Equity portfolio construction and selection using multiobjective mathematical programming

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Abstract   A multi-objective mixed integer programming model for equity portfolio construction and selection is developed in this study, in order to generate the Pareto optimal portfolios, using a novel version of the well known ε-constraint method. Subsequently, an interactive filtering process is also proposed to assist the decision maker in making his/her final choice among the Pareto solutions. The proposed methodology is tested through an application in the Athens Stock Exchange.

Keywords   Portfolio optimization · Multiobjective mathematical programming · ε-Constraint method · Equities

1 Introduction

The portfolio management process is an integrated set of steps undertaken in a consistent manner to create and maintain an appropriate portfolio (combination of assets) to meet clients' stated goals (Maginn et al. 2007). The three fundamental elements in managing any business process are: planning, execution and feedback. The same steps form the basis for the portfolio management process. In the planning step, investment objectives and policies are formulated, capital market expectations are formed and strategic asset allocations are established. In the execution step, the manager constructs the portfolio and integrates investment strategies with capital market expectations to select the specific assets for the portfolio. Finally, in the feedback step, the manager monitors and evaluates the portfolio compared with the plan. Any changes suggested by the feedback must be examined carefully to ensure that they represent long-run considerations. The emphasis in this article is laid on the portfolio construction and selection phase and we focus on equity portfolios design. The portfolio construction problem has several dimensions and as we it will be proved, the framework of multiple criteria decision making provides the solid methodological basis to resolve the inherent multicriteria nature of this problem.

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The main goal of the current study is to develop an integrated multiple-criteria portfolio construction and selection methodology, within the context of multiobjective mathematical programming. More precisely, a multiobjective mixed integer programming model for equity portfolio construction is developed, in order to generate the Pareto optimal portfolios using a novel version of the ε-constraint method. The integer (more specifically the binary) variables offer increased flexibility to the decision model, as logical conditions can also be incorporated along with the usual constraints. Afterwards, an interactive filtering process is also proposed to assist the decision maker in making his/her final selection among the solution set. The decision maker's investment policy, i.e., constraints regarding the portfolio structure, are strongly taken into account.

The paper proceeds as follows: In Sect. 2 we set the problem and we analyze the portfolio construction phase, within the frame of the portfolio management process. In Sect. 3 we provide an elaborate review of the coherent studies relevant to the portfolio selection problem, in the MCDM context. More specifically, a very large number of studies in the field of portfolio management have been compiled and classified. In Sect. 4 we present the proposed methodological framework for the portfolio selection process and in Sect. 5 we analyze the model building procedure and the corresponding application from the Athens Stock Exchange (ASE). Finally, the results and the concluding remarks are given in Sects. 6 and 7.

2 Problem setting

According to Maginn et al. (2007), portfolio management is an ongoing process in which: (1) investment objectives and constraints are identified and specified, (2) investment strategies are developed, (3) portfolio composition is decided in detail, (4) portfolio decisions are initiated by portfolio managers and implemented by traders, (5) portfolio performance is measured and evaluated, (6) investor and market conditions are monitored, and (7) any necessary rebalancing is implemented. As stated previously, the portfolio management process is an integrated set of three steps: planning, execution and feedback. More specifically, in the execution step, portfolio managers initiate portfolio decisions based on analysts' inputs, and trading desks and then implement these decisions (portfolio implementation decision). Subsequently, the portfolio is revised as investor circumstances or capital market expectations change; thus, the execution step interacts constantly with the feedback step. In making the portfolio selection/composition decision, portfolio managers may use the techniques of portfolio optimization. Portfolio optimization—quantitative tools for combining assets efficiently to achieve a set of return and risk objectives—plays a key role in the integration of strategies with expectations. The portfolio implementation decision is as important as the portfolio selection/composition decision. Poorly managed executions result in transaction costs that reduce performance. Transaction costs include all costs of trading, including explicit transaction costs, implicit transaction costs, and missed trade opportunity costs. Additionally, in order to elaborate the relationship between the decision context of the investor and the economic environment of the securities, Spronk and Hallerbach (1997) decompose the investment decision process in the following stages: (1) security analysis to determine the relevant characteristics (or attributes) of the investment opportunities, (2) portfolio analysis to delineate the set of non-dominated or efficient portfolios, (3) portfolio selection to choose the optimal portfolio from the efficient set, and (4) preference analysis.

In recent years, the development of new techniques in operations research and management science, as well as the progress in computer and information technologies gave rise to new approaches for modeling the portfolio selection problem. Several authors have developed a

new approach, using Multiple Criteria Decision Making (MCDM) for portfolio management. The multidimensional nature of the problem has been emphasized by researchers in finance, as well as by MCDM researchers (Mavrotas et al. 2008; Xidonas and Psarras 2009; Xidonas et al. 2009a, b, c, d; Steuer et al. 2005, 2006a, b, 2007a, b; Zopounidis and Doumpos 2002; Zopounidis 1999; Hurson and Zopounidis 1993, 1995, 1997; Spronk and Hallerbach 1997; Zeleny 1977, 1981, 1982; Colson and Zeleny 1979, 1980; Jacquillat 1972). In the above studies, elaborate and exhausted justifications are provided for modeling portfolio management problems within the MCDM frame. The general idea is that the analysis of the risk nature in portfolio management shows that the latter comes from various origins and its nature is multidimensional. Also, individual goals and investor's preferences cannot be incorporated in these models. MCDM provides the methodological basis to resolve the inherent multicriteria nature of portfolio selection problem. Additionally, it builds realistic models by taking into account, apart of the two basic criteria of return and risk (mean-variance model), a number of important other criteria. Furthermore, MCDM, have the advantage of taking into account the preferences of any particular investor. To manage efficiently portfolio selection, it is necessary to take into account all the factors that influence the financial markets. Then, portfolio management is a multicriteria problem. Effectively, multifactor models point out the existence of several influence factors for the determination of the stock prices. Furthermore, fundamental analysis models, commonly used in practice, underline that stock prices are also dependant on the firm health and its capacity to pay dividends. The latter problem itself is a multicriteria problem because, in order to solve it, we must appreciate the profitability of the firm, its debt level (in the short and long terms) and quality of management. Finally, in practice, an investor has a personal attitude and particular objectives.

Moreover, Hurson and Zopounidis (1995, 1997) consider that the classical approach imposes a norm to the investor's behavior that can be restrictive. Also, it cannot take into account the personal attitude and preferences of a real investor confronted with a given risk in a particular situation. However, experience has proved that the classical approach is useful, for instance concerning the diversification principle and the use of the beta as measure of risk. Thus, the use of the classical approach seems to be necessary but not sufficient, to manage portfolio selection efficiently. Some additional criteria must be added to the classical risk-return criteria. In practice, these additional criteria can be found in fundamental analysis or constructed following the personal goals of the investor. The combination of the above principles is difficult because of the complexity of multicriteria problems on the one hand and the use of criteria from different origins and of conflicting nature on the other hand. Furthermore, MCDM will facilitate and favor the analysis of compromise between the criteria. It equally permits to manage the heterogeneity of criteria scale and the fuzzy and imprecise nature of the evaluation that it will contribute to clarify. Linking the multicriteria evaluation of an asset portfolio and the research of a satisfactory solution to the investor's preferences, the MCDM methods allow taking into account the investors' specific objectives. Furthermore, these methods do not impose any normative scheme to the comportment of the investors and allows synthesizing in a single procedure the theoretical and practical aspects of portfolio management, and then it allows a non normative use of theory.

3 Review of existing studies

Portfolio selection models are at the heart of the portfolio construction phase. Since the pioneering work of Markowitz (1952, 1959) in the theory of portfolio analysis, based on the mean-variance formulation, several portfolio selection models have been proposed.

According to this formulation, an investor regards expected return as desirable and variation of return (variance) as undesirable. In the basis of the Markowitz mean-variance formulation, many researchers developed miscellaneous new methodologies. Elton et al. (2007) provide a complete overview as far as these methodologies is concerned. Apart from the mean-variance model, they cite the single index models, the multi-index models, the average correlation models, the mixed models, the utility models in which the preference function of the investor plays a key role in the construction of an optimum risky portfolio, and the models which employ different criteria such as the geometric mean return, safety first, stochastic dominance and skewness. Pardalos et al. (1994), also, provide a review and some computational results of the use of optimization models for portfolio selection.

The portfolio construction problem can be realized as a two stage process (Hurson and Zopounidis 1995, 1997): (1) evaluation of the available securities to select the ones that best meet the investor's preferences, (2) specification of the amount of capital to be invested in each of the securities selected in the first stage. The implementation of these two stages in the traditional portfolio theory is based on the mean-variance approach. Within this multidimensional context, the MCDM paradigm provides a plethora of appropriate methodologies to support the evaluation of the available securities as well as portfolio synthesis/optimization. The former (securities' evaluation) has been studied by MCDM researchers using discrete evaluation methods (outranking relations, multi-attribute utility theory, preference disaggregation analysis, rough sets). Studies conducted on this topic have focused on the modeling and representation of the investor's policy, goals and objectives in a mathematical model. The model aggregates all the pertinent factors describing the performance of the securities and provides their overall evaluation. The securities with the higher overall evaluation are selected for portfolio synthesis purposes in a latter stage of the analysis. This stage is realized within the MCDM framework as a multiple-objective mathematical programming/goal programming problem. The decision maker specifies the portfolio synthesis criteria, his objectives/goals and an iterative and interactive process is invoked to identify a portfolio that best meets his investment policy.

Zopounidis et al. (1998) classify the studies concerning the use of multicriteria analysis in portfolio selection according to their special methodological background (Pardalos et al. 1995) as follows: (1) multiobjective mathematical programming, (2) multiattribute utility theory, (3) outranking relations, and (4) preference disaggregation approach. Doumpos (2000) categorizes the research studies in portfolio management in four basic classes: (1) Models focusing on the analysis and perception of the securities' behavior, (2) Forecasting models focusing on the rapid spotting of the security trends, (3) Security evaluation methodologies focusing on modeling of the investor's preferences, and (4) Portfolio synthesis and optimization methodologies.

A sample of some significant studies in the field follows. Saaty et al. (1980) propose to construct a portfolio using the analytic hierarchy process methodology. Lee and Chesser (1980) present a goal programming model to construct a portfolio. Rios-Garcia and Rios-Insua (1983) construct a portfolio using multi-attribute utility theory and multiobjective linear programming. Evrard and Zisswiller (1983) use multi-attribute utility theory to perform a valuation of some stocks. Nakayama et al. (1983) propose a graphics interactive methodology to construct a portfolio using multiple criteria. Martel et al. (1988) perform a portfolio selection using the outranking methods ELECTRE I and ELECTRE II. Colson and de Bruyn (1989) propose a system that performs a stock valuation and allows the construction of a portfolio. Szala (1990) performs stock evaluation in collaboration with a French investment company. Khoury et al. (1993) use the outranking methods ELECTRE IS and ELECTRE III to select international index portfolios. The purpose of Colson and Zeleny (1979) is to construct an

efficient frontier in concordance with the principles of stochastic dominance. Hurson and Zopounidis (1993) propose to manage the portfolio selection by using the MINORA system that will be presented in the following section. Zopounidis et al. (1998) propose the use of the ADELAIS system to construct a portfolio using some diversification constraints, some constraints representing the investor's personal preferences and multiple stock-market criteria. Tamiz et al. (1996) propose to use goal programming for portfolio evaluation and selection. Dominiak (1997a, b) presents a procedure for security selection that uses a multicriteria discrete analysis method based on the idea of reference solution. Hurson and Ricci (1998) propose to combine Arbitrage Pricing Theory (APT) and MCDM to model the portfolio management process.

Steuer et al. (2007b) employ six categories in order to place multiple criteria oriented portfolio analysis research into perspective: (1) overall framework, (2) portfolio ranking, (3) skewness inclusion, (4) use of alternative measures of risk, (5) decision support systems, and (6) the modeling of individual investor preferences. In the first category, he classifies articles that are overview pieces such as Hallerbach and Spronk (2002a, b) and Bana et al. (2001), in which the benefits of embracing multiple criteria concepts in financial decision making are outlined. Employing tools from multiple criteria decision analysis for portfolio ranking, there are papers represented by Yu (1997), Jog et al. (1999) and Bouri et al. (2002). In the category of skewness inclusion there are papers by Stone (1973), Konno et al. (1993), Konno and Suzuki (1995) and Chunhachinda et al. (1997). With regard to alternative measures of risk, there are the efforts by Zeleny (1977), Konno and Yamazaki (1991), Feinstein and Thapa (1993) and Doumpos et al. (1999). In the category of decision support systems employing mathematical programming techniques, there are the approaches of Ballestero and Romero (1996), Ogryczak (2000), Arenas Parra et al. (2001), Ballestero and Pla-Santamaria (2003), Ehrgott et al. (2004) and Zopounidis and Doumpos (2000). In the sixth category, recognizing that some criteria may come from financial-economic theory and others may come from the individual investor, we have Spronk and Hallerbach (1997), Ballestero (1998) and Bana et al. (2004).

In the seminal studies of Zopounidis (1999), Zopounidis and Doumpos (2002), Steuer and Na (2003), Spronk et al. (2005) and Xidonas and Psarras (2009), the interested reader can find elaborate reviews regarding the field of multicriteria portfolio management.

4 Methodological framework

4.1 Multiobjective mathematical programming

MCDM is the field of Operational Research that deals with decision problems that involve multiple criteria and, especially in the last 20 years, undergoes a significant growth. In the portfolio selection case, the multiple points of view are incorporating in the decision model via Multiobjective Mathematical Programming (MMP). In MMP the concept of optimality is replaced with that of efficiency or Pareto optimality. The Pareto optimal or efficient or non-dominated solutions (portfolios in the current case) are the solutions that cannot be improved in one objective function without deteriorating their performance in at least one of the others. The general MMP model is defined as follows:

$$\max \ \left(f_1\left(x\right), f_2\left(x\right), \ldots, f_p\left(x\right)\right) \quad \text{s.t.} \quad \boldsymbol{x} \in S \tag{1}$$

where $\boldsymbol{x}$ is the vector of decision variables, $f_1(x), \ldots, f_p(x)$ are the p objective functions and S is the feasible region.

According to Hwang and Masud (1979) the methods for solving MMP problems can be classified into three categories according to the phase in which the decision maker involves in the decision making process expressing his/her preferences: The a priori methods, the interactive methods and the generation or a posteriori methods.

In a priori methods, the decision maker expresses his/her preferences before the solution process (e.g. setting goals or weights for the objective functions). The criticism about the a priori methods is that it is very difficult for the decision maker to know beforehand and to be able to accurately quantify (either by means of goals or weights) his/her preferences. In the interactive methods, phases of dialogue with the decision maker are interchanged with phases of calculation and the process usually converges after several iterations to the most preferred solution. The decision maker progressively drives the search with his answers. The drawback is that (s)he never sees the whole picture (the set of Pareto optimal solutions) or an approximation of it. Hence, the most preferred solution is "most preferred" in relation to what he/she has seen and compared so far. In a posteriori methods (or generation methods) the Pareto optimal solutions of the problem (all of them or a sufficient representation) are generated and then the decision maker involves, in order to select among them, the most preferred one. In the current study, we use a generation method (ε-constraint method) accompanied with a decision aid tool for selecting the most preferred among the Pareto optimal solutions.

4.2 Generation of the Pareto optimal solutions with ε-constraint method

The generation methods are the less popular due to their computational effort (the calculation of the efficient solutions is usually a time consuming process) and the lack of widely available software. However, they have some significant advantages. The solution process is divided into two phases: First, the generation of the efficient solutions and subsequently the involvement of the decision maker when all the information is on the table. Hence, they are favorable whenever the decision maker is hardly available and the interaction with him/her is difficult, because he/she is involved only in the second phase, having at hand all the possible alternatives (the efficient solutions of the MMP). Besides, the fact that none of the potential solutions has been left undiscovered, reinforces the decision maker's confidence on the final decision.

The most widely used generation methods are the weighting method and the ε-constraint method. In the weighting method, a weighted sum of the objective functions is optimized. By varying the weights of the objective functions, we obtain different efficient solutions. In the ε-constraint method, we optimize one of the objective functions using the other objective functions as constraints, incorporating them in the constraint part of the model as shown below (Cohon 1978; Chankong and Haimes 1983):

$$\max f_1(x)$$

$$\text{s.t.}$$

$$f_2(x) \geq e_2$$

$$f_3(x) \geq e_3$$

$$\cdots$$

$$f_p(x) \geq e_p$$

$$x \in S \tag{2}$$

By parametrical variation in the RHS of the constrained objective functions (e_i) the efficient solutions of the problem are obtained. The ε-constraint method has several advantages over the weighting method.

1. For linear problems, the weighting method is applied to the original feasible region and results to a corner solution (extreme solution), thus generating only efficient extreme solutions. On the contrary, the ε-constraint method alters the original feasible region and is able to produce non-extreme efficient solutions. As a consequence, with the weighting method we may spend a lot of runs that are redundant in the sense that there can be a lot of combination of weights that result in the same efficient extreme solution. On the other hand, with the ε-constraint we can exploit almost every run to produce a different efficient solution, thus obtaining a more rich representation of the efficient set.
2. The weighting method cannot produce unsupported efficient solutions in multiobjective integer and mixed integer programming problems, while the ε-constraint method does not suffer from this pitfall (Steuer 1989; Miettinen 1998)
3. In the weighting method the scaling of the objective functions has strong influence in the obtained results. Therefore, we need to scale the objective functions to a common scale before forming the weighted sum. In the ε-constrained method this is not necessary.
4. An additional advantage of the ε-constraint method is that we can control the number of the generated efficient solutions by properly adjusting the number of grid points in each one of the objective function ranges. This is not so easy with the weighting method (see point 1 above).

In the literature, several versions of the ε-constraint method have been appeared trying to improve its performance or adapt it to a specific type of problems (see e.g., Ehrgott and Ryan 2002; Laumanns et al. 2006; Hamacher et al. 2007). However, despite its advantages over the weighting method, the ε-constraint method has three points that need attention in its implementation: (a) the calculation of the range of the objective functions over the efficient set, (b) the guarantee of efficiency of the obtained solution, (c) the increased solution time for problems with several (more than two) objective functions. We try to address these three issues with a novel version of the ε-constraint method the augmented ε-constraint method (AUGMECON).

4.3 The augmented ε- constraint method (AUGMECON)

The detailed description of the method is beyond the scope of the paper but the interested reader can find it in Mavrotas (2007, 2009). We will just point out briefly its innovations related with the above three issues: First of all, we use lexicographic optimization in order to construct the payoff table. In this way we guarantee the efficiency of the obtained solution and a more reliable calculation of the objective functions' ranges. Second, we properly modify the objective function of the single objective problem adding a second priority term in order to guarantee that the obtained optimal solution is Pareto optimal for the original multiobjective problem (avoiding so weakly Pareto optimal solutions). Third, we accelerate the whole process by introducing the "early exit from the loop" option when the intermediate problems become infeasible. This capability saves a lot of computational time in problems with more than 2–3 objective functions. This technique is especially beneficial when there are several objective functions in the problem. In a real application problem (Mavrotas 2007) with 6 objective functions, 236 binary variables and 92 constraints, the early exit from the loops reduce the computation time by 45%. Namely, the computation time for 5 grid points per objective function ($5^5 = 3125$ optimization problems to be solved) was approximately
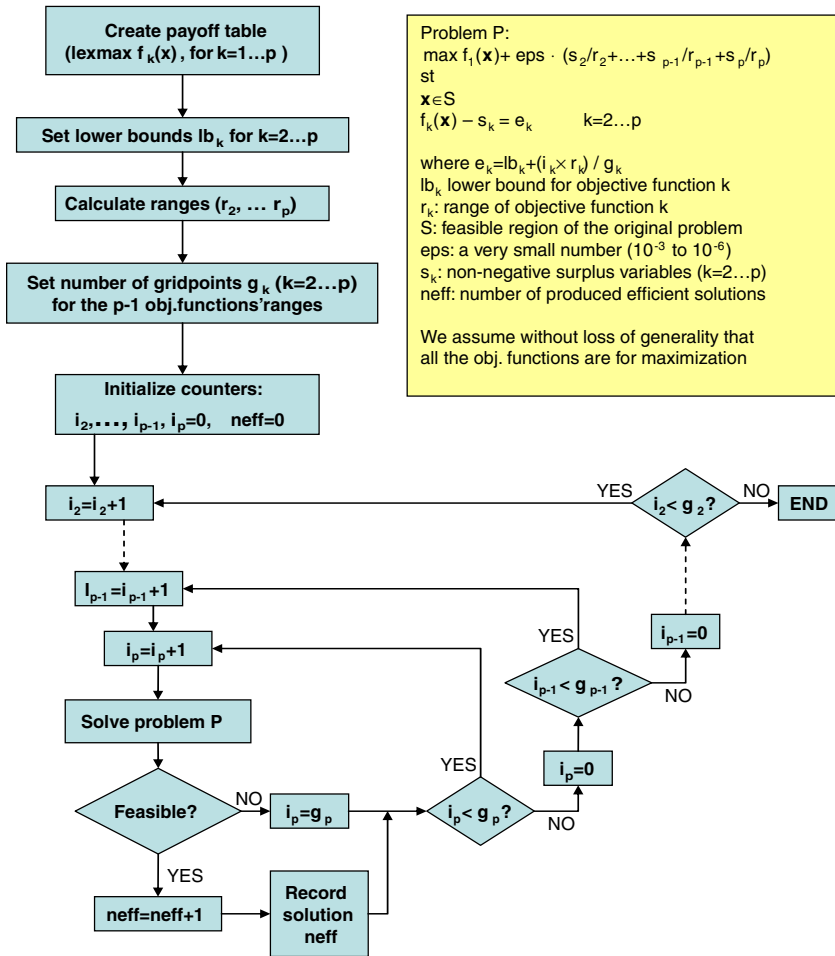
Fig. 1 Flowchart of the AUGMECON method

21 min in a Pentium M 1.7 GHz computer. Using the early exit technique the computation time reduced to 11 min and 30 s by just avoiding the redundant iterations that lead to infeasible solutions, solving actually only 1705 optimization problems.

Practically, the augmented ε-constraint method is applied as follows: From the payoff table we obtain the range of each one of the $p - 1$ objective functions that are going to be used as constraints. Then we divide the range of the i-th objective function to q_i equal intervals using $(q_i - 1)$ intermediate equidistant grid points. Thus we have in total $(q_i + 1)$ grid points that are used to vary parametrically the RHS (e_i) of the i-th objective function. The total number of runs becomes $(q_2 + 1) \times (q_3 + 1) \times \cdots \times (q_p + 1)$. A desirable characteristic of the ε-constraint method is that we can control the density of the efficient set representation by properly assigning the values to the q_i. The higher the number of grid points the more dense is the representation of the efficient set but with the cost of higher computation time. A trade off between the density of the efficient set and the computation time is always advisable. The flowchart of the algorithm is shown in Fig. 1.

The AUGMECON method has been coded in GAMS (General Algebraic Modeling System), a widely used modeling language (Brooke et al. 1998). It is an effort to provide the multicriteria community with access to very powerful optimization tools and, vice versa, to provide the many GAMS users with tools for effectively dealing with multi-objective optimization. The code is available in the GAMS library with an educational example (http://www.gams.com/modlib/libhtml/epscm.htm) and supporting documentation (Mavrotas 2007). The interested reader can use AUGMECON in his/her own problems by modifying only the part of code that has to do with the example (the specific objective functions and constraints), as well as the parameters of AUGMECON (number of grid points per objective function). The part of the code that performs the calculation of the payoff table with lexicographic optimization and the production of the Pareto optimal solutions is fully parameterized in order to be ready to use. The GAMS version of AUGMECON can be used in multiobjective linear programming (MLP), mixed integer programming (MIP) or even nonlinear programming (NLP) problems (given that the necessary solvers are installed).

4.4 Selection of the most preferred among the Pareto optimal solutions

Once we have obtained the Pareto optimal solutions, the subsequent task is to assist the decision maker in making his/her final choice among these solutions (decision support). One option is to use one of the discrete Multiple Criteria Decision Making methods (see e.g. Belton and Stewart 2002) regarding the obtained Pareto optimal solutions as the decision alternatives and the objective functions as the performance criteria. However, in the present case we incorporated a simple and straightforward selection mechanism in order to assist the decision maker in his final selection without much more computational effort. Namely, we used the technique of interactive filtering of the solutions (Steuer 1989) which is appropriate in gradual focusing on the most preferred solution, reducing the "information overload" caused by the usually great number of Pareto optimal solutions evaluated in a number of criteria. In this technique a representative, small subset of the Pareto optimal solutions is automatically calculated using the "first point outside the neighborhood" algorithm. From this subset the decision maker selects his/her most preferred efficient solution from a small sample of representative Pareto optimal solutions that is automatically produced by the method. For the decision maker it is easier to compare along a number of criteria (usually more than 2) 3–7 alternatives instead of hundreds. This selection drives the search to a reduced searching space and the procedure (selection among a representative sample) is repeated for a predetermined number of iterations. The reduction factor of the searching space is automatically adjusted according to the number of initial solutions, the number of iterations and the number of solutions in the representative set. For illustrative purposes, a small example of the interactive filtering process is shown schematically in Fig. 2.

The aim of the interactive filtering process is to select the most preferred among the 20 Pareto optimal solutions that are depicted graphically in two dimensions (Fig. 2a). The representative set of the 5 solutions is first calculated (solutions 1, 4, 7, 9 and 13 in the dark circles) using the algorithm of the first point outside the neighborhood and these five solutions are shown to the decision maker. Assume that the decision maker selects solution 9. Then the search space is contracted around solution 9 (dashed line in Fig. 2b). Now, the corresponding representative set consists of solutions 2, 6, 9, 13 and 17 as it is calculated from the method. Assume that the decision maker selects solution 17. In the final (third) iteration, the search space is contracted around solution 17 (dashed line). The five solutions 7, 8, 10, 14, 17 are shown to the decision maker and assume that he/she selects solution 14 which is the final
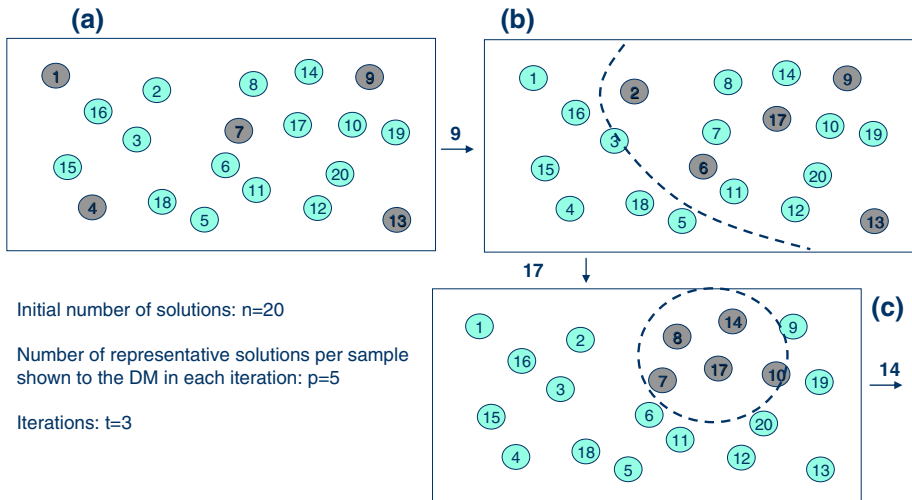
Fig. 2 Graphical representation of the interactive filtering process

output of the interactive filtering technique (the most preferred Pareto optimal solution). More details about the interactive filtering can be found in Steuer (1989).

5 Model building and application

5.1 Securities and evaluation criteria

The sample considered in the study consists of 66 securities of the ASE covering a broad spectrum of business activities, taking into account the major issue of the diversification effect (see Statman 1987; Brennan 1975; Jennings 1971; Fisher and James 1970; Evans and Archer 1968). The study period includes the record of the weekly based closing prices between 1/1/2004 and 31/6/2007. Table 1 summarizes the securities included in the sample, its distribution in the corresponding supersectors [according to the Industry Classification Benchmark (ICB) standards (www.icbenchmark.com)] and the problem's decision variables (X_i: percentage of capital to be invested in security i, $i = 1, \ldots, n$ where n is the number of securities under consideration).

The proposed methodology is based on interviews with asset management experts, as well as on the international literature, concerning the assessment of portfolio performance. The aim of the interviews was to be identified and selected these criteria that are the most appropriate to be used for the portfolio optimization process. The criteria that are finally used in the model are well established and widely used measures for portfolio selection, both in theory and practice (the sign in the parenthesis denotes the type of criterion desired utility: (+) for increasing utility and (−) for declining utility):

1. Capital return per share (+): $R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}$, where R_t is the return on a share in period t, P_t is the share price in period t, P_{t-1} is the share price in period $t - 1$ and D_t is the dividend that the share gives to the investor in period t.
2. Relative dividend yield (+): Security's dividend yield/Subsector's dividend yield, where Dividend yield of a security = Dividend in period t/Share price closed in period t. The

	Supersector	Number of securities	Decision variables
Table 1 Number of securities in the sample per supersector			
1	Food and beverage	7	$X_1 - X_7$
2	Personal and household goods	7	$X_8 - X_{14}$
3	Construction and materials	6	$X_{15} - X_{20}$
4	Industrial goods and services	6	$X_{21} - X_{26}$
5	Technology	3	$X_{27} - X_{29}$
6	Telecommunications	2	$X_{30} - X_{31}$
7	Chemicals	1	X_{32}
8	Basic resources	4	$X_{33} - X_{36}$
9	Oil and gas	2	$X_{37} - X_{38}$
10	Retail	5	$X_{39} - X_{43}$
11	Travel and leisure	5	$X_{44} - X_{48}$
12	Utilities	1	X_{49}
13	Health care	1	X_{50}
14	Financial services	8	$X_{51} - X_{58}$
15	Banks	6	$X_{59} - X_{64}$
16	Insurances	2	$X_{65} - X_{66}$

relative dividend yield was a criterion that proposed directly by the experts, instead of the conventional dividend yield ratio, as a more realistic/comparable measure (see also Hurson and Zopounidis 1995, 1997; Zopounidis et al. 1998 for the use of dividend yield as a portfolio selection criterion).

3. Mean Absolute Deviation/MAD $(-)$:

$$MAD_p = \frac{1}{T} \sum_{t=1}^{T} \left| r_{pt} - E\left(r_p\right) \right| = \frac{1}{T} \sum_{t=1}^{T} \left| \sum_{i=1}^{n} w_i r_{it} - \sum_{i=1}^{n} w_i E\left(r_i\right) \right|$$

$$= \frac{1}{T} \sum_{t=1}^{T} \left| \sum_{i=1}^{n} w_i \left[r_{it} - E\left(r_i\right) \right] \right| \tag{3}$$

where T is the number of periods across which the MAD of the portfolio is calculated, n is the number of securities, $(E(r_i))$ is the expected capital return of security i, $(E(r_p))$ is the expected capital return of the portfolio and w_i the amount invested in security i (see Konno and Yamazaki 1991; Feinstein and Thapa 1993; Doumpos et al. 1999; Michalowski and Ogryczak 2001; Angelelli et al. 2007).

4. Beta coefficient $(-)$: $COV(R_i, R_m)/VAR(R_m)$, where R_i is the return of the share i and R_m is the return of the market portfolio (see Hurson and Zopounidis 1995, 1997; Zopounidis et al. 1998).

5. Relative price earnings ratio $(-)$: Security's P/E/Subsector's P/E, where P/E = Share price in the stock market in period t / Earnings per share in period t. The price earnings ratio, in this case too, was a criterion that proposed directly by the experts, instead of the conventional price earnings ratio, as a more realistic/comparable measure (see also Hurson and Zopounidis (1995, 1997); Martel et al. (1988); Zopounidis et al. (1998) for the use of P/E as a portfolio selection criterion).

6. Marketability (+): Number of transactions of shares of a company during period *t* / Total number of shares of a company during period *t* (see Hurson and Zopounidis 1995, 1997; Zopounidis et al. 1998).

From the above criteria, the first two (capital return and relative dividend yield) reflect on a generic return/performance dimension, the following two criteria reflect on a generic risk dimension (we employ the beta coefficient for market risk and the mean absolute deviation for the security's individual risk) and finally, the last two criteria (relative price earnings ratio and marketability) reflect on a generic market acceptance dimension. Extended theoretical presentation of these criteria can be found in Sharpe et al. (1999), Bodie et al. (2004) and Reilly and Brown (2005). Table 2 summarizes the evaluation of each security in the criteria that are used in the model. We have to stress that all the necessary data that were utilized for the application of the proposed methodology are provided in the Statistical Bulletins of the Athens Stock Exchange (www.ase.gr).

5.2 Modeling constraints and the investment policy statement of the decision maker

The aim of the developed multi-objective programming model is to provide the Pareto optimal portfolios according to the six criteria which are properly translated to six objective functions. The multi-objective programming model has the following characteristics:

5.2.1 Decision variable

The decision variables of the model are both continuous and binary. The continuous variables X_i represent the weight of the *i*-th security in the portfolio ($i = 1, \ldots, 66$). The binary variables B_i represent the existence of the *i*-th security in the portfolio ($B_i = 1$) or not ($B_i = 0$). The incorporation of these binary variables offers a more realistic modelling of the real decision situation for three main reasons: First, logical conditions dealing with the interdependence of the incorporated securities can be easily formulated (e.g. a logical condition may be "if security *a* is incorporated in the model then security *b* must be also present"). Second, lower bounds can be set to the incorporated securities, so that, if security *a* is incorporated in the portfolio, its weight must be at least a minimum value. Third, with the binary variables we have complete control of the number of securities that are incorporated in the portfolio (e.g. the number of securities must be between 10 and 20). All these three types of conditions can not be modelled with conventional Linear Programming (e.g. only continuous variables). The incorporation of binary variables gives rise to a MIP model. These models are harder to solve but they provide a more realistic representation of the real decision situation, allowing the decision maker to establish more complex conditions. Moreover, nowadays the related hardware and software can provide the solution of difficult MIP problems in a few minutes, so that the solution time is no more a serious problem as it used to be in the previous decades.

5.2.2 Objective functions

The objective functions correspond to the six criteria of evaluation that were mentioned in the last paragraph. Namely, the six objective functions are:

Table 2 The evaluation of securities in the five criteria

Variables	Return	DYi/DYsub	Beta	PEi/PEsub	Marketability
X_1	0.007480	0.638	1.0420	1.000	2.910
X_2	0.003313	0.563	0.7293	0.397	7.350
X_3	0.004519	1.044	0.6641	1.000	1.650
X_4	0.003357	0.708	0.6286	1.150	60.430
X_5	0.003461	0.231	0.0789	1.385	8.030
X_6	0.002042	1.662	0.7296	0.702	0.400
X_7	0.001537	2.008	1.1769	0.832	13.060
X_8	0.001266	0.962	0.3040	1.197	0.170
X_9	0.005842	1.492	1.6577	1.717	5.850
X_{10}	0.011105	1.000	0.7505	1.000	2.140
X_{11}	0.009174	0.623	1.6403	0.490	7.420
X_{12}	0.000464	5.500	1.2448	0.288	2.390
X_{13}	0.002379	0.363	0.7785	1.098	12.140
X_{14}	0.012243	0.225	1.2573	1.110	3.250
X_{15}	0.003581	0.779	1.2201	1.216	7.520
X_{16}	0.004652	1.133	0.5374	0.650	5.150
X_{17}	0.005715	1.013	1.3323	1.249	54.610
X_{18}	0.000863	0.974	0.9993	1.000	12.030
X_{19}	0.007836	0.615	0.8022	1.200	6.170
X_{20}	0.005172	2.039	0.9667	0.696	1.540
X_{21}	0.005383	1.120	1.1489	0.376	2.110
X_{22}	0.011128	0.816	1.3550	0.342	26.490
X_{23}	0.008848	1.375	1.3576	0.530	3.840
X_{24}	0.004149	0.889	1.6912	1.212	4.100
X_{25}	0.005894	0.625	0.6885	1.089	3.020
X_{26}	0.010557	0.695	0.7599	1.519	2.880
X_{27}	0.002113	0.684	0.7796	0.793	3.910
X_{28}	0.004644	0.997	0.4699	1.000	2.700
X_{29}	0.008850	0.372	2.4350	0.033	2.320
X_{30}	0.005061	0.960	0.9235	1.000	17.080
X_{31}	0.008965	0.244	1.7540	1.577	4.180
X_{32}	0.002319	0.385	1.1249	0.978	5.180
X_{33}	0.003600	1.003	0.7647	0.901	1.320
X_{34}	0.006368	1.003	0.5428	1.000	3.880
X_{35}	0.012181	1.006	1.8008	0.929	7.940
X_{36}	0.010267	0.700	1.1979	0.713	4.530
X_{37}	0.011261	1.223	0.8683	1.326	5.890
X_{38}	0.008144	1.000	1.1620	1.197	6.440
X_{39}	0.004184	0.974	0.3728	1.000	2.910
X_{40}	0.007535	1.126	1.2989	1.854	2.130
X_{41}	0.000901	0.963	0.5525	1.026	4.350

Table 2 continued

Variables	Return	DYi/DYsub	Beta	PEi/PEsub	Marketability
X_{42}	0.006064	1.653	0.8839	0.833	2.970
X_{43}	0.007038	0.526	1.2641	1.534	6.230
X_{44}	0.000298	1.234	0.5827	0.969	0.470
X_{45}	0.006027	0.242	0.7635	1.114	0.720
X_{46}	0.005116	1.136	0.9662	0.865	6.780
X_{47}	0.005190	1.721	1.3082	0.598	3.960
X_{48}	0.002962	0.479	1.2726	1.105	6.570
X_{49}	0.007846	0.669	0.8916	1.050	3.910
X_{50}	0.013315	0.568	0.7357	0.560	1.640
X_{51}	0.001367	0.892	0.6377	1.078	1.070
X_{52}	0.001379	1.076	0.4279	0.597	1.120
X_{53}	0.001791	1.912	0.6719	0.888	0.660
X_{54}	0.002477	0.982	0.1620	1.000	3.830
X_{55}	0.002678	0.997	0.5323	1.182	2.950
X_{56}	0.002815	0.920	0.6945	0.506	4.810
X_{57}	0.008956	0.992	1.3613	1.000	6.810
X_{58}	0.008400	0.187	1.9452	0.835	17.080
X_{59}	0.004551	0.847	1.1067	0.775	6.380
X_{60}	0.001245	0.613	1.2438	0.885	1.440
X_{61}	0.006752	0.621	1.5259	1.209	4.810
X_{62}	0.005205	0.834	1.2146	1.000	4.650
X_{63}	0.010869	0.616	1.0903	0.939	2.520
X_{64}	0.007767	0.624	1.1750	0.877	5.040
X_{65}	0.004066	1.000	1.8186	1.000	1.670
X_{66}	0.006517	0.950	0.9924	1.000	7.920

(1) Maximize portfolio's return:

$$\max z_1 = \sum_{i=1}^{n} r_i X_i \tag{4}$$

where n is the number of securities ($n = 66$ in the current case) and r_i is the return of the i-th security as defined in the previous paragraph. The r_i for the 66 securities are given in the 2nd column of Table 2.

(2) Maximize portfolio's dividend yield:

$$\max z_2 = \sum_{i=1}^{n} rd_i X_i \tag{5}$$

where rd_i is the relative dividend's yield of the i-th security. The corresponding values are given in the 3rd column of Table 2.

(3) Minimize portfolio's Mean Absolute Deviation (MAD):

$$\min z_3 = \frac{1}{T} \sum_{t=1}^{T} \left| \sum_{i=1}^{n} X_i(r_{it} - E(r_i)) \right| \tag{6}$$

where T is the number of periods across which the portfolio's variation is calculated. It can be noted that the absolute value of the expression destroys the linearity of the model. In order to preserve the linearity of the model we apply the Konno and Yamazaki (1991) transformation. According to this transformation we use T additional positive continuous variables (Y_t) that represent the absolute deviation in each period and $2 \times T$ constraints as follows:

$$\sum_{i=1}^{n} X_i (r_{it} - E(r_i)) + Y_t \geq 0 \qquad \forall \, t = 1, \ldots, T \tag{7}$$

$$\sum_{i=1}^{n} X_i (r_{it} - E(r_i)) - Y_t \leq 0 \qquad \forall \, t = 1, \ldots, T \tag{8}$$

And the objective function is accordingly transformed to:

$$\min z_3 = \frac{1}{T} \sum_{t=1}^{T} Y_t \tag{9}$$

As it was mentioned, in the current case study we use weekly data from January 2004 to June 2007, which means $T = 183$. So, the cost of the linearization in the MIP model is the addition of 183 continuous variables and 366 constraints.

(4) Minimize portfolio's beta coefficient:

$$\max z_4 = \sum_{i=1}^{n} b_i X_i \tag{10}$$

where b_i is the beta coefficient of the i-th security and its values are in the 4th column of Table 2.

(5) Minimize portfolio's relative P/E ratio:

$$\max z_5 = \sum_{i=1}^{n} rpe_i X_i \tag{11}$$

where rpe_i is the relative P/E ratio for the i-th security and its values are in the 5th column of Table 2.

(6) Maximize portfolio's marketability:

$$\max z_6 = \sum_{i=1}^{n} m_i X_i \tag{12}$$

where m_i is the marketability of the i-th security and the corresponding values are given in the 6th column of Table 2.

5.2.3 Constraints

We have two kinds of constraints: the mandatory constraints and the policy constraints. The mandatory constraints are necessary for the proper formulation of the model with the minimum requirements while the policy constraints depend on the decision maker and his/her policy on the portfolio creation.

(1) Completeness constraint:

$$\sum_{i=1}^{n} X_i = 1 \tag{13}$$

which means that all the available capital is invested. It is a mandatory constraint.

(2) Auxiliary constraints: The $2 \times T$ constraints mentioned before (12) and (13) that are needed for the linearization of the expression for the MAD (3rd objective function). In the specific model they are mandatory constraints.

(3) The number of securities in the portfolio must vary between 5 and 15 (see Statman 1987; Brennan 1975).

$$5 \leq \sum_{i=1}^{n} B_i \leq 15 \qquad (14)$$

(4) The maximum share of each security in the portfolio cannot exceed the 20%.

$$X_i - 0.2B_i \leq 0 \qquad \forall i = 1, \ldots, n \qquad (15)$$

(5) If a security is incorporated in the portfolio, its share must be at least 0.1% (minimum share).

$$X_i - 0.001B_i \geq 0 \qquad \forall i = 1, \ldots, n \qquad (16)$$

(6) Upper bound for investment in specific supersectors (in the current study 5, 11 and 16):

$$\sum_{i \in Sk} Xi \leq 0.05 \qquad k = 5, 11, 16 \qquad (17)$$

These particular supersectors (i.e., the corresponding sectoral stock market indexes) performed a negative capital return for the examined study period.

(7) Lower bound for investment in specific securities (in the current study X_1, X_{10}, X_{61}):

$$X_i \geq 0.05 \qquad i = 1, \ 10, \ 61 \qquad (18)$$

(8) Lower bound for the investment amount in securities with beta less than one:

$$\sum_{i \in BLT1} X_i \geq 0.5 \qquad (19)$$

where *BLT1* is the set of securities with beta less than 1 (35 securities). The decision maker sets the lower bound of the total weight of these securities in the portfolio to 0.5. In this constraint the decision maker can set his/her preferences reflecting his/her attitude towards risk. Low values for the RHS indicate an aggressive (risk prone) behaviour while high values indicate a conservative (risk averse) behaviour. In the specific case the decision maker express a risk neutral attitude.

(9) Lower bound for the investment amount in securities with high capitalization:

$$\sum_{i \in BC} Xi \geq 0.65 \qquad (20)$$

where *BC* is the set of securities with high capitalization (41 securities).

The final model consists of 6 objective functions, 249 continuous variables ($= 66 + 183$), 66 binary variables and 509 constraints.

6 Results and discussion

6.1 Generation of the Pareto optimal portfolios

The multi-objective problem is solved with the augmented ε-constraint method using the GAMS/CPLEX solver (as it was mentioned, the augmented ε-constraint has been formulated for GAMS and is available in GAMS model's library). In the specific case we used 5 grid points for each one of the constrained objective functions which means that $5^5 = 3125$ solutions of an MIP problem are normally needed. However, due to the "early exit of the loops" feature of the augmented ε-constraint method which is activated when infeasibilities occur, only 844 from the nominal number of 3125 runs are performed while the solution time is 249 s in a Pentium Core 2 Duo 2.0 GHz. With this grid, the number of the obtained Pareto portfolios was 268 which is considered as an adequate number for further investigation. Using a denser grid, we can obtain a more rich representation of the Pareto set. However, the solution time is expected to increase almost exponentially.

The output of the GAMS model is the evaluation of each Pareto portfolio to the 6 criteria along with the number of securities in each portfolio and the weight of each security in the Pareto portfolio in the form of Table 3.

It was observed that 43 from the 66 securities are involved in the 268 Pareto portfolios. This means that about 1/3 of the total securities are not eligible in none of the Pareto portfolios according to the six criteria. The number of securities in each Pareto portfolio varies from 7 to 15. As it was expected, the portfolios with the greater number of securities (near to 15) occur when the minimum MAD is pursued (diversification). In this stage, fruitful information may be extracted for the statistics of each one security regarding its participation in the Pareto portfolios. For example, we can calculate the number of appearances of each security in the Pareto portfolios as well as the minimum, average and maximum weight of the security across all the Pareto portfolios. In this way the decision maker gets an idea of which securities are more often present and which are not. Conclusively, the actual contribution of the multi-objective programming is to aid the decision maker to express his/her preferences regarding the basic characteristics of the desired portfolios (by proper constraints) and then to reduce the search space only to the relevant Pareto optimal choices.

6.2 The interactive filtering process

In the next step, the decision maker proceeds to the implementation of the interactive filtering process, for selecting among a number of Pareto optimal portfolios his/her most preferred. We note that this application is an illustration of the proposed approach and no real decision maker (a private or an institutional investor) is implicated. For this reason, the asset management experts that collaborated in the study, stated their personal preferences. The number of iterations is set to 3 and the number of representative solutions that are shown in every iteration to the decision maker to 10. These are indicative values, as the interactive filtering process can be fully parameterized on the number of iterations and the sample size that is presented to the decision maker. The interactive filtering process has been implemented in Microsoft Excel using some coding Visual Basic for Applications.

Table 3 Output of the GAMS model

#	Return	DYi/DYsub	Beta	PEi/PEsub	Marketability	MAD	Stck/Portf	Securities weigths in the portfolios						
								$X_1(\%)$	$X_2(\%)$	$X_3(\%)$	...	$X_{64}(\%)$	$X_{65}(\%)$	$X_{66}(\%)$
1	0.011	0.768	1.047	0.810	7.920	0.021	10	5.0	0.0	0.0	...	0.0	0.0	0.0
2	0.011	1.194	1.086	0.770	8.176	0.022	8	5.0	0.0	0.0	...	0.0	0.0	0.0
3	0.009	1.621	1.058	0.712	7.355	0.021	8	5.0	0.0	0.0	...	0.0	0.0	0.0
4	0.008	2.048	1.026	0.744	5.299	0.020	10	5.0	0.0	0.0	...	0.0	0.0	0.0
5	0.004	2.475	1.033	0.740	4.287	0.019	8	5.0	0.0	0.0	...	0.0	0.0	0.0
6	0.011	0.837	0.954	0.895	7.813	0.022	9	5.0	0.0	0.0	...	0.0	0.0	0.0
7	0.010	1.194	0.954	0.873	6.772	0.022	9	5.0	0.0	0.0	...	0.0	0.0	0.0
8	0.009	1.621	0.954	0.849	4.999	0.022	8	5.0	0.0	0.0	...	0.0	0.0	0.0
9	0.008	2.048	0.954	0.806	3.265	0.021	9	5.0	0.0	0.0	...	0.0	0.0	0.0
10	0.010	0.807	0.772	1.115	3.549	0.023	8	5.0	0.0	0.0	...	0.0	0.0	0.0
11	0.009	1.194	0.772	0.961	3.349	0.021	9	5.0	0.0	0.0	...	0.0	0.0	0.0
12	0.008	1.621	0.772	0.847	3.265	0.020	10	5.0	0.0	0.0	...	0.0	0.0	0.0
13	0.005	2.048	0.772	0.828	3.265	0.016	12	5.0	0.0	0.0	...	0.0	0.0	0.0
14	0.008	0.768	0.590	1.013	3.540	0.019	10	5.0	0.0	0.0	...	0.0	0.0	0.0
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
256	0.006	1.621	0.954	0.812	10.486	0.014	15	5.0	0.0	0.0	...	0.0	0.0	0.0
257	0.007	0.818	0.772	0.812	10.486	0.014	14	5.0	4.1	0.0	...	0.0	0.0	0.0
258	0.006	1.194	0.772	0.812	10.486	0.014	14	5.0	0.0	0.0	...	0.0	0.0	0.0
259	0.005	1.621	0.772	0.812	10.486	0.014	15	5.0	0.0	3.7	...	0.0	0.0	0.0
260	0.008	0.784	0.999	0.941	17.708	0.014	15	5.0	0.0	0.0	...	0.0	0.0	0.0
261	0.007	1.194	0.983	0.909	17.708	0.014	15	5.0	0.0	0.0	...	0.0	0.0	0.0
262	0.008	0.779	0.954	0.947	17.708	0.014	15	5.0	0.0	0.0	...	0.0	0.0	0.0
263	0.007	1.194	0.954	0.919	17.708	0.014	15	5.0	0.0	0.0	...	0.0	0.0	0.0

Table 3 continued

#	Return	DYi/DYsub	Beta	PEi/PEsub	Marketability	MAD	Stck/Portf	Securities weigths in the portfolios						
								$X_1(\%)$	$X_2(\%)$	$X_3(\%)$	$\ldots$	$X_{64}(\%)$	$X_{65}(\%)$	$X_{66}(\%)$
264	0.007	0.832	0.772	0.985	17.708	0.014	14	5.0	0.0	0.0	$\ldots$	0.0	0.0	0.0
265	0.005	1.194	0.772	0.950	17.708	0.014	14	5.0	0.0	0.0	$\ldots$	0.0	0.0	0.0
266	0.006	0.824	0.772	0.965	17.708	0.014	15	5.0	0.0	0.1	$\ldots$	0.0	0.0	0.0
267	0.006	0.923	0.769	0.956	4.016	0.010	15	5.0	0.0	0.0	$\ldots$	0.0	0.0	0.0
268	0.006	0.916	0.761	0.953	4.146	0.010	15	5.0	0.0	0.0	$\ldots$	0.0	0.0	0.0
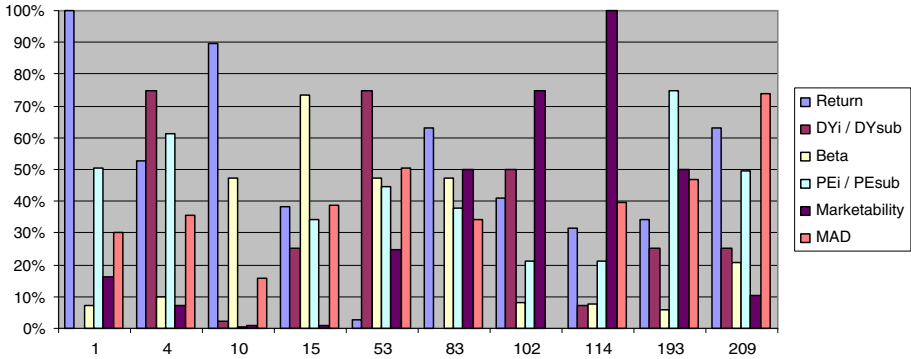
Fig. 3 Accomplishment (%) of the optimal performance per criterion for the portfolios after the 1st iteration
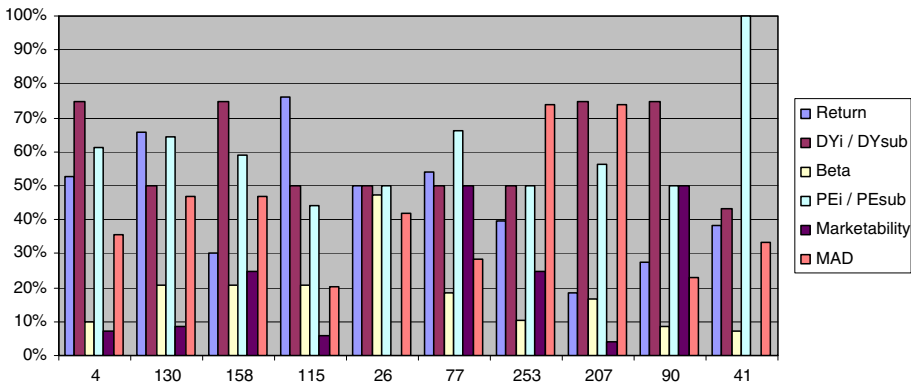
Fig. 4 Accomplishment (%) of the optimal performance per criterion for the portfolios after the 2nd iteration

6.2.1 First iteration

After the first iteration, ten representative portfolios from the complete set of the 268 Pareto solutions are proposed to the decision maker. In Fig. 3 a graphical representation (histogram) of the accomplishment (%) of the optimal performance per criterion, is provided for the ten portfolios.

This type of representation gives the decision maker the chance to obtain an, as much as possible, visual notion of the results. The choice of the experts after the first iteration was portfolio No. 4, since they expressed a categorical preference of simultaneous achievement of high performance as far as the criteria of capital return, relative dividend yield and relative P/E are concerned. Portfolio No. 4 is the one that satisfies most these preferences, in comparison to the other portfolios (see Fig. 3). Subsequently the decision space is contracted around portfolio No. 4 and the second iteration is launched.

6.2.2 Second iteration

In the second iteration ten new portfolios (including portfolio No. 4), representative of the new contracted search space are proposed to the decision maker. Again, in Fig. 4 a graphical
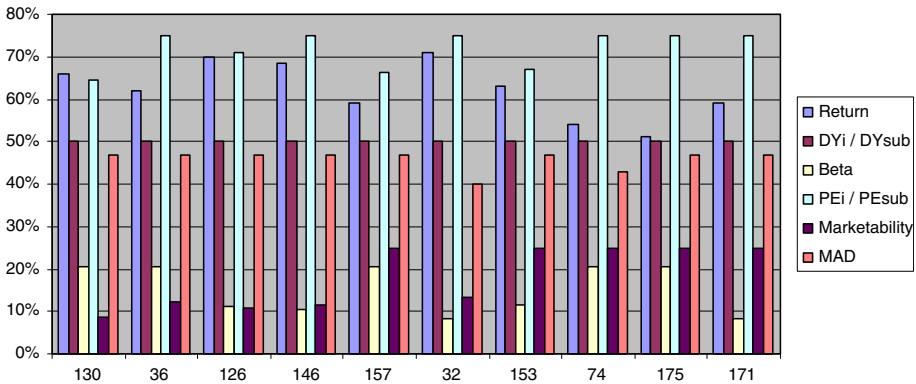
Fig. 5 Accomplishment (%) of the optimal performance per criterion for the portfolios after the 3rd iteration

representation of the accomplishment (%) of the optimal performance per criterion is provided.

The choice of the experts after the second iteration was portfolio No. 130, since they again demanded the high performance in the previously mentioned criteria to be satisfied, in addition with a new request for the as much as possible high performance in the mean absolute deviation (total risk) criterion. Indeed, portfolio No. 130 is the one that satisfies most these preferences, in comparison to the other portfolios (see Fig. 4). The search space is contracted around portfolio No. 130.

6.2.3 Third iteration

Finally, in the third iteration, ten new portfolios (representatives of the reduced search region around No. 130) are again proposed to the decision maker. Figure 5 provides a graphical representation of the accomplishment (%) of the optimal performance per criterion for the final ten portfolios.

It is observed that a number of twenty securities in total participate in the ten portfolios (which means that 46 securities are absent from the last 10 portfolios). Also, seventeen of these securities are high capitalization stocks and three securities are low-medium capitalization stocks. The minimum number of stocks that a portfolio contains is ten (portfolio No. 74) and on the other hand, the maximum number of stocks that a portfolio contains is thirteen (portfolio No. 153). Among the final ten portfolios, the asset management experts explicitly stated their final preference for portfolio No. 153, since it combines satisfactory rate of return with satisfactory relative dividend yield, relative P/E and MAD, in comparison to the other portfolios.

7 Concluding remarks

A multi-objective mixed integer programming model for equity portfolio construction has been presented, accompanied by an interactive filtering process for the assistance of the decision maker in making his/her final choice among the solution set. During the decision process, stock market objectives were used. In a multicriteria analysis framework, additional criteria were examined besides the two basic criteria of return and risk. The introduction of

binary variables in the multiobjective programming model provides more realistic and flexible modeling as it allows for expressing logical and other conditions that cannot be expressed in a conventional Linear Programming framework. Specific information about the decision maker's preferences for certain securities or particular sectors, his attitude towards risk and past experience in stock evaluation were taken into consideration. The fact that the proposed methodology is capable of helping the investor to select the portfolio that satisfies, as much as possible, his spectrum of investment desires, makes it a very powerful decision support tool. The methodology that has been presented could be a useful tool for investors (private or institutional) and professionals in the field, in constructing and designing their portfolios. The competitiveness and effectiveness of portfolio management companies, stockbrokers and experts are increased through the use of scientific methods. The contribution of the proposed methodology has to do with the facts that follow:

- Incorporation of the decision maker's (private or institutional investors) preference system by taking into account their policy constraints regarding the portfolio structure.
- Incorporation of several criteria that in a realistic basis represent the ways professionals (asset managers or trading experts) supports their client's (private or institutional investors) strategies and decisions.
- Incorporation of logical conditions through the integer variables.
- Generation of the Pareto optimal portfolios among which a rational decision maker has to select his/her final choice.
- Interactive procedures that gradually drive the decision maker to his/her most preferred choice by expressing his/her subjective preferences.

The flexibility of the method allows for:

- Expansion of the criteria set by considering a broader grid of portfolio performance measures.
- Utilization of certain stock market indexes of major significance as benchmark portfolios.
- Considering, before the proposed optimization model, of an additional multicriteria procedure for the evaluation of the corporate performance of the stocks that will participate in the optimization process, on the basis of fundamental analysis ratios.
- Taking into account, as far as the above mentioned aspect, the crucial importance issue of industry/sectoral accounting singularities.
- Expansion of the methodology so as to include additional asset classes.

Further work that may be considered in order to integrate the proposed methodology can be summarized as follows:

- Combination of the current methodology with a MCDM technique with discrete alternatives, for evaluating, ranking or sorting the resulted portfolios.
- Embodiment of the methodology in a web-based decision information system so as real time investment decisions to be supported.

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