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Preliminary Measurements on Grid Turbulence in Liquid ^{4}He

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Abstract A grid has been pulled through a column of liquid helium at speeds as high as 1 m/s and at temperatures as low as 90 mK. A 300 micrometer Ge thermometer with response time of less than 1 ms measured the temperature rise resulting from the decay of the turbulence generated. It is believed that homogeneous, isotropic quantum turbulence was formed, since mesh Reynolds numbers in excess of 100,000 were created. The rates and power spectra of the energy increases detected in the helium after grid-pulls are determined. The results are compared to other quantum and classical results, and to the theory of the Kelvin wave cascade in a viscosity-free fluid.

Keywords Superfluid ^{4}He · Turbulence · Kolmogorov · Grid

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1 Introduction

Liquid ^{4}He may exist in two states: He I and He II. He I is considered a classical liquid with all normal liquid properties, including non-zero viscosity. He II has been under intense scrutiny ever since its strange properties were observed that are attributed to superfluidity [1, 2]. He II is not a simple superfluid, which would have zero viscosity. Rather, it is composed of two interpenetrating fluids, each composed of the same set of helium atoms, represented by a quantum wave function [3–5]. These two fluids interact through mutual friction [6]. Because the viscosity of even He I is low, it is not difficult to produce turbulent flow in it. Since the normal component of He II vanishes as T approaches absolute zero [7], it was originally expected that the Reynolds number in He II would approach infinity as T approached zero, making

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non-turbulent flow non-existent at $T = 0$. At least at higher temperatures, an effective viscosity seems to exist [8] making it still necessary to produce somewhat rapid flow of the fluid to generate turbulence at low temperature. Another feature which makes He II quite different from a classical fluid is the quantized circulation of vorticity: each and every vortex has exactly the same circulation [9]

$$\kappa = \frac{h}{m_4} = 0.001\ \mathrm{cm}^2/\mathrm{s} \tag{1}$$

All these facts contrive to make liquid helium a very fertile medium in which to study turbulence.

Normal/superfluid component counter flow, generated by a heater, was initially used to produce flow in superfluid helium and, in some cases, turbulence [10]. Vibrating bodies and rotating disks have also been used to produce turbulence in the superfluid [11–14]. These all produce specialized, and in various ways, complicated flow patterns, limiting analysis, particularly when trying to compare to classical fluids. Two component counterflow does not exist in classical fluids, including He I.

Many classical studies have used flow past a grid, either in a wind tunnel or by pulling the grid through the fluid. Where as vibrating objects create specialized forms of turbulence, the flow through a grid produces turbulence which is homogenous. One obvious question, when investigating superfluid turbulence, is how does it compare to flow of a classical fluid past a grid. One experiment has been accomplished by pulling a grid through superfluid helium above 1 K, when there is significant normal component (∼1%) and viscosity [15]. Here we present preliminary measurements on flow past a grid in liquid helium below 1 K, where the normal component and viscosity become vanishingly small.

2 Experimental Details

The broad opportunities offered by studying liquid helium near absolute zero are somewhat offset by the difficulty of producing experimental data. This has resulted in simulations, in many ways, proceeding measurement. The reader is encouraged to peruse this literature [16–20]. Simulations and the few measurements available suggest that decaying turbulence in a pure superfluid will undergo something similar to a Richardson cascade, but will not end in the energy being dissipated through viscosity (which is non-existent). One idea [21] is that at the end of the Richardson cascade, another (Kelvin wave) cascade will carry the energy to smaller scales. The vortex tangle will contain many Kelvin wave vibrations, whose wavelengths will progressively shorten through a non-linear process. This non-linear process is a result of sharp kinks produced on the vortex lines by progressive intersecting and reconnection of the vortex lines. This Kelvin wave cascade thus produced will end with the turbulent energy contained in wavelengths comparable to the quadrapole frequency of the kelvin wave. The energy will then couple to phonons and result in heating.

The requirements to measuring this Kelvin wave cascade are (a) produce turbulence in a volume of helium at millikelvin temperature and (b) detect the temperature rise produced. The temperature is expected to rise some time after the turbulence begins to decay, and to last for a short time. Therefore, the challenges of the experiment are: (a) produced turbulence quickly (10 s of milliseconds) in an isolated sample of helium, without producing extraneous heat, and (b) measure the temperature of the helium be sampled at a rapid rate (milliseconds) and for a relatively long time (seconds) over a wide temperature range (10 s of millikelvin). To meet these challenges, a motor was designed and built [22] which can move the grid at 1 m/s and sub-millimeter size thermistors we've developed [23] to measure the time dependent temperature rise in the helium after the grid has stopped. These were mounted in a copper helium cell that was weakly thermally coupled to the mixing chamber of a dilution refrigerator.

The experimental procedure was to stabilize the cell temperature with a puddle of liquid in the bottom, where the grid (similar to Stalp et al. [15]) was suspended just above the thermistors which were mounted on the bottom of, but not in direct thermal contact with, the cell. The grid was then pulled upward with a rapid acceleration (average 50 m/s^2) and speeds up to 1 m/s through the helium puddle and held above the puddle for a short time until it was allowed to fall back through the puddle. During this motion the thermistor resistance was monitored with a bridge driven at 70 kHz and a time constant of about 300 μs. The motion of the grid pulling motor was monitored with a capacitive position sensor operated in a bridge driven at 90 kHz and a time constant of about 300 μs. The motor, position sensor, thermistor, and cell body temperature were all controlled and monitored by a National Instruments Board and LabView program [24]. The board had a sampling rate of 50 μs.

3 Results

3.1 Results with Empty Cell

Measurements at 0.52 K will be the focus of this preliminary report. The grid was pulled about 4 mm through the liquid helium, held above the helium surface for about 0.03 s, and then driven down to its lowest position. A graph of the speed of the grid as a function of time is shown in Fig. 1, where positive speeds represent a rising grid (and negative speeds falling). The mesh Reynolds number is

$$Re_M = V_g M\rho/\nu \tag{2}$$

where V_g is the grid velocity, M is the mesh size and ρ/ν is the kinematic viscosity. For this motion just described Re_M can be calculated if the kinematic viscosity ν/ρ is know, however this is currently a topic of debate. Fig. 1 gives two calculations of Re_M using the results of Stalp et al. [8] (right scale) and Golov et al. [25] (left scale). Either one indicates turbulent flow. These measurements can eventually help determine the kinematic viscosity through consistency arguments.

The grid was pulled in an empty cell yielding baseline measurements against which future measurements will be compared. First, at room temperature, all electronics were operated as at low temperature, except that no current was sent to the motor, as it has a resistive solenoid at room temperature. No thermal effects were seen, and a power spectrum was measured. Using the Debye T^3 heat capacity for the Ge thermistor ($T_{\text{debye}} = 480$ K), the power spectrum of Fig. 2 was observed. The best-fit power law is $f^{-1.73\pm.03}$.

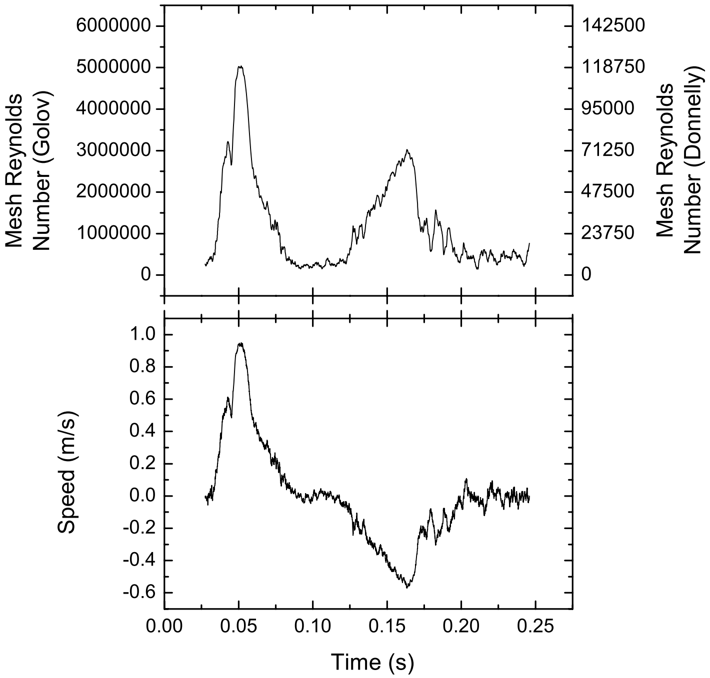

Fig. 1 Grid speed and mesh Reynolds number during the measurement at 0.52 K. Grid speed is obtained by differentiating the grid position vs. time graph, increasing "noise". The two sets of mesh Reynolds numbers use kinematic viscosities from two different experiments: Golov et al. [25] and Stalp et al. [8]

Next, at 0.6 K the grid was pulled in the empty cell, yielding the thermal response shown in Fig. 3. Also shown is the thermal response after pulling the grid through helium at 0.52 K. There is an apparent sudden rise in the thermistor temperature as soon as current is applied to the motor, which tends to a plateau, not recovering after the motor current ceases. Since the small rise occurs in both the empty cell and in liquid helium, it is not due to turbulence. And since it is constant, it doesn't effect the analysis of the heating caused by turbulence.

3.2 Results in Helium $T = 0.52$ K

A plot of the energy released into the helium after the grid is pulled starting at $T = 0.52$ K, as measured by the thermistor, is shown in Fig. 4. This is calculated from the measured temperature rise using enthalpy of ^{4}He. The grid position vs. time is also shown. It can be seen that the same immediate rise in energy occurs as current is applied to the motor as seen in the empty cell at 0.6 K. In addition, a much larger rise, with rapid oscillations, is seen to develop about 0.23 s after the beginning of the grid motion. The classical drag on the grid in helium

$$F = CAV_g^2\rho/2 \tag{3}$$

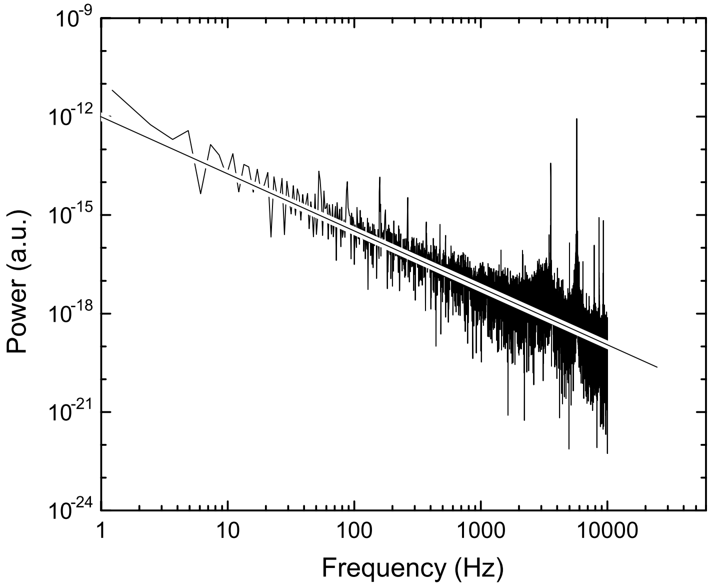

Fig. 2 This is the power spectrum for the heat measured by the thermistor at 0.6 K on the occasion when the current provided to the motor did not provide enough force to move it. The *line* is a power law fit to the data for the range of frequencies between 10 Hz and 3000 Hz. The frequency exponent derived from the slope of this fit is 1.73 ± 0.03

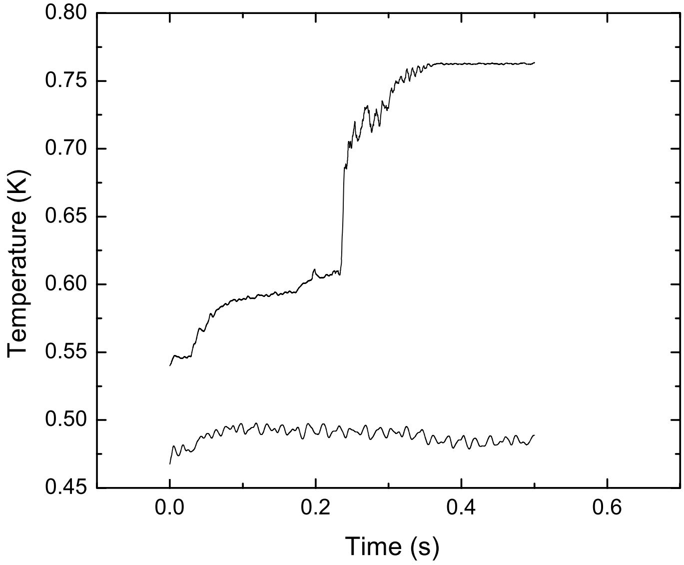

Fig. 3 This is the temperature rise measured by the thermistor after the grid is pulled for the case where there is helium in the cell (*upper curve* at initial $T = 0.52$) K and when there is not helium in the cell (*lower curve* at initial $T = 0.6$ K). Where the plots have been shifted to better differentiate the two curves

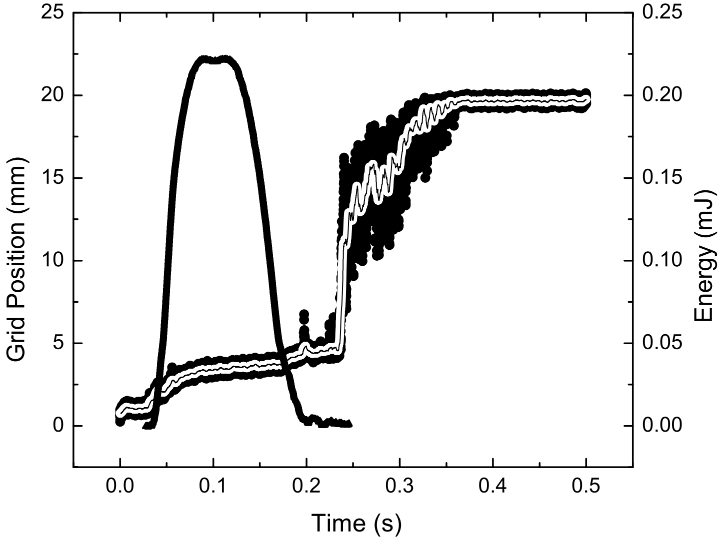

Fig. 4 Time graph of grid position and energy released into the helium of Fig. 1. As can be seen, all grid motion ceases before the energy enters the helium. The *line insert* on the energy use is an averaging representing a 3000 Hz filter. As can be read off the graph, there is a total energy rise of ∼.15 mJ

is calculated to inject ∼30 μJ into the liquid (turbulent motion), where A is the opaque area of the grid (1.78 cm^2), C is the coefficient which is taken to be unity, and ρ is helium density at 0.52 K. As can be seen from Fig. 4, there is a 150 μJ measured rise in energy. This comparison is favorable, since on warming a leak resulted in the inability to measure the precise amount of helium that was in the cell. The numbers quoted here represent a lower bound on the volume of helium under test.

4 Analysis of Data at $T = 0.52$ K

It is expected that the heating seen after the grid is pulled is caused by the decay of the generated turbulence, and not mechanical friction of the motor, as discussed in previous work [22]. Briefly, the cell has no sintered heat exchanger and all components of the motor are above the liquid helium in vacuum. We noticed a significant difference in the amount of time it took the thermistors and the cell to cool down to base temperature after we ran the motor in vacuum. The thermistors cooled much faster which told us they were in better contact with something colder outside the cell (mixing chamber) than anything inside the cell, including the cell itself. Of course, with helium in the cell, the thermistors are best thermally coupled to the turbulent helium.

Initially, two quantities may be extracted from the temperature measured against time after a grid pull: (1) the power spectrum of the energy fluctuations; and (2) the rate of rise of the average energy of the ^{4}He in the cell.

4.1 Power Spectrum of Energy Fluctuations

As can be seen in Fig. 5, the energy spectrum $E(f)$ follows the Kolmogorov prediction. Similar to the findings of Maurer and Taberling [11], the power spectrum of the measured energy follows a $-5/3$ power law. However, on further scrutiny of the data and attempting many different fits, we have observed that the best fit is very sensitive to the number of points used in the fit. This is a feature of the noise (or chaos) of the system. Changing the number of fit points by the equivalent of 50 Hz in frequency space can change the exponent by as much as 0.1. We notice a change in the structure of the spectrum beginning at about 3000 Hz in Fig. 5, and this coincides with noise observed in the empty cell measurement of Fig. 2 This leads us to cut off the fit at this frequency. The energy spectrum of the decay seen in simulations goes as f^{-1}, as compared to the Kolmogorov dependence of $f^{-3/2}$.

The energy spectrum of the thermistor taken across the entire data set is Kolmogorov like ($f^{-5/3}$), not f^{-1} as predicted by the Kelvin wave theory. However the power spectrum taken from .23 to .37 seconds of the energy rise in the thermistor (Fig. 4) has a lower exponent $k^{-1.2\pm.1}$, again with the large error due to the same type of fluctuations in the spectrum as the two shown above. This spectrum was taken over this particular region in order to focus our results on the time period when the thermistor is in fact measuring an effect from the helium. By decreasing the number of cycles of background noise used in the Fourier transform we believe that this number is a more accurate measurement of the frequency dependence of the helium in out system.

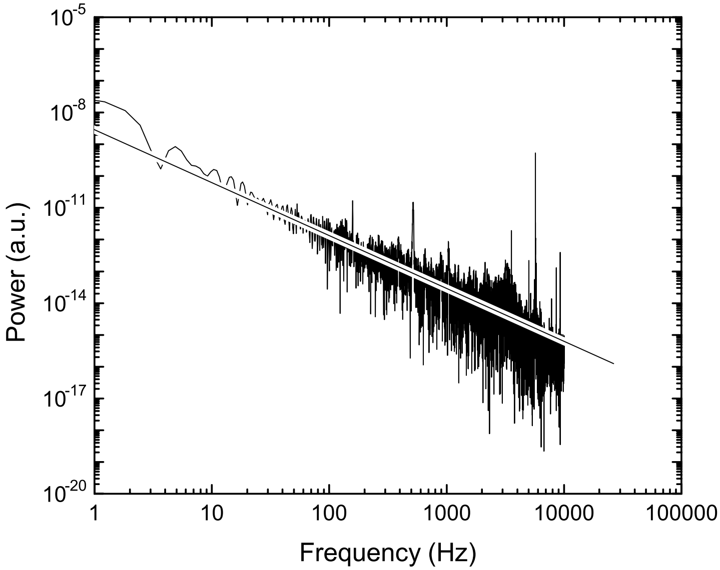

Fig. 5 This is the power spectrum for the energy measured by the thermistor on the occasion when the grid moved through the He. The *line* is a power law fit to the data for the range of frequencies between 10 Hz and 3000 Hz. The frequency exponent derived from this fit is 1.660.02, where 0.02 is the standard deviation of the linear fit and certainly an underestimate of the overall error, as detailed in the text

4.2 Rate of Energy Rise

In the absence of viscosity, one might expect that turbulence can neither be generated, nor will it decay. Experience shows that turbulence is fairly easy to create in superfluid helium, even near absolute zero, and that it does decay. At temperatures above 1 K, the decay is seen to follow a Richardson-type cascade producing a Kolmogorov energy spectrum. This is thought to come about because even a small amount of normal fluid will couple to the super component through mutual friction, making all the fluid act as a classical fluid. Near $T = 0$, the absence of the normal component leads one to expect a different decay path. The large eddies composed of many quantized vortex lines that are created by the mesh of the pulled grid will vibrate freely (Kelvin waves) because of the lack of viscous drag. As one vortex line encounters another vortex line, breaking and reconnections will produce kinks that become more and more closely spaced along the lines. This might be like a Richardson cascade, but will not be observed thermally. However, when the wavelength of the Kelvin waves reaches a certain frequency (likely that of quadrapole radiation) the vibrations of the vortex lines will be damped as the energy radiates into the sound modes. This will be observed as heat in the fluid. The rate of heating is predicted [21] to obey a $t^{-3/2}$ power law, while classically it would be t^{-2}.

The preliminary results at 0.52 K are not in agreement with the predictions and simulations of the Kelvin wave cascade (see Figs. 6, 7). The beginning time-decay of the turbulent energy is slightly faster than $t^{-3/2}$, measured as $t^{-1.6}$ (Fig. 6). But

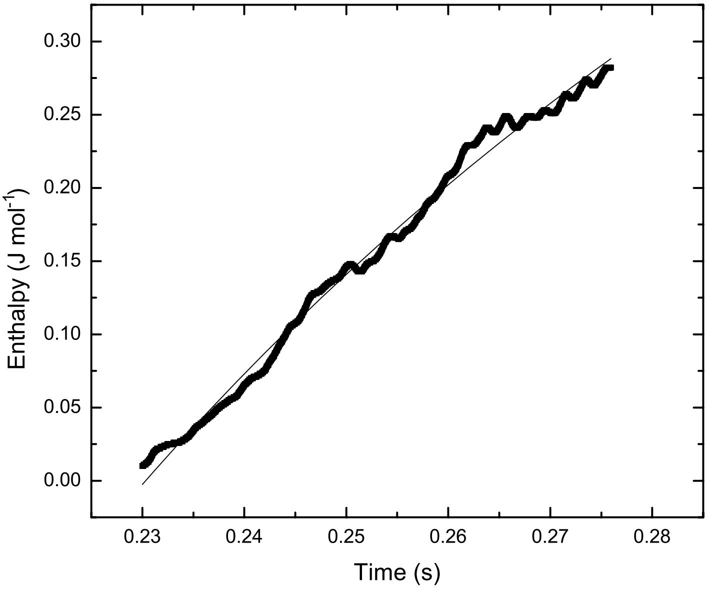

Fig. 6 This shows the time dependence of energy dissipated by the helium into the thermistor for the grid motion seen in Fig. 4 for the time period of 23 ms to 28 ms after the grid is pulled. The *rough line* is a spline interpolation of the energy rise shown in Fig. 4 and the *smooth line* is a best fit curve through the interpolated data. The time dependence for this line is $H(t) = \sim t^{1.6\pm0.1}$

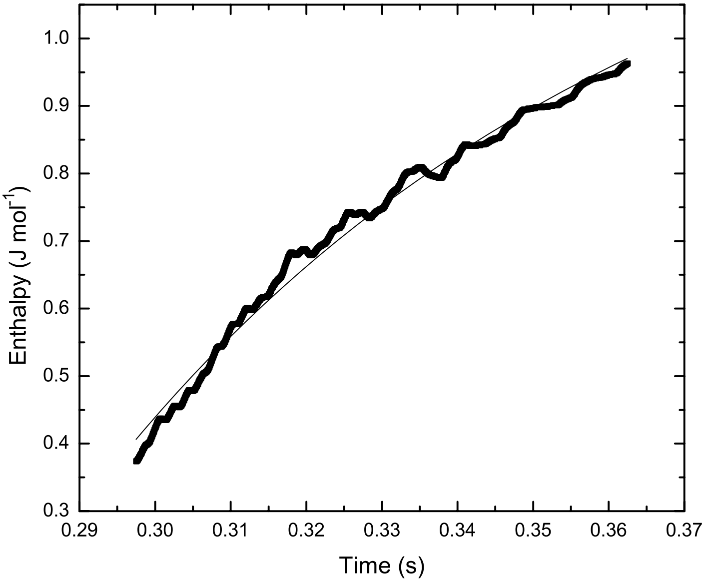

Fig. 7 This shows the time dependence of energy dissipated by the helium into the thermistor for the grid motion seen in Fig. 4. for the time period of 30 ms to 37 ms after the grid is pulled. The *rough line* is a spline interpolation of the energy rise shown in Fig. 4 and the *smooth line* is a best fit curve through the interpolated data. The time dependence for this line is $H(t) = {\sim}t^{3.6\pm0.1}$

it is composed of two segments that seem to have different exponents. The second segment has a much higher exponent $t^{-3.6}$ (Fig. 7).

Perhaps these conflicts with the Kelvin wave cascade theory and the complexity of these results is caused by the measurements being made in a transition region between the higher temperature quasi-classical behavior and the quantum behavior expected closer to absolute zero.

5 Future Work

These preliminary measurements do not definitively raise one theory over another. It is still possible that the Kelvin wave cascade is an important feature of quantum turbulence decay. But these measurements indicate that the subject of quantum turbulence may be even richer than was originally suggested, and is ripe for study. The next thermal measurements will be made near absolute zero.

The power spectra measured suggest that these might be key to understanding quantum turbulence. Pressure fluctuation measurements near absolute zero are being pursued.

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