TRANSFER PROCESSES IN RHEOLOGICAL MEDIA

NONISOTHERMAL CHANNEL FLOW OF A NON-NEWTONIAN FLUID UNDER THE CONDITIONS OF CHEMICAL TRANSFORMATIONS

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The processes of hydrodynamics and heat transfer during the flow of a non-Newtonian polymeric medium in a plane channel have been investigated with boundary conditions of the first kind. Consideration was given to a steady-state nonisothermal process with low values of the Reynolds number and high values of the Péclet number, which made it possible to neglect gravity and inertial forces, and also longitudinal thermal conductivity. The Ellis model with a viscosity dependent on temperature, pressure, and the degree of chemical transformation was used as the rheological model. A high-viscosity medium was investigated. Therefore, it is the dissipative term that was taken account of in the energy equation. With the above rheological model, an expression in explicit form was obtained from the equation of motion of such a medium for the velocity profile of its flow. It has been established that the dependence of the viscosity of a medium on its temperature, pressure, and the degree of transformation has an appreciable effect on the distribution of the temperature of the medium and the velocity of its flow along the channel's length. Taking account of the temperature dependence of the viscosity of the medium leads to a decline in the role of dissipation in the process of its warmup. The dependence of the medium's viscosity on pressure, conversely, considerably enhances the dissipative effect, which appreciably accelerates the course of a chemical reaction.

Keywords: *non-Newtonian fluid, heat transfer, nonisothermal flow, dissipation.*

Introduction. The processes of nonisothermal flow of non-Newtonian media in various channels have been the focus of numerous publications whose partial review was given in [1]. Use is frequently made of the power law as a rheological model [2, 3]. However, the power-law model for a pseudoplastic medium has a substantial drawback: at low rates of shear (shear stresses), this model gives an infinitely high viscosity. In this connection, the Ellis rheological model [4, 5] is preferred. For flow in a plane channel, this model is of the form

$$\frac{\partial v_x}{\partial y} = \frac{1}{\mu}\,(1 + k|\tau_{xy}|^{m-1})\tau_{xy}\;. \tag{1}$$

At high rates of shear (shear stresses), it represents the power law, and at very low rates (stresses) demonstrates the Newtonian behavior. In [6], the error in using the power-law model when problems of flow and heat transfer in a plane channel are solved has been evaluated in [6]. Calculations for the power law and the Ellis model were compared. It has been noted that the power-law model yields a great error in predicting the fields of temperature and velocity, and also of stress on the wall. Here, an analysis of the literature showed the absence of works on describing flow of an Ellis fluid with a temperature-dependent viscosity under nonisothermal conditions. Therefore, in the present work, consideration is given to nonisothermal flow of a high-viscosity non-Newtonian medium with a variable viscosity in a plane channel (Fig. 1). The medium′s temperature T_0 and the channel-wall temperature T_w do not coincide, with the quantity T_w being considered larger than T_0. This means that with flow, the compound will warm up both from the hot channel walls and due to the dissipation of energy

The action of pressure on the polymer viscosity is among the most important theoretical and technical issues. In actual practice, in various processes of treatment of high-viscosity polymers, we are dealing with pressures of the order of
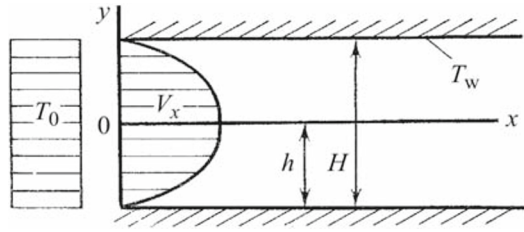

Fig. 1. Diagram of the process.

hundreds (in extrusion) and thousands (in pressure casting) of atmospheres that exert a substantial influence on the viscosity of a polymer and cause its parameters to deviate from technological ones calculated without account of this effect. Partial review of the works on this issue has been given in [7, 8]. Noteworthy is the absence of works that would take account of the dependence of the viscosity on pressure for the Ellis model.

Wall layers of liquid flow in a channel warm up most intensely from its hot walls. The dissipative function of flow in this region takes on maximum values. Furthermore, wall layers stay in the channel much longer compared to the axial zone. A combination of these factors may result in the premature beginning of the reaction of hardening of thermosetting plastics or vulcanization (prevulcanization) of rubber mixes. Therefore, it is necessary to take into account the fact that in the low of a liquid, there is a chemical reaction which may strongly affect its rheological properties.

In the present work, an effort was made to develop a model of nonisothermal channel flow of a non-Newtonian fluid, that is relatively simple for engineering applications and takes account of the viscosity of the fluid on its temperature, pressure, and the degree of chemical transformation.

Mathematical Model. A study is made of a polymeric medium of high viscosity whose flow in a channel is implemented at low values of the Reynolds number. In this case the hydrodynamic initial portion of flow does not need to be considered, the profile of flow velocity at entry into the channel may be assumed to be developed, and inertial terms may be eliminated from the equation of motion. Also, it is assumed that there are no transverse (secondary) flows in the channel, i.e., flow has only one longitudinal velocity component v_x. We write the Ellis rheological equation (1) in dimensionless form

$$\frac{\partial V_x}{\partial Y} = \frac{1}{\overline{\mu}} (1 + K|\sigma_{xy}|^{m-1})\sigma_{xy} , \tag{2}$$

where $Y = y/h$, $V_x = v_x/\overline{v}_x$, $\overline{\mu} = \mu/\mu_w$, $\sigma_{xy} = \dfrac{\tau_{xy}h}{\mu_w\overline{v}_x}$, and $K = \dfrac{k\mu_w^{m-1}\overline{v}_x^{m-1}}{h^{m-1}}$.

Rheological equation (2) is supplemented with the equation of motion in dimensionless form

$$\frac{\partial \sigma_{xy}}{\partial Y} = \frac{dP}{dX} , \tag{3}$$

where $P = \dfrac{ph}{\mu_w\overline{v}_x}$ and $X = x/h$, and with the condition of constant flow rate

$$\int_0^1 V_x dY = 1 . \tag{4}$$

Boundary conditions are of the form

$$Y = 0 : \quad \frac{\partial V_x}{\partial Y} = 0 , \quad \frac{\partial \theta}{\partial Y} = 0 , \tag{5}$$

$$Y = 1 : \quad V_x = V_s , \quad \theta = 0 , \tag{6}$$

$$X = 0 : \quad \theta = 1 , \tag{7}$$

where $\theta = T_w - T/T_w - T_0$.

Upon the first integration of the equation of motion (3), we obtain the relation

$$\sigma_{xy} = \frac{dP}{dX} Y \, . \tag{8}$$

Substituting (8) into (2), we have

$$\frac{\partial V_x}{\partial Y} = \frac{1}{\mu} \left(\frac{dP}{dX} Y - K \left(-\frac{dP}{dX} \right)^m Y^m \right) . \tag{9}$$

Upon the repeated integration with account of boundary condition (6), we obtain the expression for the velocity profile

$$V_x = V_s - \frac{dP}{dX} \int_Y^1 \frac{Y}{\mu} \, dY + K \left(-\frac{dP}{dX} \right)^m \int_Y^1 \frac{Y^m}{\mu} \, dY \, . \tag{10}$$

The unknown pressure gradient can be found by using the condition of constant flow rate (4)

$$-\frac{dP}{dX} \int_0^1 F_1(X, Y) \, dY + K \left(-\frac{dP}{dX} \right)^m \int_0^1 F_2(X, Y) \, dY = 1 - V_s \, ,$$

$$F_1(X, Y) = \int_Y^1 \frac{Y}{\mu} \, dY \, , \quad F_2(X, Y) = \int_Y^1 \frac{Y^m}{\mu} \, dY \, . \tag{11}$$

For the nonisothermal case the hydrodynamic problem is solved simultaneously with finding the temperature field. High-viscosity polymer compounds are of low thermal conductivity. As a result, flow of such media is implemented at high values of the Péclet number (Pe > 100) as a rule. This enables us to neglect, in the energy equation, the axial thermal conductivity with convective heat transfer. The energy equation written with account of the energy dissipation is of the form

$$V_x \frac{\partial \theta}{\partial X} = \frac{1}{Pe} \frac{\partial^2 \theta}{\partial Y^2} + \frac{Br}{Pe} \sigma_{xy} \frac{\partial V_x}{\partial Y} \, , \tag{12}$$

where $Pe = \dfrac{\overline{v}_x h}{a}$ and $Br = \dfrac{\mu_w \overline{v}_x^2}{\lambda(T_w - T_0)}$. We take into account the fact that in the process of flow of a polymer compound in the channel there can be a chemical reaction that can strongly affect the compound's rheological properties. Account taken of the dependence of a polymer on the degree of its transformation can fundamentally change the entire hydrodynamic and thermal situation in the channel. Therefore, the formulated problem is solved for the case where the viscosity of a polymer depends on its temperature, pressure, and the degree of transformation

$$\overline{\mu} = \exp \left[\psi \theta + b_2 P + \frac{b_3 \beta}{\beta_* - \beta} \right] , \tag{13}$$

where $\psi = b_1(T_0 - T_w)$. It can be seen from (13) that the viscosity of the polymer compound monotonically grows and tends to infinity as $\beta \to \beta_*$. Thus, there is no clear boundary between the solid and fluid phases.

Taking account of dependence (13) requires that a kinetic equation of the chemical reaction be introduced into the mathematical model. It is assumed that the chemical reaction is single-stage and can be described using one parameter, i.e., the degree of transformation

$$V_x \frac{\partial \beta}{\partial X} = Da(1 - \beta)^r \exp \left(-\frac{\overline{U}}{\theta + \gamma} \right) , \tag{14}$$

where $Da = \dfrac{k_0 h}{\overline{v}_x}$, $\overline{U} = \dfrac{U}{R(T_0 - T_w)}$, and $\gamma = \dfrac{T_w}{T_0 - T_w}$. Equation (14) reflects in many cases the actual picture of the process and enables us to abandon multiparametric systems of kinetic equations. Thus, it has been established experimentally [9]
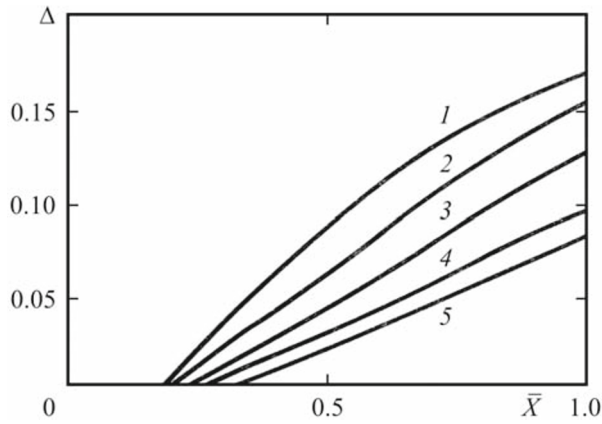
Fig. 2. Development of the thickness of the hardened polymer layer along the channel length at Br = –3.5, Da = 4, and Pe = $1.8 \cdot 10^3$ (1), $2.2 \cdot 10^3$ (2), $2.7 \cdot 10^3$ (3), $3.6 \cdot 10^3$ (4), and $4.2 \cdot 10^3$ (5).
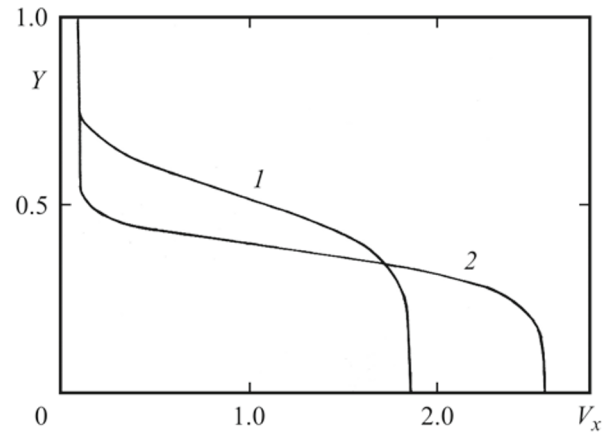
Fig. 3. Velocity profiles of polymer flow at exit from the channel at Br = 0 (1) and –10 (2).

that many reactions of hardening of thermosetting plastics or vulcanization of rubber mixes are single-stage. Here, most of the grades of rubber mixes have the reaction order $r = 1.0$–1.6. Equation (14) also describes the reaction of hardening of phenolformaldehyde resins with a reaction order $r = 1.42$. It should be noted that in the reactions of formation of polyurethanes, we have $r = 2$ as a rule [10]. In various works, Eq. (14) was also used to describe the kinetics of hardening of epoxy resins.

The mass-mean temperature of a polymer compound in dimensional and dimensionless forms is determined by the relations

$$T_{\mathrm{m}} = \frac{\int_0^h T v_x \, dy}{\int_0^h v_x \, dy}, \quad \theta_m = \int_0^1 \theta V_x \, dY. \tag{15}$$

The Nusselt number characterizing the local transfer on a channel wall is determined, in dimensional and dimensionless forms, by the relations

$$\mathrm{Nu} = \frac{\alpha h}{\lambda} = \frac{h}{|T_{\mathrm{m}} - T_{\mathrm{w}}|} \left(\frac{\partial T}{\partial y} \right)_{y=h}, \quad \mathrm{Nu} = \frac{1}{\theta_{\mathrm{m}}} \left(\frac{\partial \theta}{\partial Y} \right)_{Y=1}. \tag{16}$$

The problem was solved numerically by the iteration scheme with the finite-difference method. Initially, at the zero iteration step, use was made of the velocity and pressure-gradient distributions along the channel length for the case of flow of an isothermal fluid with a constant viscosity, which are easily obtained from (10) and (11). Here, the slip velocity on the wall V_{s} was determined from the formula obtained in [11]. Next, energy equation (12) and kinetic equation (14) were solved. Upon finding the $T(x, y)$ and $\beta(x, y)$ fields, at the next iteration step, refined recalculation of the distributions of velocity, pressure, and stresses of polymer flow was carried out with account taken of the dependence of its viscosity on temperature, pressure, and the degree of transformation (13).

Discussion of Results. The calculations were carried out in dimensionless form for a polymer compound with properties characteristic of cast grades of rubber mixes that are used in pressure casting. It was assumed that wall layers of polymer flow in the channel heat up first due to the hotter channel wall and by dissipation. Furthermore, wall flow layers stay in the channel longer than in the axial zone. A chemical reaction near the channel wall may result because of the excess warmup of the polymer. The premature beginning of the reaction may lead to an appreciable growth in the polymer viscosity in this region. Due to the strong dependence of the viscosity of the polymer on the degree of its transformation, the

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stagnation of the wall layers occurs, which, at a constant total flow rate, causes the flow in the axial zone to accelerate. The flow-velocity profile becomes sharply stretched and with a bend. Here, a pronounced wall layer of the hardening polymer (or prevulcanized rubber) moving with a constant velocity along the wall appears. In connection with the fact that the polymer compound slips over channel walls, this layer begins to form at a certain distance from the channel inlet. Here, the greater the flow rate, the larger the distance from the inlet, at which the layer is formed, as seen from Fig. 2 which shows the development of the dimensionless thickness of the hardened layer $\Delta = \delta/n$ on the channel′s dimensionless length $\bar{X} = x/l$ at a varying volumetric flow rate (at different Péclet numbers).

Analyzing the influence of the nonisothermicity of the reactive fluid on the process of its flow, we must primarily reflect the role of dissipation, since during the flow of a high-viscosity polymer compound, dissipative heat releases may turn out to be a factor sharply accelerating the chemical reaction of hardening of the polymer. Figure 3 shows velocity profiles of polymer flow at exit from the channel ($\bar{X} = 1$) that have been calculated with account and without account taken of dissipation at fixed values of Damköhler and Péclet numbers. Taking account of the dissipative factor leads to higher local temperatures of the flow and hence a faster reaction of hardening (vulcanization) of the polymer. This is manifested as the accelerated buildup in the thickness of its hardened wall layer and accordingly as the stronger stretching of the flow-velocity profile, although it is no secret that in the given example, Da and Pe numbers were selected so as to emphasize the dissipation effect determined by the Brinkmann number.

Conclusions. We were the first to formulate the problem on nonisothermal flow of a non-Newtonian polymer medium in a plane channel and to solve it based on the Ellis rheological model. The use of this model enables us to obtain, in explicit form, an expression for the velocity profile of flow of such a fluid with account of dissipative heat release and the dependence of the viscosity of the fluid on its temperature, pressure, and the degree of chemical transformation. We have calculated the temperature and velocity profiles of flow of the medium in question and have shown the substantial influence of its volumetric flow rate and of energy dissipation on the rate of growth in the hardened wall layer, which in turn leads to a substantial transformation of the velocity profile.

NOTATION

a, thermal diffusivity; b_1, b_2, and b_3, empirical rheological constants; h, channel′s half-height; k and m, rheological constants of the Ellis model; p, pressure; Pe, Nu, Br, and Da, Péclet, Nusselt, and Damköhler numbers; r, rheological-reaction order; R, universal gas constant; T, polymer temperature; T_m, mass-mean polymer temperature in a given cross section of the channel; T_w, channel-wall temperature; T_0, polymer temperature at entry into the channel; U, activation energy of the chemical reaction; v_x, axial component of the flow velocity; $\bar{v}_x$, average flow velocity; x and y, longitudinal and transverse coordinates; α, heat-transfer coefficient on the channel wall; β, dimensionless (relative) degree of chemical transformation; β_*, degree of transformation at which a polymer compound loses flowability; λ, thermal conductivity; μ, polymer viscosity; μ_w, polymer viscosity at the temperature T_w; τ_{xy}, shear stress. Subscripts: m, mean; s, slip; w, wall.

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