dependences are smooth at  $T = 1.03$  K and can be described by the Lorentzian, but they exhibit irregularities and disruptions as the temperature goes lower. Two examples of such dependences taken at different temperatures are illustrated in Fig. 3.

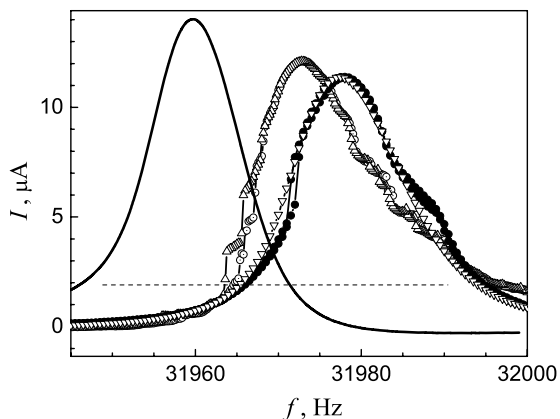
It is seen in Fig. 3 that the first irregularities (jumps) appear in the resonance curves even at  $T = 0.71$  K. As the temperature decreases, the number of jumps increases and they shift off the resonance frequency. Moreover, the jumps at the lower-frequency and higher-frequency sides of the resonance are rather different. Besides, on a further decrease in the temperature such jumps start to appear even at low excitation voltage. When frequency scanning was repeated, the jumps reappeared practically at the same single amplitudes (see Fig. 3). At  $T = 0.35$  K frequency scanning shifted in both directions ( $\circ$  increasing frequency,  $\Delta$  decreasing frequency).

Note that no such behavior was observed in pure  $^4\text{He}$ . For comparison, Fig. 3 shows (solid line) the data measured on  $^4\text{He}$  at  $T = 0.35$  K and the excitation voltage used for the mixture was the same as for  $^4\text{He}$ .

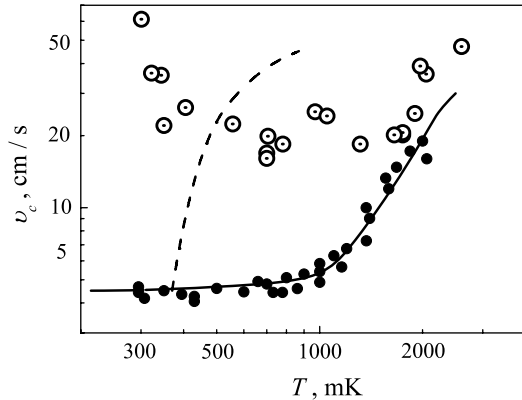
The jumps in the curve taken on the mixture may point to the influence produced by the  $^3\text{He}$  impurity. The critical signal amplitude corresponding to the transition to turbulence is shown in Fig. 3 (dashed curve).

It is known that at low temperatures  $^3\text{He}$  is adsorbed at the quantized vortex cores [23]. Previously, the adsorption of  $^3\text{He}$  at the vortex core was observed experimentally in [24, 25]. It was shown that in solutions with higher  $^3\text{He}$  concentrations such  $^3\text{He}$  adsorption in vortex core took place at higher temperatures. By extrapolation the data of [24, 25] to the 5%  $^3\text{He}$  in  $^4\text{He}$  solution we can conclude that the  $^3\text{He}$  adsorption at vortex cores occurs at  $T \leq 0.9$  K. This is clearly seen in Fig. 3 in which the first jump in the amplitude is observed at  $T = 0.71$  K.

**Fig. 3** The amplitude-frequency characteristics of a tuning fork immersed into a 5%  $^3\text{He}$ - $^4\text{He}$  solution at  $T = 1.03$  K ( $\nabla$ ), 0.71 K ( $\bullet$ ), 0.35 K ( $\circ$  increasing frequency,  $\Delta$  decreasing frequency). The exciting voltage is 20 V. Solid line HeII,  $T = 0.35$  K; the dashed line describes the critical amplitude above which turbulence starts in the investigated mixture



**Fig. 4** The temperature dependences of the critical velocity of the laminar-turbulence transition in 5%  $^3\text{He}$ - $^4\text{He}$  solution ( $\odot$ ) and pure  $^4\text{He}$  ( $\bullet$ ). The region with anomalous feature in the resonance curves is to the left of the dashed line



Thus, the irregularities appearing in the amplitude-frequency dependence are most likely related to  $^3\text{He}$  adsorption at the vortex core. A decrease in the temperature assists  $^3\text{He}$  adsorption at the vortex and irregularities start to appear at progressively smaller signal amplitudes.

It should be noted that at higher excitation control voltages the tuning fork can release heat and develop nonequilibrium conditions in the course of measurement.

## 5 Critical Flow Velocity on Transition to Turbulence

The critical flow velocity on a laminar-turbulence transition was estimated from the intersection of linear and quadratic dependencies in the  $I$ - $V$  characteristics (like in [19]). The current-velocity constant was early defined in  $^4\text{He}$ , in same way as was done in [19]. Later this constant for same tuning fork was used in the measurements of the 5%  $^3\text{He}$  mixture. The temperature dependence of the critical velocity is shown in Fig. 4 for the investigated solution along with the data for pure  $^4\text{He}$ , where the solid line is drawn through the experimental points. The dashed line separates the temperature and velocity regions in which the features of the resonance curves were present (on the left) or absent (on the right). These features in the amplitude-frequency dependences below 0.5 K were also observed in the laminar flow of the solution (see Fig. 4). They may indicate that quantized vortices can be present in a superfluid solution before the onset of turbulence.

It is seen that in solution the critical flow velocities are higher than in pure  $^4\text{He}$ , and its temperature dependence is nonmonotonous. This behavior of the critical velocity as a function of temperature can be attributed to the rather large quantity of the normal component density which at  $T \leq 1$  K is determined mainly by the  $^3\text{He}$  impurities.

## 6 Conclusions

This series of experiments has provided the first information about the specific features of the transition to turbulence in a dilute superfluid  $^3\text{He}$ - $^4\text{He}$  solution. In contrast to pure  $^4\text{He}$ , the solution enhances the laminar flow stability and the critical

velocity of the transition in it's several times higher, especially at low temperatures. Further experimental and theoretical investigations are called for adequate interpretation of the processes observed.

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