HYDROGASDYNAMICS IN TECHNOLOGICAL PROCESSES

BENDING OF A VISCOUS JET EMANATING FROM A CAPILLARY

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The bending of the jet of a viscous fluid, outflowing from a capillary, under the joint action of the inertial, viscous, and surface tension forces of the fl uid in it, was investigated. A linear model of the bending of such a jet is proposed. The dispersion relations have been obtained for the rate of increasing the disturbances of this jet. It is shown that the jet bends spontaneously. A qualitative analysis of the influence of the viscosity of a fluid and the velocity of its outfl ow from a capillary on the angle of deviation of the fl uid jet formed from the capillary axis has been performed.

Keywords: viscous fl uid jet, jet bend, capillary hydrodynamics, stability loss, drip refrigerator-emitter.

Introduction. Over the past decade, significant progress has been achieved in the development of space technologies: networks of low-orbit satellites, space tugs, and systems of service in the outer space have been realized, and work has been carried out to create technologies for the assembling of large antennas in orbit, to use powerful nuclear installations in the outer space, and to organize a large-scale production in orbit. To completely realize the indicated and other important projects, it is necessary to substantially increase the power of the energy installations of spacecrafts. This problem is directly related to the problem on the removal of the low-potential heat from these installations which is usually solved with the use of panel refrigerators-emitters. An increase in the power of such an installation leads to an increase in its emitting surface area, mass, and vulnerability to meteorites. The use of drip refrigerators-emitters (DRE), whose operation is based on the radiative cooling of a droplet flow that is formed in a special way, propagates in the outer space, and is then caught [1], for overcoming the limitations imposed on the energy installations of spacecrafts, is the concern of the present work.

For a DRE of importance is the parallelism of the fluid jets formed in a droplet sheet produced by a drip emitter, which was attained before through the improvement of the technology of fabrication of the capillary holes in the die of the emitter [2–5]. However, experimental investigations have shown that the direction of propagation of the fluid jets emanating from capillary channels is distorted not only by the unevenness of these channels but also by their bend (Fig. 1).

The phenomenon of bending of fluid jets has been investigated over several decades because of its importance for many applications. At present there are methods of calculating the shape of fluid jets in a number of cases, among which is the inertial outflow of a fluid with no viscous, surface tension, and mass forces from a capillary [6], the outflow of a viscous fluid with weak mass and inertial forces from a capillary [7], the multistable inertial-gravitational incidence of a fluid jet on a surface $[8]$, the outflow of a fluid from a capillary with a predominant action of the surface-tension force on it $[9]$, and the aerodynamic interaction of a fluid with the surrounding medium [10].

The bending of the fluid jet in a capillary of a drip refrigerator-emitter substantially influences the movement of the jet through the liquid film formed on the outer surface of the die of the emitter (Fig. 1a and b), the removal of the capillary meniscus at the instant the emitter is put on, and the deviation of the jet from the direction of the fluid outflow from the capillary channel. This deviation is mainly determined by the joint action of the inertial, dynamic pressure, viscous, and surface tension forces of the fluid in the jet.

Formulation of the Problem. A bending outflow of a viscous fluid from a capillary into a vacuum under conditions where the action of the mass forces on the fluid jet formed can be disregarded is considered. It is known from the experimental

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Fig. 1. Bending of VM1-S jets at a temperature of 60° C: a) instant an emitter with a plane die in which holes of diameter 0.35 mm are made is put on: b) interaction of jets of diameter 0.27 mm, moving with a velocity of 8 m/s, with a film; c) removal of the capillary meniscus on the milled protrusions in an emitter with capillary holes of diameter 0.5 mm at a liquid outflow velocity of 2 m/s; d) interaction of the liquid jets, emanating from a capillary tube of radius 0.7 mm with a velocity of 0.4 m/s, with the meniscus on the tube.

observations that the bend of such a fluid jet is substantially dependent on its interaction with the meniscus formed near the capillary hole.

In [6], the problem on the bending outflow of a viscous fluid from a capillary was solved for the case where the direction of propagation of the fluid jet formed is mainly determined by the quasi-stationary dynamic pressure and inertial forces of the fluid, and the model equations involve only the derivatives of the deviation of the jet from the capillary axis δ with respect to its coordinate. The solution of this problem for the case where the direction of propagation of the fluid jet is determined mainly by the nonstationary viscous force of the fluid and the model equations are local and involve none of the indicated derivatives are presented in [7]. We have obtained quasi-one-dimensional nonlocal nonstationary solutions of the problem on the outflow of a viscous fluid from a capillary, defining the bending of the fluid jet formed in the linear approximation.

Fig. 2. Diagram (a) and photograph (b) of a bending outflow of a viscous fluid from a capillary needle: 1) axis of the needle; 2) axis of the fluid jet; 3) center of the capillary hole.

Figure 2a shows a diagram of the bending outflow of a viscous fluid from a capillary needle. The bend of the fluid jet formed is planar, and the axes of needle 1 and jet 2 pass through the plane of the pattern. The center of the capillary hole 3 is at the axis of rotation of the jet. The bend of the fluid jet is due to the action, on it, of the moment M of the viscous, inertial, surface tension, and dynamic pressure forces of the fluid. It is assumed that near the capillary hole there arises a meniscus of radius larger by *p* times than the jet radius *r*.

If the axis of the jet is shifted from the axis of the needle through distance δ , the moment of the dynamic pressure forces *M*b.d.p is determined as the product of the jet shift and the force equivalent to the rate of change in the momentum of the jet $[6]$:

$$
\mathcal{M}_{\text{b.d.p}} = -\rho V^2 S \delta = -\rho V^2 \pi r^2 \delta \,. \tag{1}
$$

The viscous force bending the jet can be estimated by the relation

$$
F_{\rm v} = -2\pi r \mu V \ .
$$

The bending moment of the viscous forces $M_{b,v}$ is equal to

$$
\mathcal{M}_{b,v} = -2\pi r \mu V \delta \,. \tag{2}
$$

The bending moment of the capillary forces $M_{b,c}$ is determined as the product of the capillary pressure in the jet into its cross-sectional area and arm:

$$
\mathcal{M}_{b.c} = -\pi r \sigma \delta \tag{3}
$$

The inertial, surface tension, and viscous forces acting in the fluid jet prevent its bending. If the jet is bended, its velocity field changes so that the particles on the outer side of the bend move with a velocity higher than the velocity of movement of the particles on its inner side. The velocity field of the fluid jet changes with change in the pressure in it, and this change can be determined by the Bernoulli equation

$$
\frac{1}{2}\,\rho V^2 + P_0 = \frac{1}{2}\,\rho u(z)^2 + P(z)\,.
$$

The movement of the fluid in the jet under the action of the centrifugal force is defined by the relation

$$
-\frac{\rho V(z)^2}{R_\infty}=\frac{\partial P}{\partial z}.
$$

In the case where R_{∞} is much larger than the radius of the jet *r* (Fig. 2b), we have the relations

$$
u(z) = V\left(1 + \frac{z}{R_{\infty}}\right), \quad P(z) = P_0 - \frac{\rho V(z)^2}{R_{\infty}},
$$

where $P_0 = \sigma/r$ is the capillary pressure in the internal space of the jet. In this case, the moment of the inertial force preventing the bending of the jet will be equal to [6]

$$
\mathcal{M}_{\text{pr.b}} = \frac{\rho V^2}{R_{\infty}} \iint_{S} z^2 dS.
$$

Since at a small deviation of the jet from the axis of the capillary, $1/R_\infty = -d^2\delta/dx^2$, we may write the equation

$$
\mathcal{M}_{\text{pr.b}} = -\delta'' \rho V^2 \iint\limits_{S} z^2 dS = \delta'' \rho V^2 \frac{1}{2} \pi r^4 \,. \tag{4}
$$

The surface tension force of the fluid in the jet also prevents its bending. In the case of bending of the jet, the pressure in it changes by the value

$$
\Delta P = \frac{\sigma}{R_{\infty}} = -\sigma \delta''.
$$

Under the action of the pressure in the jet, in it there arises the moment

$$
\mathcal{M}_{\text{pr.b}} = \iint_{S} \Delta P z dS = -\sigma \delta'' \iint_{S} z dS = \frac{2}{3} \pi r^3 \sigma \delta'' \ . \tag{5}
$$

The influence of the viscous force acting in the jet on its bending can be determined by the rate of deformation of the jet edges [6]

$$
v = 2rx^* \frac{d}{dt} \left(\frac{\delta}{x^{*2}} \right),
$$

where the expression differentiated with respect to the time represents the local radius of the curvature of the particle paths in the core of the fluid flow. Of practical interests is the bend of the jet in the region of relaxation of its velocity of characteristic length *x** determined from the theory of the boundary layer near the capillary hole:

$$
\frac{x^*}{r_{\rm b}} = \frac{V\rho}{\mu} \left(\frac{p}{0.7}\right)^2 r_{\rm b}.
$$

In view of this relation, the viscous force preventing the bending of the jet can be represented in the following form:

$$
F = \mu \frac{v}{x^*} \pi r^2 = \frac{2}{x^{*2}} \pi r^3 \mu \dot{\delta}.
$$

The moment of this force $M_{\text{pr.v}}$ is equal to

$$
\mathcal{M}_{\text{pr.b}} = -\frac{2}{x^{*2}} \pi r^4 \mu \dot{\delta} \tag{6}
$$

In the final analysis we write the moment equation defining the bending of the jet near the capillary hole

$$
\rho V^2 \pi r^2 \delta + 2\pi r_b \mu V \delta + \pi r \sigma \delta + \delta'' \rho V^2 \frac{1}{2} \pi r^4 + \frac{2}{3} \pi r^3 \sigma \delta'' = \frac{2\pi \mu^3}{V^2 \rho^2} \left(\frac{0.7}{p}\right)^4 \dot{\delta} \,. \tag{7}
$$

We introduce into consideration the dimensionless variables $\delta = r\delta'$, $x = rx'$, and $t = rt'/V$ as well as the following dimensionless similarity criteria: the parameter *M* and the Ohnesorge number determined by the relation

$$
Oh = \frac{\mu}{\sqrt{\sigma r \rho}}.
$$

The parameter *M* represents the ratio between the velocity of the jet and the minimum attainable velocity of a stable outflow of the fluid from the capillary hole V_{min} :

$$
\frac{2\sigma}{r} = \frac{\rho V_{\text{min}}^2}{2} \ .
$$

Hence,

$$
M = \frac{V_0}{V_{\text{min}}} = \frac{V_0}{2} \sqrt{\frac{\rho r}{\sigma}}.
$$

Equation (7) expressed in new variables takes the following form at $p = 1$:

$$
\text{Oh}^3 \dot{\delta} = \frac{2}{3} (1 + 3M^2) \delta'' + (1 + 4M^2 + 4M \text{ Oh}) \delta \,. \tag{8}
$$

Stationary Bending of a Jet. The stationary approximation of Eq. (8) has the form

$$
\frac{2}{3} (1 + 3M^2) \delta'' + (1 + 4M^2 + 4M \text{ Oh}) \delta = 0.
$$
 (9)

If $d\delta/dx = 0$ at the outlet of a capillary, the axis of the fluid jet emanating from it will deviate from the capillary axis by the law

$$
\delta = \mathcal{D}\left[\cos\left(\sqrt{1 + \frac{M^2 + 4M \text{ Oh}}{1 + 3M^2}} x\right) - 1\right].\tag{10}
$$

It follows from this relation that the length of the bend wave of the jet is equal to

$$
\lambda = 2\pi \sqrt{\frac{1 + 3M^2}{1 + 4M^2 + 4M \text{ Oh}}}.
$$

The dependence of this length on the parameter *M* at different values of the Ohnesorge number is presented in Fig. 3a. It is seen from this figure that the higher the viscosity of the fluid in the jet, the smaller the length of its bend wave and the larger the possible angular deviation of the jet from the capillary axis. The quantity *D* is determined by the parameters of the capillary meniscus and it can be estimated by two ways, in particular, on the supposition that the initial curvature of the jet is equal in order of magnitude to the curvature of the capillary-meniscus surface: $d^2\delta/dx^2 \sim 1/p$. In this case,

$$
\mathcal{D} = \frac{1}{p} \left[1 + \frac{M^2 + 4M \text{ Oh}}{1 + 3M^2} \right]^{-1} . \tag{11}
$$

Graphs of dependence (11) at $p = 1$ and different values of Oh are presented in Fig. 3b. It is seen from this figure that an increase in the viscosity of the fluid in the jet leads to an increase in its deviation from the axis of the capillary.

It can be also assumed that the maximum deviation of the jet from the axis of the capillary comprises $\delta_{\text{max}} = (p-1)$. In this case, the curvature of the jet will be equal in order of magnitude to $\delta_{\text{max}}/x^{*2}$. With the use of (10), the following relation can be obtained:

$$
\mathcal{D} = \frac{1}{16} \frac{p-1}{p^2} \frac{\text{Oh}^2}{M^2} \frac{1+3M^2}{1+4M^2+4M \text{ Oh}}.
$$
\n(12)

If the deviation of the jet from the capillary axis, determined by (12), is attained at a distance $\lambda/2$, one can estimate the angle of the jet deviation φ . The dependence of this angle on the parameter *M* at different Ohnesorge numbers is

Fig. 3. Dependence of the bend length of a fluid jet emanating from a capillary (a), the amplitude of the deviation of the jet from the capillary axis (b), the angle of this deviation (c), and the maximum wave number of the jet (d) on the parameter *M* at $p = 2$: 1) Oh = 0.1; 2) 0.25; 3) 0.5; 4) 1.

presented in Fig. 3c. The value of the parameter *p* was taken to be equal to 2 because, at this value of *p*, the angle φ reaches a maximum value. It is seen from the indicated figure that an increase in Oh leads to a large increase in the angle φ which can reach several degrees.

Nonstationary Bending of a Jet. The rate of increasing the disturbances of the fluid jet emanating from a capillary was determined from the solution of Eq. (8) on the condition that $\delta \sim \exp(i k x + \omega t)$. Substitution of this expression into (8) gives the relation

$$
\omega = \frac{2}{3} \frac{1 + 3M^2}{Oh^2} \left(\frac{3}{2} \frac{1 + 4M^2 + 4M \text{ Oh}}{1 + 3M^2} - k^2 \right).
$$
 (13)

It follows from the dispersion relation that the shear deformations of the jet, which cause no bends in it, increase with a maximum rate. The factor of increasing the shear of the jet is determined from the relation

$$
\omega_{\text{max}} = \frac{1}{\text{Oh}^2} (1 + 4M^2 + 4M \text{ Oh}) .
$$

The minimum possible value of *M* for a DRE can be estimated at 1.5. In this case, the limiting value of ω_{max} is equal to

$$
\omega_{max} > \frac{1}{Oh^2} (10 + 6 \text{ Oh}) .
$$

Of practical interest is the maximum Ohnesorge number equal to 0.5. In this case, $\omega_{\text{max}} > 50$. Because of the thermal fluctuations in the jet, its axis can displace for a distance of the order of $(k_BT/\sigma)^{1/2}$ ~ 1 nm. The characteristic radius of the jet is equal to $r \sim 0.1$ mm, i.e., $\delta_0 \sim 10^{-5}$. This initial disturbance of the jet could increase to a macroscale for the dimensionless time equal to ~12. In the case where $r = 100 \mu m$ and $V = 2 \mu s$, the thermal fluctuations in the jet could increase for the time equal to ~5⋅10⁻³ s. For this time, the axis of the jet shifts from the center of the capillary channel to a new stationary position determined by the geometry of the capillary meniscus. According to (13), the maximum wave number of the jet bend is equal to

$$
k_{\max} = \sqrt{\frac{3}{2} \frac{1 + 4M^2 + 4M \text{ Oh}}{1 + 3M^2}}.
$$
 (14)

The dependence of k_{max} for different values of Oh is presented in Fig. 3d. It is seen from this figure that the higher the viscosity of the fluid in the jet, the smaller the radius of its bend and the larger the deviation of the jet from the direction of outflow of the fluid from the capillary determined by its axis. However, as *M* increases, the maximum wave number of the jet bend rapidly approaches its asymptotic value equal to $k_{\text{max}} = 2^{0.5}$.

Effect of Mass Forces. The bending moment of the gravity force of the fluid in the jet emanating from a capillary in the horizontal direction is determined by the relation

$$
\mathcal{M}_{\text{b.g.}} = \frac{1}{2} \pi r^2 \rho g l^2.
$$

Comparing this moment with the moment of the dynamic pressure forces in the jet (1) at δ equal to the jet radius, we obtain the ratio between these moments $g l^2/(2V^2 r)$. In the case where the radius of the jet is equal to 0.1 mm and the velocity of the fluid outflow from the capillary is 2 m/s, the influence of the gravity on the jet becomes significant at $l > 1$ cm (~100 radii of the jet), which is larger by an order of magnitude than the characteristic length of the bend wave of the jet. Therefore, the direct influence of the gravity on the bending of the jet can be disregarded. However, the gravity indirectly influences the jet bending through the change in the shape of the capillary meniscus.

Interaction of Bending Jets. In the case where a fluid outflows from the capillaries positioned in *N* parallel rows in the die of a dip emitter, near the capillary holes there arises a film that determines the interaction of the fluid jets (Fig. 4). If the jets are bended only in the plane of disposition of the axes of the capillaries, the system of equations for the *N* values of δ*i* has the form

$$
\text{Oh}^{3} \dot{\delta}_{i} = \frac{2}{3} (1 + 3M^{2}) \delta_{i}^{*} + (1 + 4M^{2} + 4M \text{ Oh}) \delta_{i} + St_{i} , \qquad (15)
$$

where St_i accounts for the interaction of the *i*th jet with the other jets and is a function of all the values of δ_i and the derivatives $d\delta_i/dt$, $d\delta/dx$, and $d^2\delta/dx^2$. The interaction of bending fluid jets can be due to the action of the viscous and capillary forces in them as well as due to the change in the pressure field in the capillary meniscus on the surface of the die caused by the change in the dynamic pressure of the fluid in the jets. Moreover, in the system of fluid jets there takes place a long-range interaction: if jets intersect at a considerable distance from the surface of the die, the point of linkage of two jets can "fall," due to the action of the capillary forces, on the surface of the capillary meniscus.

Frames of a video recording of the fluid jets emanating from the capillary holes in the die of a drip emitter in the gravity field are presented in Fig. 4. Figure 4a shows the process of "disposal-floating" of a capillary meniscus on the surface of the die, realized due to the outflow of the fluid in the form of a knee. It is seen from Fig. 4b that at the site of the knee removed there arises a new overlap and not a new jet. We call the reader's attention to the presence of several "knees" in the jets formed (in the leftmost jet in Fig 4a and in the third jet from the right in Fig. 4b). The characteristic time period of a "knee" in a jet, representing the ratio between the length of the knee and the velocity of the jet, comprises $\sim 10^{-3}$ s. The development of the fluid jets emanating from the capillary holes in the die is shown in Fig. 4e and d. These jets can break down as a result of their nonlinear interactions and change the direction of their propagation.

Comparison of Calculation and Experimental Data. The results of our investigations can be used for the explanation of experimental data on the bending of the water jets emanating slowly from a capillary tube under the microgravity

Fig. 4. Outflow of a fluid with a velocity of 8 m/s from a drip emitter having a plane die with holes of radius 330 μ m spaced 1.5 mm apart at the instants of time $t = 0$ (a), 0.114 (b), 0.419 (c), and 0.425 (d).

conditions. In calculating the parameter *M*, the radius of a water jet was assumed to be equal to the radius of the capillary tube. Photographs of such jets emanating from the capillary holes in the die of a drip emitter with different velocities are presented in [11]. Droplets were formed in the emitter only at $M = 0.85$, 1.25, and 2.2. The formation of droplets was mainly determined by the bending vibrations of the jets. At $M = 0.85$, the droplets formed flew away within a cone with an angular opening $\varphi \approx 5^\circ$. The angle of such a cone was 1.5^o at *M* = 1.25 and was smaller than 1^o at *M* = 2.2. A decrease in the angle of flying away of droplets with increase in the velocity of the water outflow from the capillary tube corresponds to the theoretical model presented in Fig. 3c. The angle of flying away of droplets observed in an experiment was larger than that determined in the realization of the corresponding theoretical model constructed in the linear quasi-stationary approximation. The vibrations of the fluid jet in the experiment were nonlinear, with a complex dynamics determined by the processes of thinning and rupture of the jet. Moreover, in the experiment, droplets moved under the action of the aerodynamic effects. A good agreement has been obtained between the calculation and experimental data on the length of the bend wave of a fluid jet. It follows from (10) that the minimum length of the bend wave of a fluid jet with $M \to 0$ and Oh $\to 0$ comprises $\lambda_{\min} \approx 5.2r$. This result agrees with the experimental data obtained in [10].

Conclusions. A model of calculating the characteristics of the bending and shift of the jet of a viscous fluid outflowing from a capillary hole in the die of a drip emitter has been developed. It was established that the bending of the jet is due to its interaction with the capillary meniscus formed on the surface of the die. The bending vibrations of such a jet arise spontaneously, and their amplitude increases rapidly with increase in the Ohnesorge number of the jet and decrease in the velocity of its propagation. A fluid jet produced by a drip emitter with a plane die, in which the jet interacts with the film formed on the surface of the die at the instant the emitter is put on, experiences the largest angular deviation from the direction of the fluid outflow from a capillary in the emitter die. For the formation of a flow of droplets moving in the form of parallel jets, drip emitters with capillary tubes are best.

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NOTATION

 D , amplitude of deviation of a fluid jet from the axis of a capillary; *g*, free fall acceleration; *k*, wave number; k_B , Boltzmann constant; *l*, length of a jet region; *M*, dimensionless velocity of the fluid outflow from the capillary; *p*, ratio between the radius of the meniscus near the capillary hole and the jet radius *r*; *P*, pressure; *R*, radius of the meniscus at the outlet of the capillary channel; R_{∞} , radius of the jet-axis curvature; *r*, steady radius of the fluid jet; r_{out} , outer radius of a capillary needle; *S*, cross-sectional area of the jet; *T*, temperature; *t*, time; *u*, local velocity of movement of the fluid in the jet; *V*, mean mass velocity of propagation of the jet; *x*, coordinate axis coincident with the jet axis; *z*, transverse coordinate; δ, distance between the axes of the needle and the jet; λ, length of the bend wave of the jet; μ, dynamic viscosity of the fluid; ρ, density of the fluid; σ, surface tension of the fluid; $φ$, angle of deviation of the droplet path from the axis of the capillary; ω, factor of increasing the disturbance of the jet. Subscripts: v, viscous; out, outer; g, gravity; d.p, dynamic pressure; b, bend; c, capillary; pr, prevention.

REFERENCES

- 1. A. A. Koroteev, A. A. Safronov, and N. I. Filatov, Influence of the structure of a droplet sheet on the power of frameless cosmic emitters and the efficiency of energy installations, *Teplofiz. Vys. Temp.*, **54**, No. 5, 817–820 (2016).
- 2. A. A. Koroteev, *Drip Refrigerators-Emitters of New-Generation Cosmic Energy Installations* [in Russian], Mashinostroenie, Moscow (2008).
- 3. V. G. Konyukhov and G. V. Konyukhov, *Thermal Physics of Nuclear Energy Installations* [in Russian], Yanus-K, Moscow (2009).
- 4. A. V. Bukharov, *Thermophysical Problems of Obtaining Stable Flows of Droplets with Minimum Scatters in Velocity and Size*, Doctoral Dissertation (in Engineering), MÉI, Moscow (2016).
- 5. D. B. Wallace, D. J. Hayes, and J. M. Bush, *Study of Orifice Fabrication Technologies for the Liquid Droplet Radiator*, MicroFab Technologies, Inc. Piano, Texas (1991).
- 6. A. Bejan, On the buckling property of inviscid jets and the origin of turbulence, *Lett. Heat Mass Transf.*, **8**, No. 3, 187–194 (1981).
- 7. M. L. Merrer, D. Quere, and C. Clanet, Buckling of viscous filaments of a fluid under compression stresses, *Phys. Rev. Lett. Amer. Phys. Soc*., **109**, No. 6, Article ID 064502 (2012).
- 8. N. M. Ribe, M. Habibi, and D. Bonn, Stability of liquid rope coiling, *Phys. Fluids*, **18**, Article ID 084102 (1994).
- 9. T. Jingxuan, N. M. Ribe, X. Wu, and H. S. Shum, Steady and unsteady buckling of viscous capillary jets and liquid bridges, *Phys. Rev. Lett*., **125**, No. 10, Article ID 104502 (2020).
- 10. V. M. Entov and A. L. Yarin, Dynamics of free jets and films of viscous and rheologically complex fluids, *Itogi Nauki Tekh*., **18**, 112–197 (1984).
- 11. F. Sunol and R. Gonzalez-Cinca, Liquid jet breakup and subsequent droplet dynamics under normal gravity and in microgravity conditions, *Phys. Fluids*, **27**, ID Article 077102 (2015).