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ORIGINAL RESEARCH

Study of the Absorption Rate of Lower Hybrid Wave Energy by High Energetic Ions in Tokamak

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Abstract Lower hybrid driven current has been proven to be the most efficient way to drive current in tokamak, and it will be applied in ITER. However high energetic ions generated by fusion reactions, injection of neutral beams, and RF acceleration can absorb lower hybrid wave (LHW) reducing the CD efficiency. The absorption rates of three kinds of high energetic ions to LHW under ITER's parameters are studied for comparison in the paper. The absorption rate of protons with 1 MeV is almost the same level as that of alpha particles, but the absorption rate of deuterons is lower than those of protons and alpha particles. Generally the absorption rate increases with the parallel refractive index as well as the toroidal magnetic field, and increases with the LHW frequency and then decreases with it. The background plasma temperature has little influence on the absorption rate, but it increases fast at first and then descends slightly with the background plasma density.

Keywords High energetic ion · Lower hybrid wave · Absorption coefficient

Introduction

The efficiency of current driven by low hybrid waves is of great importance in fusion plasmas. In ITER, LHCD in combination with other H & CD would be a key tool for the following tasks: to sustain AT steady-state plasmas, to extend the plasmas' duration in the intermediate H-mode and to save volts-seconds in the current ramp-up phrase. A 20 MW/one antenna LHCD system is allowed to install in the 'second phase' of ITER operation [1, 2]. In ITER, there are many high energetic ions generated by fusion reactions, injection of neutral beams, and RF acceleration. It is found that the high energetic ions of MeV can absorb the power of LHW, which may degrade the efficiency of current driven by LHW. In the past decades, the evaluation of the interaction of the high energetic ions with LHW has been concerned. Karney [3, 4] investigated the stochastic dynamics of particle acted by LHW. Imai et al. [5] showed the fast ions were accelerated by LHW during the injection of neutron beam and LHW for heating plasma of JT-60. Nemoto et al. [6] investigated the interaction of Lower Hybrid Wave with fast ions injected by Neutral Beam on the JT-60 Tokamak, and found that the critical coupling density, above which the LH power is absorbed mostly by ions rather than electrons, depends strongly on the LH frequency but not on the refractive index parallel to the toroidal magnetic field. Barbato and Santini [7] obtained the rate of the absorption of LHW by alpha particles by using the distribution function of alpha particles from quasi-linear theory. Heikkine and Kiviniemi [8] studied the minority ions generated by ion cyclotron resonance heating (ICRH) interacted with LHW based on the two-dimensional Fokker–Planck equation. Testa et al. [9] analyzed the mechanism of the coupling of LHW to the protons of Mev in JET. Ziebell [10] numerically observed the quasi-linear interaction of high energetic ions with LHW in tokamak device. Schneider et al. [11] applied two large codes, i.e. SPOT and DELHINE to examine the interaction of alpha particles with LHW under the parameters of ITER.

In this paper, we compared the absorption rate of three kinds of high energetic ions: proton, deuteron, and alpha

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particle with ITER parameters similar to that of the ITER steady-state reference scenario 4 [11].The paper is organized as follows. In section "Calculating Model of the Absorption Rate of LHW by Energetic Ions", the model of the interaction of the high energetic ions with LHW is introduced. In section "Numerical Results", absorption rates are studied under different plasma and LHW parameters in ITER, such as LHW parallel refractive index, LH frequency, electron temperature, plasma density of background and the toroidal magnetic field. The comparison of the absorption rates of three kinds of high energetic ions: proton, deuteron and alpha particle is also presented in this section. The conclusions are given in section "Conclusions".

Calculating Model of the Absorption Rate of LHW by Energetic Ions

In order to obtain the absorption rate of LHW by energetic ions, we start from the cold plasma dielectric tensor, which is given by [11]

$$\vec{K} = \begin{bmatrix} \varepsilon_\perp & -i\varepsilon_{xy} & 0 \\ i\varepsilon_{xy} & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\perp \end{bmatrix}, \tag{1}$$

with

$$\begin{cases} \varepsilon_\perp = 1 - \sum_{j=1}^{N_{spec}} \frac{\omega_{p,j}^2}{\omega^2 - \omega_{B,j}^2} \\ \varepsilon_\parallel = 1 - \sum_{j=1}^{N_{spec}} \frac{\omega_{p,j}^2}{\omega^2} \\ \varepsilon_{xy} = \sum_{j=1}^{N_{spec}} \frac{s_j \omega_{p,j}^2 \omega_{B,j}}{\omega\left(\omega^2 - \omega_{B,j}^2\right)} \end{cases}, \tag{2}$$

where $\varepsilon_\perp$, $\varepsilon_\parallel$ and ε_{xy} are the elements of plasma dielectric tensor. N_{spec} is the total number of ion species ω is the frequency of the electromagnetic wave, and $\omega_{B,j}$, $\omega_{p,j}$ are the cyclotron and the plasma frequencies for species j respectively. The plasma dispersion relation is written as

$$D\left(\omega, \vec{k}\right) = P_4 n_\perp^4 + P_2 n_\perp^2 + P_0 = 0, \tag{3}$$

with

$$\begin{cases} P_0 = \varepsilon_\parallel \left[\left(n_\parallel^2 - \varepsilon_\perp\right)^2 - \varepsilon_{xy}^2\right] \\ P_2 = \left(\varepsilon_\parallel + \varepsilon_\perp\right)\left(n_\parallel^2 - \varepsilon_\perp\right) + \varepsilon_{xy}^2 \\ P_4 = \varepsilon_\perp \end{cases}, \tag{4}$$

where $n_\parallel$ is the refractive index parallel to the toroidal magnetic field. Including the correction of ε_{xx} to hot plasma and ignoring the term with k_z, one obtains

$$\varepsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{Be}^2} - \frac{3\omega^2 \omega_{pe}^2}{\left(\omega^2 - \omega_{Be}^2\right)\left(\omega^2 - 4\omega_{Be}^2\right)} \frac{v_{th}^2}{c^2} n_\perp^2, \tag{5}$$

Here ε_{xx} is the same as $\varepsilon_\perp$ in Eqs. (1) and (2).

In Ref. [12], authors have shown that the finite electron temperature has a significant effect on the propagation of LHW and should be included in the real part of the dielectric tensor as well as the imaginary part. In this paper, the dispersion relation with thermal correction [12, 13] is also used for calculation the absorption rate, namely

$$D\left(\omega, \vec{k}\right) = P_6 n_\perp^6 + P_4 n_\perp^4 + P_2 n_\perp^2 + P_0 = 0, \tag{6}$$

with

$$\begin{cases} P_0 = \varepsilon_\parallel \left[\left(n_\parallel^2 - \varepsilon_\perp\right)^2 - \varepsilon_{xy}^2\right] \\ P_2 = \left(\varepsilon_\parallel + \varepsilon_\perp\right)\left(n_\parallel^2 - \varepsilon_\perp\right) + \varepsilon_{xy}^2 \\ P_4 = \varepsilon_\perp \\ P_6 = -\left(\frac{3\omega_{pi}^2}{\omega^2}\frac{v_{ti}^2}{c^2} + \frac{3\omega^2 \omega_{pe}^2}{4\omega_{Be}^4}\frac{v_{te}^2}{c^2}\right) \end{cases}, \tag{7}$$

where $v_{ti} = \sqrt{T_i/M_i}$ and $v_{te} = \sqrt{T_e/M_e}$ are the thermal velocities of ion and electron respectively.

Since the high energetic ions will collide with background ions, the distribution function of the high energetic ions is assumed obeying the slowing down distribution [14, 15],

$$f(v) = \begin{cases} \frac{S\varepsilon_0^2 M_i M}{n_e Z_i^2 Z^2 e^4 \ln\Lambda}\left(\frac{1}{1 + v^3/v_{Crit}^3}\right) & v < v_0 \\ 0 & v > v_0 \end{cases}, \tag{8}$$

with

$$v_{Crit}^3 = 3\sqrt{\frac{\pi}{2}} Z \frac{M_e}{M} v_{te}^3, \tag{9}$$

where M_i, M_e and M are the mass of ions, electrons of background, and the mass of the high energetic ion respectively. Z_i and Z are the charge numbers of the background ion and the high energetic ion respectively. n_e is the background plasma density. ε_0 is the permittivity of free space. ln Λ is the Coulomb logarithm, which changes with the plasmas to be studied and is set to 17 here. S is the source rate, which means the number of the ion of the slowing down distribution produced in unit volume at unit time. v_0 is the birth velocity of the high energetic ions. v_{Crit} is the beam-ion critical velocity at which the contributions of ions and electrons of background to the slowing down rate are exactly equal to one another. Here the high energetic ions' slowing down velocity distribution is only vivid when the confinement time for them is much longer than their slowdown time. We also assume that the velocity distribution of them is not change, for the Lower Hybrid Wave power density is so low that it does not affect the velocity distribution of the high energetic ions. Here the distribution has no spatial dependence, for the plasma is ideal homogenous with the magnetic field. The density of the ion of the slowing down distribution is obtained after taking the integration in velocity space.

$$N_i = \frac{4}{3}\pi v_{Crit}^3 ln\left(1 + \frac{v_0^3}{v_{Crit}^3}\right)\frac{S\varepsilon_0^2 M_i M}{n_e Z_i^2 Z^2 e^4 ln\Lambda}, \tag{10}$$

The general formula of the absorption rate γ of the plasma waves is defined as the imaginary part of the wave frequency

$$\omega = \omega_r - i\gamma \tag{11}$$

Then γ can be obtained by linearization of D ($\omega_r - i\gamma$) in the weak absorption limit ($\gamma \ll_r$), which is expressed as follows [11]:

$$\gamma = \frac{Im(D(\omega, k))}{\partial Re(D(\omega, k))/\partial\omega}, \tag{12}$$

where $D(\omega, k) = 0$ is the plasma dispersion relation. Since the imaginary part of $D(\omega, k)$ is from ε_{xx}. Here γ is only valid when the absorption process is linear with low LH wave power density.

The absorption rate γ can be rewritten as

$$\gamma = \frac{\left(\varepsilon_\perp - n_\parallel^2\right)(\varepsilon_\perp - n^2)}{\partial Re(D(\omega, k))/\partial\omega} Im(\varepsilon_{xx}), \tag{13}$$

Here we can let

$$\beta = \frac{\left(\varepsilon_\perp - n_\parallel^2\right)(\varepsilon_\perp - n^2)}{\partial Re(D(\omega, \mathbf{k}))/\partial\omega}, \tag{14}$$

We neglect the contribution of the high energetic ions to the real part of the dielectric tensor since the density of the high energetic ions is much less than that of the background ions. According to Refs. [11, 16], $Im(\varepsilon_{xx})$ is expressed as

$$Im(\varepsilon_{xx}) = -2\pi\frac{\omega_p^2\omega}{k_\perp^3}\int_{v_{min}}^{v_{max}}\frac{dv_\perp}{\sqrt{v_\perp^2 - v_{min}^2}}\int_0^{v_{\parallel max}} 2\frac{\partial f}{\partial v_\perp}dv_\parallel, \tag{15}$$

where ω_p is the plasma frequency of the high energetic ions and $v_{min} = c/n_\perp$ is resonant velocity.v_{max} corresponds to v_0 of the slowing down model. $v_{\parallel max} = \sqrt{v_0^2 - v_\perp^2}$ is the maximum parallel velocity. The interaction between high energetic ions and LHW occurs through perpendicular Landau damping and satisfies the resonance condition $v_{min} = \frac{c}{n_\perp} < v_{max}$. Combining with dispersion relation $n_\perp^2 = -\frac{P_2}{P_4}$, we can deduce the parallel refractive index of LHW for Landau damping between the LHW and the high energetic ions should satisfied

$$n_\parallel > \sqrt{\varepsilon_\perp + \frac{\varepsilon_{xy}^2}{|\varepsilon_\parallel|} + \frac{\varepsilon_\perp c^2}{|\varepsilon_\parallel| v_{max}^2}}, \tag{16}$$

Numerical Results

We calculate the absorption rate γ with different plasma and LHW parameters in ITER in this section. We study the influence of the refractive index parallel to the toroidal magnetic field and LH frequency, the influence of the parameters of plasmas, i.e. the temperature and the density as well as the magnitude of the toroidal magnetic field. The parameters similar to that of the ITER steady-state reference scenario 4 [11] are used in the following calculation.

Effects of the Parallel Refractive Index on the Absorption Rate

We calculate the absorption rate γ under the following parameters: $E_{max} = 1\,\text{MeV}$ of the energy of the high energetic ions ($E_{max} = 3.5$ MeV for alpha particles), $\omega = 5$ GHz of the frequency of LHW, $T_e = 12$ keV of the temperature of electrons, $n_e = 7 \times 10^{19}\,\text{m}^{-3}$ of the density of electrons, $B = 5$ T of the magnitude of the toroidal magnetic field. Different parallel refractive indexes at damping place are chosen for calculating the absorption rate. Results are shown in Fig. 1. Different parallel refractive index affects the absorption rate.

From Fig. 1, the curves of the absorption rate γ increase sharply with $n_\parallel$ at low values and smoothly at high value. The absorption rate γ of alpha particle is larger than that of proton when $n_\parallel > 2.3$. The absorption rate γ of deuterons is much smaller than those of the two other ions.

We can explain the results through perpendicular refractive index $n_\perp$. From the plasma dispersion relation we can yield

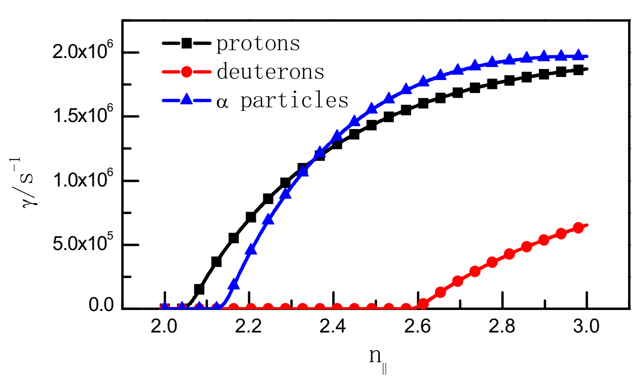

Fig. 1 The absorption rates γ of three kinds of high energetic ions as a function of $n_\parallel$

$$n_{\perp} = \sqrt{\frac{|\varepsilon_{\parallel}|}{\varepsilon_{\perp}} n_{\parallel}^2 - |\varepsilon_{\parallel}| - \frac{\varepsilon_{xy}^2}{\varepsilon_{\perp}}}. \quad (17)$$

From Eq. (17), larger $n_{\parallel}$ leads $n_{\perp}$ larger, which results in the perpendicular phase velocity of LHW lower, then the number of resonant high energetic ions in the slowing down model with LHW becomes more, so the absorption rate γ increases with $n_{\parallel}$ as shown in Fig. 1.

Compared three kinds of ions, the absorption rate of deuterons is much less than those of the two other ions, and the absorption rates of protons and alpha particles are comparable, which means deuterons is more difficult to interact with LHW under these parameters. The maximum velocity of deuterons is less than that of protons, and the amount of deuterons interacting resonantly with LHW is less than that of protons. For alpha particles, the value of the absorption rate is fairly big due to their large energies. So the absorption rate of deuterons is not considered in the following study.

Effects of LHW Frequency on the Absorption Rate

The parameters of plasmas are the same as those in 3.1, and $n_{\parallel}$ of LHW is 2.5 for study. The relationship between the absorption rate of protons and alpha particles and the LH frequency is shown in Fig. 2.

The absorption rate increases slightly with the LH frequency increasing first, and then it decrease with it. There are peaks at 2.75 and 3.5 GHz for two kinds of ions respectively. The absorption rate of protons is larger than alpha particles' when the LH frequency is below 3.7 GHz, and then it is smaller than the other.

Since β of expression (14) and the perpendicular refractive index $n_{\perp}$ affect the absorption rate deduced from Eqs. 13 to 15, their change with LH frequency is also estimated, as shown in Fig. 3. β decreases with LH frequency, while $n_{\perp}$ increases with LH frequency. This tendency is the same for protons and alpha particles.

Landau Damping rate is decreased when $n_{\perp}$ decreases, which will lead the absorption rate to decrease, while increasion of β will lead the absorption rate to increase. Their mutual competition leads to excite a peak for the absorption rate of two kinds ions respectively.

We also calculated the absorption rates of 500 keV protons and deuterons by using the same parameters as Fig. 2 except the energy of protons and deuterons. They are much smaller than 1 MeV energy of theirs. The absorption rate is zero when LH frequency is larger than 4.7 GHz for 500 keV protons, and it is zero when the frequency of LHW is larger than 3.78 GHz for 500 keV deuterons. In order to compare the absorption rate with alpha particles, we choose the energy of protons is 1 MeV and the LH frequency is 5 GHz during our calculation.

Effects of Electron Temperature of Background Plasma on the Absorption Coefficient

Under the same plasma parameters as section "Effects of the Parallel Refractive Index on the Absorption Rate", the absorption rate changing with plasma temperature is calculated. Here $n_{\parallel}$ of LHW is 2.5. Results are shown in Fig. 4. The curve of absorption rate γ of proton increases mildly with the rise of the electron temperature,while the γ of alpha particle is higher than that of the proton and also increased with electron temperature first, and then decreased slowly when the electron temperature is about 11 keV. It is lower than that of protons when the electron temperature is about 16 keV. But the influence of electron temperature on the absorption rate is not obviously for these two particles.

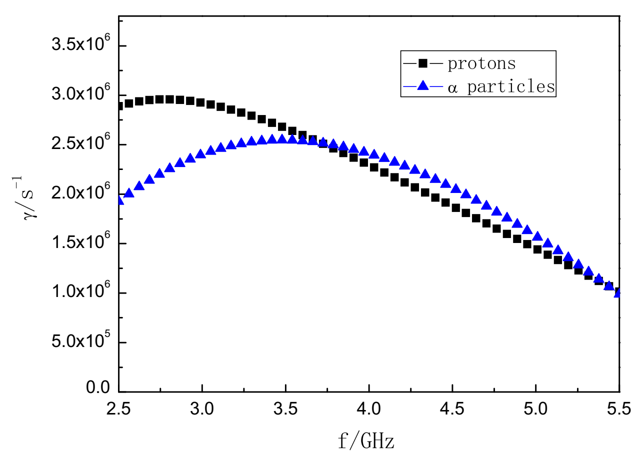

Fig. 2 The absorption rates γ of protons and alpha particles as a function of LH frequency

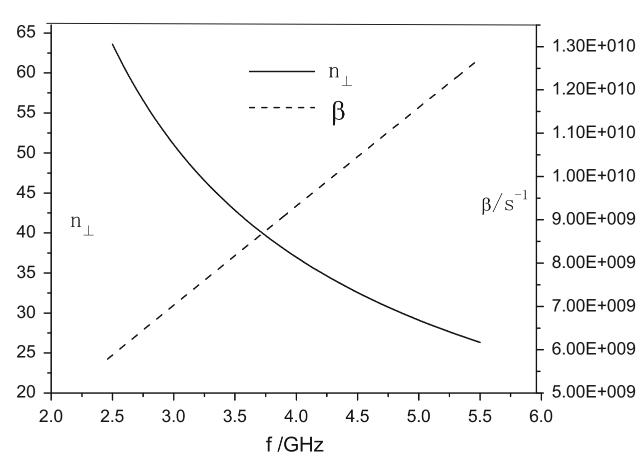

Fig. 3 β and $n_{\perp}$ as a function of LH frequency

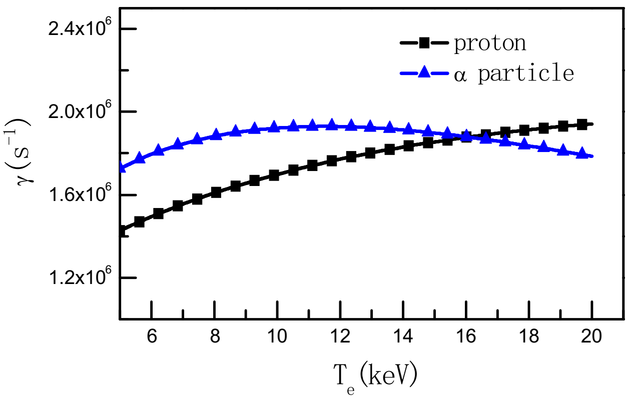

Fig. 4 The absorption rates γ of protons and alpha particles as a function of electron temperature

In the slowing down model, the electron temperature of background affects the critical velocity. From the critical velocity in Eq. (9), the critical velocity increases with the rise of electron temperature of background which indicates that ions will be more easily to have interactions with the rise of electron temperature of background. The shapes of distribution function of proton and alpha particle are different because of the difference of the critical velocities of them, which leads to the dissimilarity of their absorption rates.

Effects of Electron Density of Background Plasma on the Absorption Rate

We calculate the absorption rate γ under the same plasma parameters as section "Effects of the Parallel Refractive Index on the Absorption Rate" except for electron density of background. The absorption rate γ depends on the background plasma density as shown in Fig. 5.

From Fig. 5, the curves of absorption rate γ of proton and alpha particle are increased obviously with n_e at low density and then decrease slightly when $n_e > 6.0 \times 10^{19}$. The value of the γ of alpha particle is larger than that of proton when $n_e > 4.0 \times 10^{19}$.

The values of β in Eq. (14) and $n_\perp$ change obviously when the background plasma density changes, which are shown in Fig. 6.

The value of $n_\perp$ increases with the background plasma density increasing but the speed of the increasing becomes slower gradually, while the value of β decreases linearly with the background plasma density. The increase of $n_\perp$ leads to decrease of perpendicular phase velocity of lower hybrid wave, and the intensification of Landau damping leads to increase of the absorption rate, while the decrease of β directly leads to the decrease of the absorption rate. The value of $n_\perp$ directly has a great impact on the Landau damping. When the background plasma density is low, the absorption rates increase as the increase of the background plasma density. However, as the further increasing of the background plasma density, the speed of the increasing becomes slower because the change of β has the major impact which leads to the downward trend of the absorption rate. Since the changes of $n_\perp$ and β is not so large, the absorption rates change very slowly with the high background plasma density.

Effects of Toroidal Magnetic Field on the Absorption Rate

We calculate the absorption rates γ of protons and alpha particles under the same parameters as section "Effects of the Parallel Refractive Index on the Absorption Rate" except toroidal magnetic field. Results are shown in Fig. 7.The absorption rate γ is near zero when toroidal magnetic field is lower than 2.5T, but the absorption rates γ increase with the toroidal magnetic field, and the increasing speed becomes slower gradually. When B is about 3.3T, the absorption rates of proton and alpha particle intersect and then the absorption rate of alpha particles is larger than that of proton.

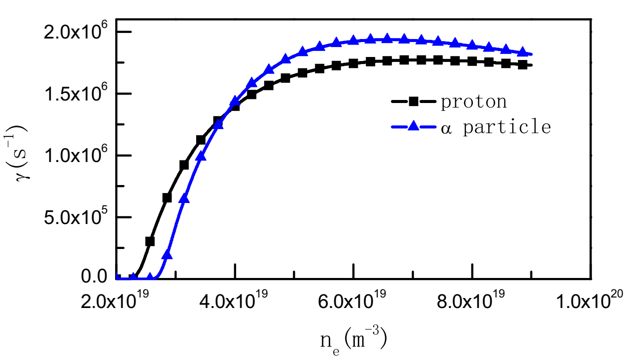

Fig. 5 The absorption rates γ of protons and alpha particles as a function of background plasma density

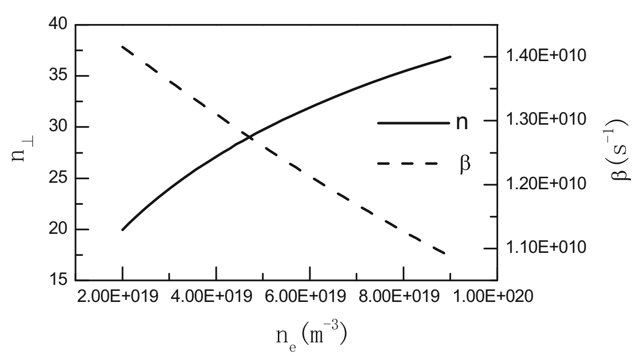

Fig. 6 The value of β and $n_\perp$ as a function of the background plasma density

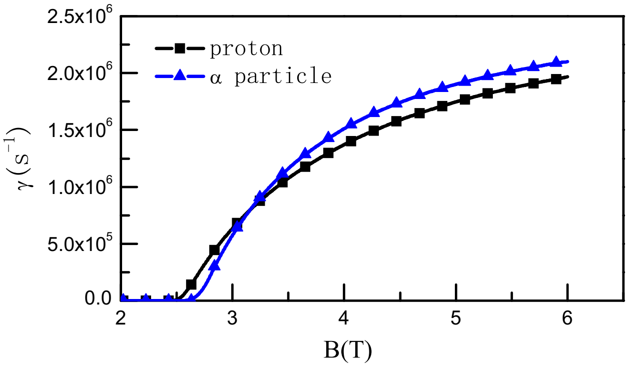

Fig. 7 The absorption rates γ of protons and alpha particles as a function of the intensity of toroidal magnetic field

The toroidal magnetic field affects the dispersion relation through affecting ions' cyclotron frequency. From Eq. (2), the increase of B leads the value of $\varepsilon_{\perp}$ to zero, and it leads to increase the value of $n_{\perp}$ and then to increase the value of the absorption rate.

Conclusions

It is now increasingly admitted that LH waves have a unique capability to drive the current efficiently far off-axis as required in ITER steady-state scenarios. However, many high energetic ions generated by fusion reactions, injection of neutral beams, and RF acceleration in ITER may damp the wave energy, thus reducing the CD efficiency. The effects of parallel refractive index, LH frequency, electron temperature, plasma density and toroidal magnetic field on the absorption rate of LHW by high energetic ions have been carried out for the ITER steady-state reference scenario 4. Three kinds of high energetic ions are compared in this paper.

In this paper, we assume that the absorption process is linear, high energetic ions are well confined and thermalized and the LH power density is so low that it does not affect the velocity distribution of the high energetic ions. The study does not consider spatial dependence and only study an ideal homogenous plasma with a magnetic field.

The absorption rate of the 1 MeV protons has the same level with that of 3.5 MeV alpha particles, but the absorption rate of the 1 MeV deuterons is much smaller compared with them. So we mainly compared the absorption rates of protons and alpha particles. The absorption rate all increases in some degree with parallel refractive index, plasma density and toroidal magnetic field, but it increases first and then decrease with LH frequency. The peaks are at 2.75 and 3.5 GHz for protons and alpha particles respectively. The influence of electron temperature on the absorption rate is small relatively.

The perpendicular refractive index $n_{\perp}$ of LHW is an important parameter that affects the absorption rate. Generally, Landau Damping rate is decreased when the perpendicular refractive index $n_{\perp}$ decreases, which will lead the absorption rate to decrease. The perpendicular refractive index is related the parallel refractive index of the launched LHW. On the other hands, seen from Eq. (13), parameters in the expression $\beta = \frac{\left(\varepsilon_{\perp} - n_{\parallel}^2\right)\left(\varepsilon_{\perp} - n^2\right)}{\partial Re(D(\omega, \boldsymbol{k}))/\partial\omega}$ also compete with the perpendicular refractive index $n_{\perp}$, and it affects the absorption rate.

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