J Low Temp Phys (2008) 150: 415–419
DOI 10.1007/s10909-007-9562-z

Numerical Simulations of Superfluid Turbulence under Periodic Conditions

Luiza P. Kondaurova ·
Vladimir A. Andryuschenko ·
Sergey K. Nemirovskii

Received: 29 June 2007 / Accepted: 28 September 2007 / Published online: 28 November 2007

Abstract This paper is devoted to numerical simulation of vortex tangle dynamics in superfluid helium. The problem is solved on the base of the so called reconnection ansatz consisting of the equation of motion for vortex lines plus reconnection of a loop. A new algorithm, which is based on consideration of crossing lines, is used for the reconnection processes. Calculations are performed for a cubic box. Periodic boundary conditions are applied in all directions. We use the 4th order Runge-Kutta method for the integrations in time. The dynamics of quantized vortices with various counterflow velocities is studied. The density of vortex lines and number of reconnections as functions of vortex line density are calculated.

Keywords Vortex tangle · Superfluid turbulence · Reconnections

PACS 67.40.Vs · 98.80.Cq · 7.37.+q

1 Introduction

A vortex tangle (VT), which consists of chaotic vortex loops, influences many properties of superfluid helium [1, 2]. The whole dynamics of this system is very complicated to be describe analytically, therefore numerical simulations are almost the only source of knowledge about vortex tangle. There are many works devoted to numerical investigations of vortex tangles and to turbulent state of helium, (e.g., see [3–7]). These works had been fulfilled in the similar manner based on pioneering paper by Schwarz [3]. Namely the calculations have been done with application of the so called reconnection ansatz. The main idea of the reconnection ansatz is that the equation of

L.P. Kondaurova · V.A. Andryuschenko · S.K. Nemirovskii (✉)
Kutateladze Institute of Thermophysics SB RAS, 630090 Novosibirsk, Russia
e-mail: nemir@itp.nsc.ru

motion in the local induced approximation is supplemented with the forced reconnection whenever some criteria are reached. Schwarz himself proposed this criterion in the form that the vortex lines reconnect whenever the minimal distance between the pair of vortices, Δ becomes $\Delta = 2R/[c\ln(R/a_0)]$, where R is the radius of curvature at the given point, c is the constant ($\simeq$1), and a_0 is the radius of the vortex core. Other authors (e.g. [7]) had chosen the mesh size ξ as a minimal distance. Thus, in the cited works only the distance between the points of a vortex line was chosen as the criterion for reconnection. In our opinion, this is slightly incorrect approach, because the elements of filaments can go away from each other and the crossing may not occur. This can distort real dynamics of vortex filaments and lead to unadequate results. For instance, in papers [3, 6] degeneration of stochastic picture occurred and the vortex tangle changed into a state with lines stretching from wall to wall with poor dynamics and rare events. In contrast we offer a new algorithm of reconnection events, which is based on consideration of crossing lines. We demonstrated that in this case the problem of degeneration of vortex tangle is removed. We completed several numerical experiments and explored some physical quantities, such as the vortex line density and the rate of reconnections.

2 Statement of Problem

Let us consider the main steps of problem formulation such as the equation of motions, reconnection procedure and conditions of numerical experiment. We start with dynamics of the vortex loops. In the presence of the friction force of vortices on the normal component of helium, velocity $\dot{\mathbf{s}}$ of the vortex line elements is [3]:

$$\dot{\mathbf{s}} = \dot{\mathbf{s}}_0 + \alpha\big(\mathbf{s}' \times (\mathbf{v}_{ns} - \dot{\mathbf{s}}_0)\big) - \alpha'\mathbf{s}' \times \big[\mathbf{s}' \times (\mathbf{v}_{ns} - \dot{\mathbf{s}}_0)\big] \quad (1)$$

where $\mathbf{s}(\xi, t)$ is the radius-vector of the vortex line points, ξ is the arc length, $\mathbf{s}'$ is derivative with respect to the arc length ξ. α and α' are the friction coefficients. Quantity $\dot{\mathbf{s}}_0$ is the self induced velocity, which is taken as the local induced approximation. Let us consider now the reconnection of lines (see [8]). The first step in modeling the reconnection process is selection of the point pairs, which are the candidates for reconnection. After the pairs was defined, it was assumed that the line segments between each of the pair were moving with a constant velocity (according V_i, V_j) during

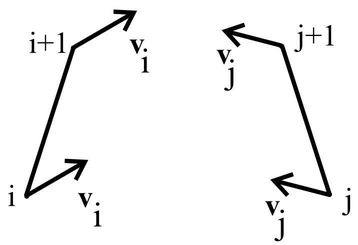

Fig. 1 Schematic picture for crossing line elements

the time step, as illustrated in Fig. 1. From the compatibility of equations

$$\begin{aligned} x_i + V_{x,i}h + (x_{i+1} - x_i)s1 &= x_j + V_{x,j}h + (x_{j+1} - x_j)s \\ y_i + V_{y,i}h + (y_{i+1} - y_i)s1 &= y_j + V_{y,j}h + (y_{j+1} - y_j)s \\ z_i + V_{z,i}h + (z_{i+1} - z_i)s1 &= z_j + V_{z,j}h + (z_{j+1} - z_j)s, \quad 0 \le s1 \le 1;\ 0 \le s \le 1 \end{aligned} \tag{2}$$

the meeting of these line segments during the time step was determined. Here $(x_i, y_i, z_i, x_{i+1}, y_{i+1}, z_{i+1})$; $(x_j, y_j, z_j, x_{j+1}, y_{j+1}, z_{j+1})$ are the coordinates of the first and the second pairs of points, accordingly; $V_{x,i}, V_{y,i}, V_{z,i}$; $V_{x,j}, V_{y,j}, V_{z,j}$ are the projections of the velocities of the points and the line segments on the coordinate axis. If the line segments meet, the reconnection occurs. Thus, if originally the points belonged to the same loop, a pair of new loops was generated. Otherwise the confluence of the loops occurs. The equation of motion (1) with the reconnection condition (2) is the base for our numerical simulation of the stochastic vortex line dynamics. We run the calculations at the temperature $T = 1.6$ K (which corresponds to choice $\alpha = 0.098$, $\alpha' = 0.012$) in the computing cubic box $50 \times 50 \times 50\ \mu\text{m}^3$. In our calculations we usually start with an initial vortex configuration of twenty four vortex rings as it is shown in Fig. 2 (upper left). The periodic conditions in all three direction were taken. Counterflow velocities were taken to be equal 6, 8, 12 cm/s. The initial condition was chosen to make the total impulse of the system equal to zero. For the integration of the motion equation (1) in time we used the classical 4th order Runge-Kutta method.

3 Results

The outcome of a typical numerical experiment is shown in Fig. 2. Here the initial configuration of vortex loops evolves in the periodical box accordingly to the equation of motion (1) with the reconnection condition (2). The obtained numerical results demonstrate that initially smooth vortex rings transform into highly chaotic vortex tangle with a uniform structure. We controlled the development of the process by monitoring one of the key characteristics of vortex tangle, namely the vortex line density $L(t)$. In Fig. 3 we depicted the time evolution of quantity $L(t)$. It is seen that after some transient period the vortex line density takes more less steady state but with essential fluctuations. The transient time essentially depends on the applied driving counterflow velocity $\mathbf{v}_{ns}$. Note that in the similar work [6] the authors did not obtain the steady stationary case at all. Stationary situation had been reached in work [3] only after the use of the special “mix” procedure. We think that these failures of mentioned works is related to the unsuitable reconnection procedure. The very important question is dependence of the vortex line density on the applied velocity $\mathbf{v}_{ns}$ in the stationary regime. Analyzing the data in Fig. 3 we found out that with good accuracy this dependence is $L(\mathbf{v}_{ns}) = \gamma^2 \mathbf{v}_{ns}^2$ with $\gamma \approx 280$. As it was asserted in many previous works (e.g., see reviews [1, 2]) quantity L is indeed proportional to the squared counterflow velocity.

We also explored the behavior of some reconnections $N(t)$ as the function of time. We distinct the reconnections leading to breakdown of vortex loops to a fusion of loops.

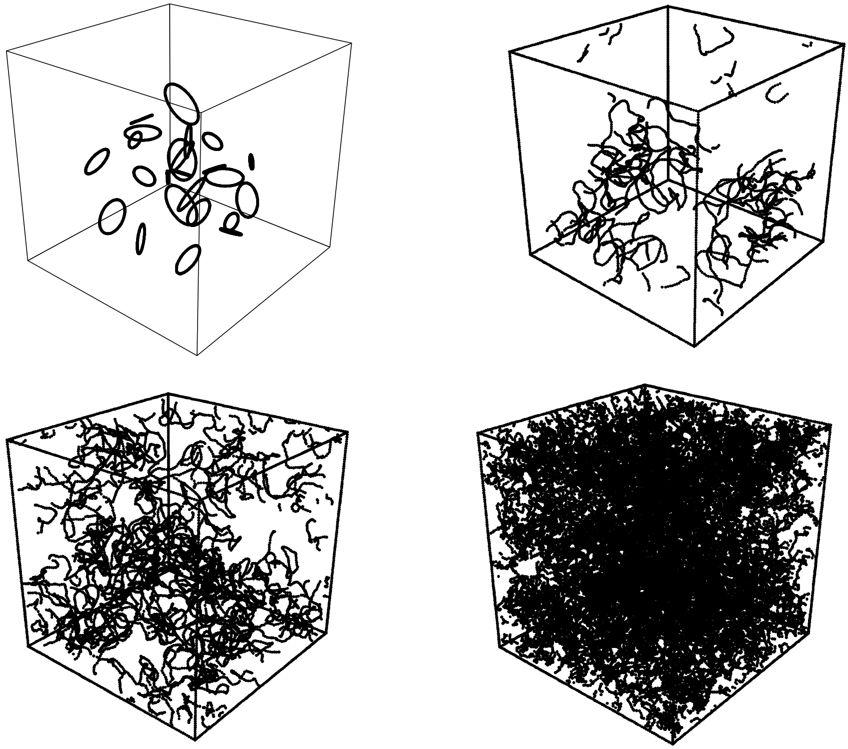

Fig. 2 Development of a vortex tangle in the periodical box with edge 50 μm. Here $\alpha = 0.098$, $\alpha' = 0.012$ corresponds to temperature $T = 1.6$ K. Counterflow velocity is taken 12 cm/s. *Upper left* $t = 0$, *upper right* $t = 2$ ms, *lower left* $t = 7$ ms, *lower right* $t = 10.8$ ms

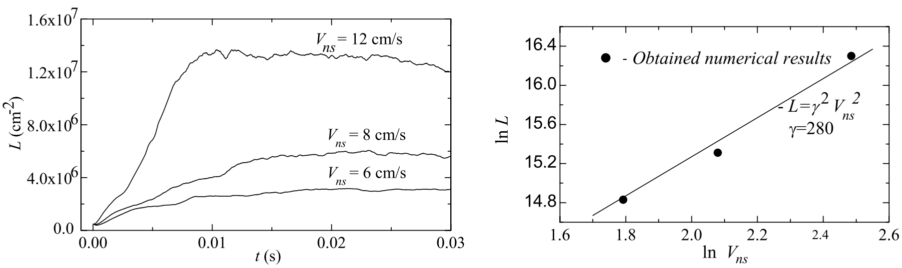

Fig. 3 Vortex line density as a function of time for different driving velocities $\mathbf{v}_{ns}$ equal to 6, 8, 12 cm/s

In Fig. 4 (left) we depicted quantities $N(t)$ for the breakdown and fusion processes for various applied velocities. As it is seen they are very close to each other, although the breakdown processes slightly prevail. From these graphs we are able to extract the rate of reconnections $dN/dt = N'(t)$ (the number of events per unit volume and unit time) in the stationary regime. Analyzing the data in Fig. 4 (right) we concluded

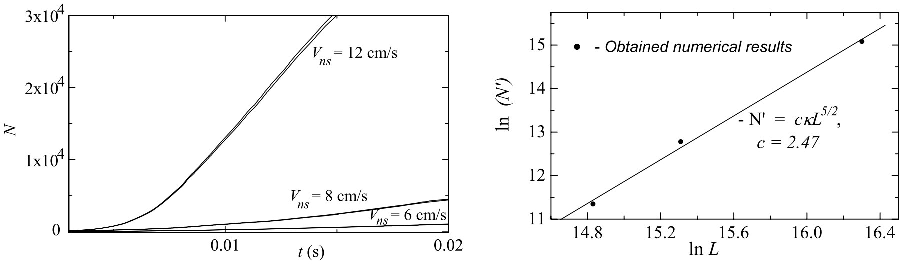

Fig. 4 (*left*) Number of reconnections as a function of time for the breakdown and fusion processes for the various applied velocities. (*right*) Rate of reconnections in the stationary regime

that

$$N' \approx C\kappa L^{5/2} \tag{3}$$

with $C \approx 2.47$. This dependence was early predicted in numerical [7, 9] and analytical [10] works. The upper lines are responsible for the breakdown processes, and the lower ones—for the fusion processes.

4 Conclusions

The obtained numerical results demonstrate that the initially smooth vortex rings transform into a highly chaotic vortex tangle. In spite of that the total length fluctuates around a constant value; we think that we reached the stationary state. The use of new reconnection algorithm removed the problem of degeneration of the vortex tangle. The density of vortex lines and number of reconnections as the functions of vortex line density were calculated. The results obtained satisfactory agree with the ones obtained or predicted in early works.

Acknowledgements This work was partially supported by grants 05-08-01375 and 07-02-01124 from the RFBR and grant of President RF NSH-6749.2006.8.

References

1. R.J. Donnelly, *Quantized Vortices in Helium II* (Cambridge University Press, Cambridge, 1991)
2. S.K. Nemirovskii, W. Fiszdon, Rev. Mod. Phys. **67**, 37 (1995)
3. K.W. Schwarz, Phys. Rev. B **31**, 5782 (1985)
4. T.F. Buttke, J. Comput. Phys. **76**, 301 (1988)
5. D.C. Samuels, Phys. Rev. B **46**, 11714 (1992)
6. R.G.M. Aarts, A numerical study of quantized vortices in HeII, Technische Universiteit Eindhoven, Eindhoven, 1993
7. M. Tsubota, T. Araki, S.K. Nemirovskii, Phys. Rev. B **62**, 11751 (2000)
8. L.P. Kondaurova, S.K. Nemirovskii, J. Low Temp. Phys. **138**, 531 (2005)
9. C.F. Barenghi, D.C. Samuels, J. Low Temp. Phys. **136**, 281 (2004)
10. S.K. Nemirovskii, Phys. Rev. Lett. **96**, 015301 (2006)