DOI 10.1007/s10891-021-02444-0
Journal of Engineering Physics and Thermophysics, Vol. 94, No. 6, November, 2021

MISCELLANEA

MAGNETOTHERMOELASTIC WAVES IN A ROTATING ORTHOTROPIC MEDIUM WITH DIFFUSION

A. K. Yadav UDC 536.21

In this paper, the governing partial differential equations for a rotating orthotropic magnetothermoelastic medium with diffusion are proposed on the basis of the Lord–Shulman theory of generalized thermoelasticity and the velocity equation is obtained. The plane wave solution of this equation is indicative of the existence of four quasi-plane waves, namely, quasi-longitudinal displacement (qLD), quasi-thermal (qT), quasi-mass diffusion (qMD), and quasi-transverse displacement (qTD) waves. The real values of the wave speeds are calculated for a particular material, and the effects of anisotropy, as well as of the diffusion, magnetic, and rotation parameters and the angle of incidence on the speeds are shown graphically.

Keywords: *thermoelasticity, orthotropic medium, diffusion, rotation, magnetic field, speed, plane waves.*

Introduction. Biot [1] developed the classical theory of dynamical coupled thermoelasticity. To remove the paradox of an infinite speed of thermal waves following from [1], Lord and Shulman [2] and Green and Lindsay [3] extended the classical dynamical coupled theory of thermoelasticity to the theory of generalized thermoelasticity. The details of these generalized theories can be found in the works of Hetnarski and Ignaczak [4] and Ignaczak and Ostoja-Starzewski [5]. Dhaliwal and Sherief [6] extended the Lord and Shulman generalization of the thermoelasticity theory to an anisotropic case given in [2]. Jeffreys [7] studied thermodynamics of an elastic solid. Gutenberg [8] invesigated the energy relation of reflected and refracted seismic waves. A. N. Sinha and S. B. Sinha [9] studied the reflection of thermoelastic waves in a solid half-space with thermal relaxation. Schoenberg and Censor [10] considered the plane wave propagation in a rotating isotropic medium and revealed three plane waves in it. Singh and Yadav [11, 12] analyzed the wave propagation in an anisotropic medium and concluded that the effect of rotation does not increase the number of waves in a transversely isotropic medium, but significantly affects their speeds. Chandrasekharajah and Srinath [13] studied thermoelastic plane waves in a rotating isotropic solid. The problems on the propagation of waves in rotating isotropic and anisotropic bodies with electric, magnetic, and thermal effects have been studied in [14–16]. Singh and Yadav [17] investigated the effect of rotation on the wave propagation in a magnetized monoclinic anisotropic medium. Abo-Dahab and Biswas [18] studied the effects of rotation and the thermal relaxation times on the Rayleigh waves in an anisotropic medium subjected to a magnetic field. Shaw and Othman [19] considered the propagation of the magnetoelastic Rayleigh waves in an orthotropic thermoelastic medium. Biswas and Mukhopadhyay [20] analyzed the thermal shock behavior and thermoelastic wave propagation in an orthotropic medium, subjected to a magnetic field, on the basis of three theories.

Diffusion is the movement of particles from a region with a greater number of particles per unit volume to a region with their smaller number. Thermodiffusion process plays an important role in many fields and objects, like satellites, returning space vehicles, geophysics, chemical industry, and the fabrication of integrated circuits in MOS (metal–oxide–silicon) transistors to prepare base, emitter, and dope polysilicon gates. Diffusion is also used to prepare a controlled amount of "dopants" in a semiconductor substrate. In addition, thermodiffusion is applied for more efficient extraction of oil from oil deposits. Using one relaxation time for a finite speed of the thermal waves propagation, Sherief et al. [21] developed the theory of generalized thermoelastic diffusion on the basis of the theory developed by Nowacki [22] and Dudziak and Kowalski [23]. Singh [24] investigated the problem of generalized thermodiffusion in the case of the reflection of P and SV

Shishu Niketan Model Senior Secondary School, Sector 22-D, Chandigarh, India; Department of Mathematics, M. M. (Deemed to be University) Mullana, Ambala, Haryana, India; email: yadavanand977@gmail.com. Published in Inzhenerno-Fizicheskii Zhurnal, Vol. 94, No. 6, pp. 1663–1672, November–December, 2021. Original article submitted March 1, 2020; revision submitted May 4, 2021.

waves from a free surface of an elastic solid. Aouadi [25] developed the micropolar theory of thermodiffusion and derived constitutive equations, using the Lord–Shulman theory. Lotfy et al. [26] investigated the effect of photothermal diffusion in the reflection of waves in a semiconductor medium. Mabrouk et al. [27] studied the effect of dual-phase-lag in photothermal process for a magneto-rotating diffusive medium. However, so far the wave propagation in a rotating, perfectly conducting, thermoelastic, orthotropic medium with diffusion in the presence of a magnetic field has not been studied in more detail. Yadav [28] studied the reflection of plane waves from a free surface of a rotating, orthotropic, magnetothermoelastic solid half-space with diffusion but have not calculated the wave speeds numerically. In this work, the governing equations for a rotating, orthotropic, magnetothermoelastic, perfectly conducting medium with diffusion are formulated in the context of the Lord–Shulman theory of generalized thermoelasticity. The velocity equation is obtained and solved and its solution is indicative of the existence of four quasi-plane waves, namely, quasi-longitudinal displacement (qLD), quasi-thermal (qT), quasi-mass diffusion (qMD), and quasi-transverse displacement (qTD) waves. The wave speeds are computed for a particular material, and their dependences on the diffusion, magnetic, and rotation parameters, as well as on the angle of incidence are shown graphically.

Basic Equations. We consider a homogeneous, thermoelastic, diffusive medium rotating with the angular rate $\mathbf{\Omega} = \Omega\mathbf{n}$ and subjected to a magnetic field with the induction $\mathbf{B}$ (here $\mathbf{B} = \mu_e\mathbf{H}$) at the initial temperature T_0, where $\mathbf{n}$ is the unit vector representing the direction of the axis of rotation and $\mathbf{\Omega} = (0, \Omega, 0)$. The basic governing equations for a homogeneous, rotating, orthotropic, thermoelastic solid with diffusion in the absence of the body forces and heat and mass diffusion sources in the generalized theory of thermoelasticiy [2] are the following:

equation of motion

$$B_{ijkm}e_{km,i} + a_{ij}T_{,i} + b_{ij}C_{,i} + (\mathbf{J} \times \mathbf{B})_i = \rho\{\ddot{u}_i + (\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}))_i + (2\mathbf{\Omega} \times \dot{\mathbf{u}})_i\} , \tag{1}$$

heat conduction equation

$$K_{ij}T_{,ij} = -\beta_{ij}T_0(\dot{e}_{ij} + \tau_0^t\ddot{e}_{ij}) + \rho C_E(\dot{T} + \tau_0^t\ddot{T}) + aT_0(\dot{C} + \tau_0^t\ddot{C}) , \tag{2}$$

mass diffusion equation

$$-\alpha_{ii}^* b_{km}e_{km,ij} - \alpha_{ii}^* b[C_{,ij}] + \alpha_{ii}^* a[T_{,ij}] = -(\dot{C} + \tau_0^c\ddot{C}) . \tag{3}$$

The Maxwell equations describing the electromagnetic field effect without the charge density and displacement current are

$$\text{curl } \mathbf{H} = \mathbf{J} , \quad \text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} , \quad \mathbf{B} = \mu_e\mathbf{H} , \quad \text{div } \mathbf{B} = 0 ; \tag{4}$$

and the Ohm law in a generalized form is

$$\mathbf{J} = \sigma[\mathbf{E} + (\dot{\mathbf{u}} \times \mathbf{B})] . \tag{5}$$

Here the magnetic field strength is taken as $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$, where $\mathbf{H}_0 = (0, H_0, 0)$, the thermal gradient effect on the conduction current $\mathbf{J}$ is neglected, and for a perfectly conducting medium $\sigma \to \infty$. We linearize the basic equations, neglecting the products of $\mathbf{h}$, $\mathbf{u}$, and their derivatives as the induced magnetic field strength $\mathbf{h}$ is very small.

The constitutive relations take the form:

$$\tau_{ij} = B_{ijkm}e_{km} + a_{ij}T + b_{ij}C , \quad \rho S = -a_{ij}(e_{ij} + \tau_0^t\dot{e}_{ij}) + \frac{\rho C_E}{T_0}(T + \tau_0^t\dot{T}) + a(C + \tau_0^t\dot{C}) ,$$

$$P^* = bC - b_{km}e_{km} - aT , \quad q_i = K_{ij}T_{,j} , \quad q_i = \rho\dot{S} , \quad \eta_i = -\alpha_{ij}^* P_{,j}^* ,$$

$$B_{ijkm} = B_{kmij} = B_{jikm} = B_{ijmk} , \quad a_{ij} = a_{ji} , \quad b_{ij} = b_{ji} ,$$

$$K_{ij} = K_{ji} , \quad \alpha_{ij}^* = \alpha_{ji}^* , \quad a_{ij} = -\beta_i^t\delta_{ij} , \quad b_{ij} = -\beta_i^c\delta_{ij} , \quad K_{ij} = K_{ii}\delta_{ij} , \quad \alpha_{ij}^* = \alpha_{ii}^*\delta_{ij} ,$$

$$\tau_{ij} = \frac{\partial W}{\partial e_{ij}} - \beta_i^t T \delta_{ij} - \beta_i^c C \delta_{ij} , \quad e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) ,$$

$$W = \frac{1}{2}(B_{11} e_{xx}^2 + B_{22} e_{yy}^2 + B_{33} e_{zz}^2 + B_{12} e_{xx} e_{yy} + B_{13} e_{xx} e_{zz} + B_{21} e_{yy} e_{xx}$$

$$+ B_{31} e_{zz} e_{xx} + B_{44} e_{yz}^2 + B_{55} e_{zx}^2 + B_{66} e_{xy}^2) ,$$

$$1 \Leftrightarrow 11 , \quad 2 \Leftrightarrow 22 , \quad 3 \Leftrightarrow 33 , \quad 4 \Leftrightarrow 23 , \quad 5 \Leftrightarrow 13 , \quad 6 \Leftrightarrow 12 ,$$

where P^* is the chemical potential per unit mass and η_i is the flow of a diffusing mass.

Formulation of the Problem. Let a uniform constant magnetic field of the strength $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$ is applied to a homogeneous, rotating, orthotropic, thermoelastic, diffusive medium. Due to the rotation and applied magnetic field, a change in the basic magnetic field occurs, so that an induced magnetic field with the strength $\mathbf{h} = (0, h, 0)$ and an induced electric field with the strength **E** develop in the medium. Let the medium is rotating about the y axis with the rotational rate $\mathbf{\Omega} = (0, \Omega, 0)$, centripetal acceleration $(\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}))$, and the Coriolis acceleration $\left(2\mathbf{\Omega} \times \frac{\partial \mathbf{u}}{\partial t}\right)$. Here the dynamic displacement vector **u** is $\mathbf{u} = (u, 0, w)$ and $\frac{\partial}{\partial y} = 0$. Using Eqs. (4) and (5) and the relation $(\mathbf{J} \times \mathbf{B})_i = \mu_e(\text{curl } \mathbf{h} \times \mathbf{H})$, we obtain the following equations:

$$(\mathbf{J} \times \mathbf{B})_1 = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z}\right), \quad (\mathbf{J} \times \mathbf{B})_2 = 0 , \quad (\mathbf{J} \times \mathbf{B})_3 = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2}\right). \tag{6}$$

Using the constitutive relations and Eq. (6) and following Dhaliwal and Sherief [6], Lord and Shulman [2], and Schoenberg and Censor [10], we can write the linear governing equations (1)– (3) in the xz plane as

$$B_{11} \frac{\partial^2 u}{\partial x^2} + (B_{13} + B_{55}) \frac{\partial^2 w}{\partial x \partial z} + B_{55} \frac{\partial^2 u}{\partial z^2} - \beta_1^t \frac{\partial T}{\partial x} - \beta_1^c \frac{\partial C}{\partial x} + \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z}\right) = \rho \left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t}\right), \tag{7}$$

$$B_{55} \frac{\partial^2 w}{\partial x^2} + (B_{13} + B_{55}) \frac{\partial^2 u}{\partial x \partial z} + B_{33} \frac{\partial^2 w}{\partial z^2} - \beta_3^t \frac{\partial T}{\partial z} - \beta_3^c \frac{\partial C}{\partial x} + \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2}\right) = \rho \left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t}\right), \tag{8}$$

$$K_{11} \frac{\partial^2 T}{\partial x^2} + K_{33} \frac{\partial^2 T}{\partial z^2} = \rho C_E \left(\frac{\partial T}{\partial t} + \tau_0^t \frac{\partial^2 T}{\partial t^2}\right) + a T_0 \left(\frac{\partial C}{\partial t} + \tau_0^t \frac{\partial^2 C}{\partial t^2}\right)$$
$$+ \beta_1^t T_0 \left(\frac{\partial^2 u}{\partial t \partial x} + \tau_0^t \frac{\partial^3 u}{\partial t^2 \partial x}\right) + \beta_3^t T_0 \left(\frac{\partial^2 w}{\partial t \partial z} + \tau_0^t \frac{\partial^3 w}{\partial t^2 \partial z}\right), \tag{9}$$

$$\alpha_{11}^* \beta_1^c \frac{\partial^3 u}{\partial x^3} + \alpha_{11}^* \beta_3^c \frac{\partial^3 w}{\partial x^2 \partial z} + \alpha_{33}^* \beta_1^c \frac{\partial^3 u}{\partial x \partial z^2} + \alpha_{33}^* \beta_3^c \frac{\partial^3 w}{\partial z^3} + \alpha_{11}^* a \frac{\partial^2 T}{\partial x^2}$$
$$+ \alpha_{33}^* a \frac{\partial^2 T}{\partial z^2} - \alpha_{11}^* b \frac{\partial^2 C}{\partial x^2} - \alpha_{33}^* b \frac{\partial^2 C}{\partial z^2} + \left(\frac{\partial C}{\partial t} + \tau_0^c \frac{\partial^2 C}{\partial t^2}\right) = 0 . \tag{10}$$

Solution of the Problem. The solutions of Eqs. (7)–(10) are sought in the following form:

$$(u, w, T, C) = (A, B, R, S) \exp\{ik(x \sin\theta + z \cos\theta - vt)\} , \tag{11}$$

where A, B, R, and S are constant, v is the phase speed, and k is the wavenumber. Using Eq. (11) in Eqs. (7)–(10), we get four homogeneous equations for A, B, R, and S:

$$(\Upsilon_1 - \Omega^*\zeta)A + \left(\Upsilon_2 - 2i\,\frac{\Omega}{\omega}\,\zeta\right)B + \frac{i}{k}\,\beta_1^t \sin\theta R + \frac{i}{k}\,\beta_1^c \sin\theta S = 0\,, \tag{12}$$

$$\left(\Upsilon_2 + 2i\,\frac{\Omega}{\omega}\,\zeta\right)A + (\Upsilon_3 - \Omega^*\zeta)B + \frac{i}{k}\,\beta_3^t \cos\theta R + \frac{i}{k}\,\beta_3^c \cos\theta S = 0\,, \tag{13}$$

$$\varepsilon^t\zeta \sin\theta A + \bar{\beta}^t\varepsilon^t\zeta \cos\theta B + \frac{i}{k}\,\beta_1^t(D_5 - \zeta)R - \frac{i}{k}\,\varepsilon^*\zeta S = 0\,, \tag{14}$$

$$\beta_1^c D_6 \sin\theta A + \beta_3^c D_6 \cos\theta B - \frac{i}{k}\,aD_6 R + \frac{i}{k}\,(bD_6 - \tau^*\zeta)S = 0\,, \tag{15}$$

where $\beta_1^c = (B_{11} + B_{12})\alpha_{1c} + B_{13}\alpha_{3c}$, $\beta_3^t = 2B_{13}\alpha_{1t} + B_{33}\alpha_{3t}$.

For the existence of a nontrivial solution of Eqs. (12)–(15), the determinant of the coefficients at A, B, R, and S is required to be equal to zero, i.e.,

$$\check{Z}_0\zeta^4 + \check{Z}_1\zeta^3 + \check{Z}_2\zeta^2 + \check{Z}_3\zeta + A_4 = 0\,, \tag{16}$$

where

$$\check{Z}_0 = \Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2,$$

$$\check{Z}_1 = -\left[\Omega^*(\Upsilon_1 + \Upsilon_3 + \varepsilon^t \sin^2\theta + (\bar{\beta}^t)^2\varepsilon^t \cos^2\theta) + (D_5 + \bar{D}_6 + \bar{\bar{D}}_6)\left(\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right)\right],$$

$$\check{Z}_2 = (\Upsilon_1\Upsilon_3 - \Upsilon_2^2) + \Omega^*(\Upsilon_1 + \Upsilon_3)(D_5 + \bar{D}_6 + \bar{\bar{D}}_6) + D_5\bar{D}_6\left(\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right)$$

$$+ \varepsilon^t \sin^2\theta(\Upsilon_3 + \Omega^*\bar{D}_6) - 2\bar{\beta}^t\varepsilon^t\Upsilon_2 \sin\theta\cos\theta + 2\varepsilon^*\Omega^*\bar{D}_6\,\frac{\beta_1^c}{b}\,(\sin^2\theta + \bar{\beta}^t\bar{\beta}^c \cos^2\theta)$$

$$+ \varepsilon^t(\bar{\beta}^t)^2(\Upsilon_1 + \Omega^*\bar{D}_6)\cos^2\theta - \Omega^*\bar{D}_6\,\frac{(\beta_1^c)^2}{b}\,(\sin^2\theta + (\bar{\beta}^c)^2\cos^2\theta)\,,$$

$$\check{Z}_3 = (\Upsilon_2^2 - \Upsilon_1\Upsilon_3)(D_5 + \bar{D}_6 + \bar{\bar{D}}_6) - \Omega^* D_5\bar{D}_6(\Upsilon_1 + \Upsilon_3) - \bar{\beta}^t\bar{D}_6\Upsilon_1 \cos^2\theta\left(\bar{\beta}^t\varepsilon^t + 2\bar{\beta}^c\varepsilon^*\,\frac{\beta_1^c}{b}\right)$$

$$+ (\bar{\beta}^c)^2\,\frac{(\beta_1^c)^2}{b}\,\bar{D}_6 \cos^2\theta(\Upsilon_1 + \Omega^* D_5) - \Upsilon_3\bar{D}_6 \sin^2\theta\left(\varepsilon^t + 2\varepsilon^*\,\frac{\beta_1^c}{b}\right) + \frac{(\beta_1^c)^2}{b}\,\bar{D}_6 \sin^2\theta(\Upsilon_3 + \Omega^* D_5)$$

$$+ 2\bar{\beta}^t\bar{D}_6\check{D}_2\left(\varepsilon^t + \varepsilon^*\,\frac{\beta_1^c}{b}\right)\sin\theta\cos\theta + 2\bar{\beta}^c\,\frac{\beta_1^c}{b}\,\bar{D}_6\Upsilon_2(\varepsilon^* - \beta_1^c)\sin\theta\cos\theta +$$

$$+ \bar{\beta}^t \varepsilon^t \bar{D}_6 \frac{(\beta_1^c)^2}{b} (\bar{\beta}^t - \bar{\beta}^c) \sin^2\theta \cos^2\theta ,$$

$$\check{Z}_4 = (\Upsilon_1 \Upsilon_3 - \Upsilon_2^2) D_5 \bar{D}_6 - D_5 \bar{D}_6 \frac{(\beta_1^c)^2}{b} ((\bar{\beta}^c)^2 \Upsilon_1 \cos^2\theta + \Upsilon_3 \sin^2\theta - 2\bar{\beta}^c \Upsilon_2 \sin\theta\cos\theta) ,$$

$$\Upsilon_1 = B_{11}\sin^2\theta + B_{55}\cos^2\theta + \mu_e H_0^2 \sin^2\theta, \quad \Upsilon_2 = (B_{13} + B_{55} + \mu_e H_0^2)\sin\theta\cos\theta ,$$

$$\Upsilon_3 = B_{55}\sin^2\theta + B_{33}\cos^2\theta + \mu_e H_0^2\cos^2\theta , \quad D_4 = K_{11}\sin^2\theta + K_{33}\cos^2\theta , \quad D_5 = \frac{D_4}{\tau_0^{t*} C_E} ,$$

$$D_6 = \alpha_{11}^* \sin^2\theta + \alpha_{33}^*\cos^2\theta , \quad \tau_0^{t*} = \tau_0^t + \frac{i}{\omega} , \quad \tau_0^{c*} = \tau_0^c + \frac{i}{\omega} , \quad \bar{D}_6 = \frac{bD_6}{\tau^*} , \quad \bar{\bar{D}}_6 = \frac{a^2 T_0 D_6}{\rho C_E \tau^*} ,$$

$$\varepsilon^t = \frac{(\beta_1^t)^2 T_0}{\rho C_E} , \quad \varepsilon^* = \frac{\beta_1^t a T_0}{\rho C_E} , \quad \Omega^* = 1 + \left(\frac{\Omega}{w}\right)^2 , \quad \bar{\beta}^t = \frac{\beta_3^t}{\beta_1^t} , \quad \bar{\beta}^c = \frac{\beta_3^c}{\beta_1^c} , \quad \bar{\beta}^{t2} = (\bar{\beta}^t)^2 ,$$

$$\frac{\tau_0^{c*}}{\rho} = \tau^* , \quad \omega = kv , \quad \zeta = \rho v^2 .$$

The four roots $\zeta_s = \rho v_s^2$ (s = 1–4) of Eq. (16) correspond to the complex phase speeds v_s of quasi-plane waves, namely, of the quasi-longitudinal displacement (qLD), quasi-thermal (qT), quasi-mass diffusion (qMD), and quasi-transverse displacement (qTD) waves. We present v_s as $v_s^{-1} = (V_s^*)^{-1} + i\omega^{-1} Q_s$, where k can be written as $k = \frac{\omega}{V^*} + iQ$ and V^* and Q are real. The real part Re (v) ≥ 0, then the real parts of v_1, v_2, v_3, and v_4 obtained from the four mentioned roots represent the speeds of the wave propagation for the qLD, qT, qMD, and qTD waves. The relation Im (v) = 0 refers to undamped time harmonic waves and Im (v) < 0, to damped waves.

Particular Cases. *Magnetothermoelastic waves in a rotating orthotopic medium.* Neglecting the diffusion parameters, we have $\beta_1^c = \beta_3^c \to 0, \bar{\beta}^c = 1$, $a = 0$, $b = 0$, $\alpha_{11}^* = 0, \alpha_{33}^* = 0$, $D_6 = 0, \bar{D}_6 = 0$, $\bar{\bar{D}}_6 = 0$, and Eq. (16) reduces to

$$R_0 \zeta^{*3} + R_1 \zeta^{*2} + R_2 \zeta^* + R_3 = 0 , \tag{17}$$

where

$$R_0 = \Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2 , \quad R_1 = -\left[\Omega^*(\Upsilon_1 + \Upsilon_3 + \varepsilon^t \sin^2\theta + (\bar{\beta}^t)^2 \varepsilon^t \cos^2\theta) + D_5\left(\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right)\right],$$

$$R_2 = (\Upsilon_1\Upsilon_3 - \Upsilon_2^2) + \Omega^*(\Upsilon_1 + \Upsilon_3)D_5 + \varepsilon^t \Upsilon_3 \sin^2\theta - 2\bar{\beta}^t \varepsilon^t \Upsilon_2 \sin\theta\cos\theta + \varepsilon^t (\bar{\beta}^t)^2 \Upsilon_1 \cos^2\theta ,$$

$$R_3 = (\Upsilon_2^2 - \Upsilon_1\Upsilon_3) D_5 , \quad \Upsilon_1 = B_{11}\sin^2\theta + B_{55}\cos^2\theta + \mu_e H_0^2 \sin^2\theta ,$$

$$\Upsilon_2 = (B_{13} + B_{55} + \mu_e H_0^2)\sin\theta\cos\theta ,$$

$$\Upsilon_3 = B_{55}\sin^2\theta + B_{33}\cos^2\theta + \mu_e H_0^2 \cos^2\theta , \quad D_4 = K_{11}\sin^2\theta + K_{33}\cos^2\theta , \quad D_5 = \frac{D_4}{\tau_0^{t*} C_E} ,$$

$$\tau_0^{t*} = \tau_0^t + \frac{i}{\omega} , \quad \tau_0^{c*} = \tau_0^c + \frac{i}{\omega} , \quad \varepsilon^t = \frac{(\beta_1^t)^2 T_0}{\rho C_E} , \quad \Omega^* = 1 + \left(\frac{\Omega}{w}\right)^2 , \quad \bar{\beta}^t = \frac{\beta_3^t}{\beta_1^t} .$$

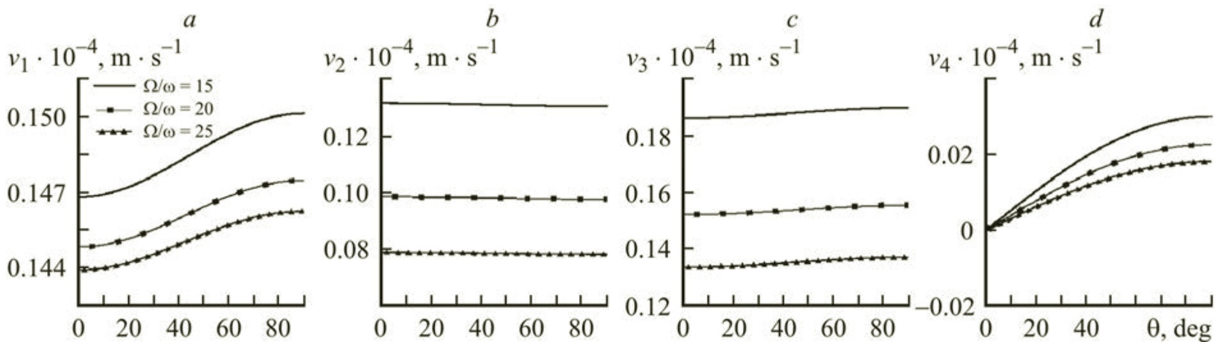

Fig. 1. Speeds of the qLD (a), qT (b), qMD (c), and qTD (d) waves against the angle of incidence at H_0 = 5 A/m and different values of Ω/ω.

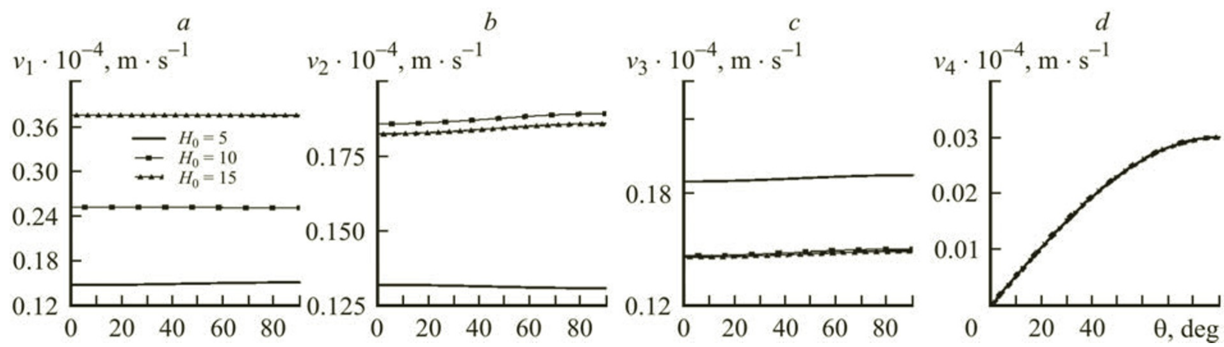

Fig. 2. Speeds of the qLD (a), qT (b), qMD (c), and qTD (d) waves against the angle of incidence at Ω/ω = 15 and different values of H_0.

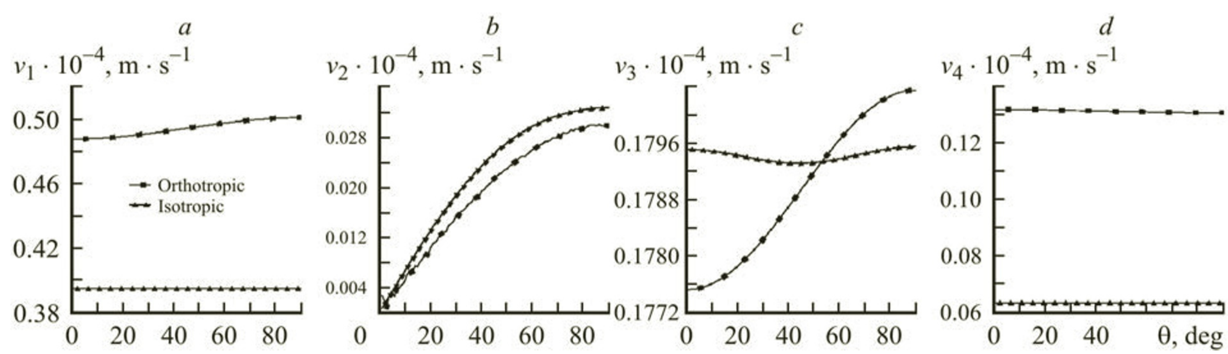

Fig. 3. Speeds of the qLD (a), qT (b), qMD (c), and qTD (d) waves against the angle of incidence at H_0 = 5 A/m and Ω/ω = 15 for orthotropic and isotropic cases.

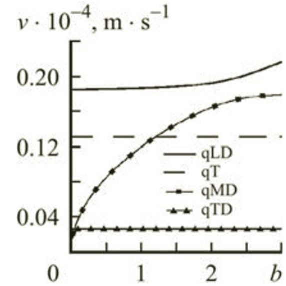

Fig. 4. Speeds of the qLD, qT, qMD, and qTD waves against the diffusion parameter at H_0 = 5 A/m, Ω/ω = 15, and θ = 60°.

The three roots $\zeta_p^* = \rho v_p^{*2}$ $(p = 1, 2, 3)$ of Eq. (17) correspond to the complex phase speeds v_p^* of the qLD, qT, and qTD waves, respectively, and the real parts of these roots represent the speeds of the wave propagation.

Thermoelastic waves in a rotating isotropic medium. Neglecting the orthotropic and magnetic effects, we have

$$B_{11} = \lambda + 2\mu\,, \quad B_{13} = \lambda\,, \quad B_{55} = \mu\,, \quad \Omega^* = 1\,, \quad \Omega = 0\,, \quad \alpha_{11}^* = \alpha_{33}^* = D^{**}\,,$$

$$\beta_1^t = \beta_3^t = \beta^t\,, \quad \bar{\beta}^t = 1\,, \quad B_{33} = \lambda + 2\mu\,, \quad H_0 = 0\,, \quad \beta_1^c = \beta_3^c = \beta^c\,, \quad \bar{\beta}^c = 1\,,$$

$$H_0 = 0\,, \quad \beta_1^c = \beta_3^c = \beta^c\,, \quad \bar{\beta}^c = 1\,, \quad \varepsilon^t = \frac{(\beta^t)^2 T_0}{\rho C_E}\,, \quad \varepsilon^* = \frac{\beta^t a T_0}{\rho C_E}\,, \quad K_{11} = K_{33} = K\,,$$

and Eq. (16) reduces to

$$N_0 l^4 + N_1 l^3 + N_2 l^2 + N_3 l + N_4 = 0\,, \tag{18}$$

where

$$N_0 = 1\,, \quad N_1 = -(D_1^{**} + D_3^{**} + \varepsilon^t + D_5^{**} + \bar{D}^* + \bar{\bar{D}}^*)\,,$$

$$N_2 = (D_1^{**} D_3^{**} - D_2^{**2}) + (D_1^{**} + D_3^{**})(D_5^{**} + \bar{D}^{**} + \bar{\bar{D}}^{**}) + D_5^{**}\bar{D}^{**} + \varepsilon^t \sin^2\theta(D_3^{**} + \bar{D}^{**})$$

$$- 2\varepsilon^t D_2^{**} \sin\theta\cos\theta + 2\varepsilon^* \bar{D}^{**} \frac{\beta^c}{b} + \varepsilon^t (D_1^{**} + \bar{D}^{**}) \cos^2\theta - \bar{D}^{**} \frac{(\beta^c)^2}{b}\,,$$

$$N_3 = (D_2^{**2} - D_1^{**} D_3^{**})(D_5^{**} + \bar{D}^{**} + \bar{\bar{D}}^{**}) - D_5^{**}\bar{D}^{**}(D_1^{**} + D_3^{**}) - \bar{D}^{**} D_1^{**} \cos^2\theta \left(\varepsilon^t + 2\varepsilon^* \frac{\beta^c}{b}\right)$$

$$+ \frac{(\beta^c)^2}{b} \bar{D}^{**} \cos^2\theta(D_1^{**} + D_5^{**}) - D_3^{**}\bar{D}^{**} \sin^2\theta \left(\varepsilon^t + 2\varepsilon^* \frac{\beta^c}{b}\right) + \frac{(\beta^c)^2}{b} \bar{D}^{**} \sin^2\theta(D_3^{**} + D_5^{**})$$

$$+ 2\bar{D}^{**} D_2^{**} \sin\theta\cos\theta \left(\varepsilon^t + \varepsilon^* \frac{\beta^c}{b}\right) + 2\frac{\beta^c}{b} \bar{D}^{**} D_2^{**} \sin\theta\cos\theta(\varepsilon^* - \beta^c)\,,$$

$$N_4 = (D_1^{**} D_3^{**} - D_2^{**2}) D_5^{**} \bar{D}^{**} - D_5^{**}\bar{D}^{**} \frac{(\beta^c)^2}{b} (D_1^{**} \cos^2\theta + D_3^{**} \sin^2\theta - 2D_2^{**} \sin\theta\cos\theta)\,,$$

$$D_1^{**} = (\lambda + 2\mu)\sin^2\theta + \mu\cos^2\theta\,, \quad D_2^{**} = (\lambda + \mu)\sin\theta\cos\theta\,,$$

$$D_3^{**} = \mu\sin^2\theta + (\lambda + 2\mu)\cos^2\theta\,, \quad D_4^{**} = K\,, \quad D_5^{**} = \frac{K}{\tau_0^{t*} C_E}\,, \quad D_6 = D^{**}\,, \quad \tau_0^{t*} = \tau_0^t + \frac{i}{\omega}\,,$$

$$\tau_0^{c*} = \tau_0^c + \frac{i}{\omega}\,, \quad \bar{D}^{**} = \frac{bD^{**}}{\tau^*}\,, \quad \bar{\bar{D}}^{**} = \frac{a^2 T_0 D^{**}}{\rho C_E \tau^*}\,, \quad \varepsilon^t = \frac{\beta^{t2} T_0}{\rho C_E}\,, \quad \varepsilon^* = \frac{\beta^t a T_0}{\rho C_E}\,, \quad \frac{\tau_0^{c*}}{\rho} = \tau^*\,.$$

The four roots $l_s = \rho v_s^{**2}$ $(s = 1\text{–}4)$ of Eq. (18) correspond to the complex phase speeds v_s^* of the P, T, MD, and SV plane waves, respectively, and the real parts v_1^*, v_2^*, and v_3^* of these roots represent the speeds of the wave propagation.

Numerical Results and Discussion. For numerical illustration, the following relevant elastic and thermal constants of cobalt at 27° are used [20]:

$$\rho = 7.4 \cdot 10^3 \ \text{kg} \cdot \text{m}^{-3} , \quad B_{11} = 3.071 \cdot 10^{11} \ \text{N} \cdot \text{m}^{-2} , \quad B_{13} = 1.027 \cdot 10^{11} \ \text{N} \cdot \text{m}^{-2} ,$$

$$B_{33} = 3.581 \cdot 10^{11} \ \text{N} \cdot \text{m}^{-2} , \quad B_{55} = 1.510 \cdot 10^{11} \ \text{N} \cdot \text{m}^{-2} , \quad B_{12} = 1.650 \cdot 10^{11} \ \text{N} \cdot \text{m}^{-2} ,$$

$$K_{11} = 0.69 \cdot 10^4 \ \text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1} , \quad K_{33} = 0.701 \cdot 10^4 \ \text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1} , \quad C_E = 0.3814 \cdot 10^3 \ \text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} ,$$

$$\beta_1^t = 7.04 \cdot 10^6 \ \text{N} \cdot \text{m}^{-2} \cdot \text{K}^{-1} , \quad \beta_3^t = 6.9 \cdot 10^6 \ \text{N} \cdot \text{m}^{-2} \cdot \text{K}^{-1} ,$$

$$\tau_0^t = 0.005 \ \text{s}, \quad \tau_0^c = 0.04 \ \text{s} .$$

The values of the diffusion parameters are as follows:

$$a = 0.24 \cdot 10^4 \ \text{kg}^{-1} \cdot \text{m}^2 \cdot \text{s}^{-2} , \quad b = 1.60 \cdot 10^5 \ \text{kg}^{-1} \cdot \text{m}^5 \cdot \text{s}^{-2} , \quad \alpha_{11}^* = 0.95 \cdot 10^{-8} \ \text{kg} \cdot \text{s} \cdot \text{m}^{-3} ,$$

$$\alpha_{33}^* = 0.90 \cdot 10^{-8} \ \text{kg} \cdot \text{s} \cdot \text{m}^{-3} , \quad \alpha_{1c} = 2.1 \cdot 10^{-4} \ \text{m}^3 \cdot \text{kg}^{-1} , \quad \alpha_{3c} = 2.5 \cdot 10^{-4} \ \text{m}^3 \cdot \text{kg}^{-1} .$$

With the help of a FORTRAN program we calculate the speeds v_1, v_2, v_3, and v_4 of the qLD, qT, qMD, and qTD waves from the solution of Eq. (16). These speeds are plotted against the angle of incidence for different values of the rotational parameter Ω/ω and the magnetic field parameter H_0 in the orthotropic and isotropic cases (see Figs. 1–3).

The speeds v_1, v_2, v_3, and v_4 of the studied waves vs. the angle of incidence are shown for H_0 = 5 A/m and different values of Ω/ω in Fig. 1. It is seen that the speeds of all the waves increase with the angle (except for the qT waves where they are almost constant) and with decrease in Ω/ω. For example, the velocity v_1 (see Fig. 1a) of the qLD waves at $\Omega/\omega = 15$ increases from $1.4389 \cdot 10^3$ m·s^{-1} at $\theta = 0^o$ to $1.464 \cdot 10^3$ m·s^{-1} at $\theta = 90^o$.

The speeds of the qLD, qT, qMD, and qTD waves are plotted vs. the angle of incidence for $\Omega/\omega = 15$ and different values of H_0 in Fig. 2. For the qLD waves (see Fig. 2a), v_1 increases with H_0 (from $1.4701 \cdot 10^3$ m·s^{-1} at H_0 = 5 A/m to $3.7529 \cdot 10^3$ m·s^{-1} at H_0 = 15 A/m for $\theta = 0^o$) and increases slowly with the angle. For the qTD waves, the speed increases with the angle and is practically independent of H_0.

The speeds of the qLD, qT, qMD, and qTD waves are plotted vs. the angle of incidence for the orthotropic and isotropic cases at $\Omega/\omega = 15$ and H_0 = 5 A/m in Fig. 3. It is seen (see Fig. 3a) that v_1 for the orthotropic case increases from $4.8788 \cdot 10^3$ m·s^{-1} at $\theta = 0^o$ to $5.0123 \cdot 10^3$ m·s^{-1} at $\theta = 90^o$. For the isotropic case, v_1 is independent of the angle and equal to $3.9490 \cdot 10^3$ m·s^{-1}.

The speeds of the qLD, qT, qMD, and qTD waves are plotted against the diffusion parameter b at Ω/ω = 15, H_0 = 5 A/m, and $\theta = 60^o$ in Fig. 4. It is seen that the speeds of the qLD and qMD waves increase, respectively, from $1.8531 \cdot 10^3$ to $2.1668 \cdot 10^3$ m·s^{-1} and from $0.2609 \cdot 10^3$ to $1.7917 \cdot 10^3$ m·s^{-1} as b increases from zero to 3. At the same time the waves of the qT and qTD waves remain constant: $1.3092 \cdot 10^3$ and $0.2610 \cdot 10^3$ m·s^{-1}, respectively.

Conclusions. The solutions of the equations for plane waves in the xz plane are indicative of the existence of four quasi-plane waves, namely quasi-longitudinal displacement (qLD), quasi-thermal (qT), quasi-mass diffusion (qMD), and quasi-transverse displacement (qTD) waves. The numerical values of the speeds of these waves are shown to depend significantly on the angle of propagation, anisotropy, and the rotation and magnetic field parameters. These values will be helpful in estimating the correct arrival times.

NOTATION

a, diffusion constant, K^{-1}·m^2·s^{-2}; **B**, magnetic induction, N·A^{-1}·m^{-1}; B_{ij}, elastic constants, N·m^{-2}; b, diffusion constant, kg^{-1}·m^5·s^{-2}; C, concentration; C_E, specific heat at constant strain, J·kg^{-1}·K^{-1}; **E**, electric field strength, V·m^{-1}; **H**, magnetic field strength, A·m^{-1}; **h**, perturbation of the magnetic field strength, A·m^{-1}; **J**, current, A·m^{-2}; K_{11} and K_{33}, thermal conductivities, W·m^{-1}·K^{-1}; **n**, unit vector; T, temperature, K; t, time, s; **u**, displacement vector, m; u and w, components of the displacement vector, m; v, wave speed, m/s; W, strain energy function, N·m^{-2}; x, y, z, coordinates, m; α_{1t} and α_{3t}, coefficients of linear thermal expansion, K^{-1}; α_{1c} and α_{3c}, coefficients of linear diffusion expansion, m^3·kg^{-1}; α_{11}^*, α_{33}^*, diffusion constants, kg·s·m^{-3}; β_1^t, β_3^t, thermal coefficients, N·m^{-2}·K^{-1}; β_1^c, β_3^c, diffusion coefficients, m^2·s^{-2}; θ, angle of propagation measured from the normal to the half-space, deg; λ and μ, Lame′s constants, N·m^{-2};

μ_e, magnetic permeability, H·m^{-1}; ρ, density, kg·m^{-3}; σ, electric conductivity, S·m^{-1}; τ_0^t, thermal relaxation time, s; τ_0^c, diffusion relaxation time, s; **Ω**, angular velocity, Hz; ω, circular frequency, Hz.

REFERENCES

1. M. A. Biot, Thermoelasticity and irreversible thermodynamics, *J. Appl. Phys*., **27**, 240–253 (1956).
2. H. W. Lord and Y. Shulman, A generalized dynamical theory of thermoelasticity, *J. Mech. Phys. Solids*, **15**, 299–309 (1967).
3. A. E. Green and K. A. Lindsay, Thermoelasticity, *J. Elast*., **2**, 1–7 (1972).
4. R. B. Hetnarski and J. Ignaczak, Generalized thermoelasticity, *J. Therm. Stresses*, **22**, 451–476 (1999).
5. J. Ignaczak and M. Ostoja-Starzewski, *Thermoelasticity with Finite Wave Speeds*, Oxford University Press, Oxford (2009).
6. R. S. Dhaliwal and H. H. Sherief, Generalized thermoelasticity for anisotropic media, *Quart. Appl. Math.*, **38**, 1–8 (1980).
7. H. Jeffreys, The thermodynamics of an elastic solid, *Proc. Camb. Phil. Soc*., **26**, 101–106 (1930).
8. R. Gutenberg, Energy relation of reflected and refracted seismic waves, *Bull. Seismol. Soc. Am*., **34**, 85–102 (1944).
9. A. N. Sinha and S. B. Sinha, Reflection of thermoelastic waves at a solid half-space with thermal relaxation, *J. Phys. Earth*, **22**, 237–244 (1974).
10. M. Schoenberg and D. Censor, Elastic waves in rotating media, *Quart. Appl. Math*., **31**,115–125 (1973).
11. B. Singh and A. K. Yadav, Plane waves in a transversely isotropic rotating magnetothermoelastic medium, *J. Eng. Phys. Thermophys*., **85**, 1226–1232 (2012).
12. B. Singh and A. K. Yadav, Reflection of plane waves in a rotating transversly isotropic magneto-thermoelastic solid half-space, *J. Theor. Appl. Mech*., Sofia, **42**, 33–60 (2012).
13. D. S. Chandrasekharaiah and K. R. Srikantiah, Thermoelastic plane waves in a rotating solid, *Acta Mech*., **50**, 211–219 (1984).
14. L. Knopoff, The interaction between elastic wave motion and a magnetic field in electrical conductors, *J. Geophys. Res*., **60**, 441–456 (1955).
15. M. I. A. Othman and N. T. Mansour, 2-D problem of magneto-thermoelastic medium under the effect of different fields with two-temperature and 3PHL model, *Am. J. Nano Res. Appl*., **4**, 33–42 (2016).
16. D. Chand, J. N. Sharma, and S. P. Sud, Transient generalized magneto-thermoelastic waves in a rotating half-space, *Int. J. Eng. Sci*., **28**, 547–556 (1990).
17. B. Singh and A. K. Yadav, Plane waves in a rotating monoclinic magnetothermoelastic medium, *J. Eng. Phys. Thermophys*., **89**, 428–440 (2016).
18. S. M. Abo-Dahab and S. Biswas, Effect of rotation on Rayleigh waves in magneto-thermoelastic transversely isotropic medium with thermal relaxation times, *J. Electromagn. Waves Appl.*, **31**, 1485–1507 (2017).
19. S. Shaw and M. I. A. Othman, Characteristics of Rayleigh wave propagation in orthotropic magneto-thermoelastic half-space: An eigenfunction expansion method, *Appl. Math. Model*., **67**, 605–620 (2019).
20. S. Biswas and B. Mukhopadhyay, Eigenfunction expansion method to analyze thermal shock behaviour in magneto-thermoelastic orthotropic medium under three theories, *J. Therm. Stresses*, **41**, 366–382 (2018).
21. H. H. Sherief, H. Saleh, and F. Hamza, The theory of generalized thermoelastic diffusion, *Int. J. Eng. Sci*., **42**, 591–608 (2004).
22. W. Nowacki, Dynamical problems of thermodiffusion in solids. Part I, *Bull. Pol. Acad. Sci., Ser. Sci. Technol.*, **22**, 55–64 (1974).
23. W. Dudziak and S. J. Kowalski, Theory of thermodiffusion for solids, *Int. J. Heat Mass Transf.*, **32**, 2005–2013 (1989).
24. B. Singh, Reflection of P and SV waves from free surface of an elastic solid with generalized thermodiffusion, *J. Earth Syst. Sci*., **114**, 159–168 (2005).
25. M. Aouadi, Theory of generalized micropolar thermoelastic diffusion under Lord–Shulman model, *J. Therm. Stresses*, **32**, 923–942 (2009).
26. K. Lotfy, S. M. Abo-Dahab, and R. Tantawy, Thermomechanical response model on a reflection photothermal diffusion waves (RPTD) for semiconductor medium, *Silicon*, **12**, 199–209 (2020).

27. N. Mabrouk, Moh′d Yasein, K. Lotfy, and A. A. El-Bary, Effect of magneto-rotator-diffusive waves on the photothermal excitation medium of a dual-phase-lag model and variable thermal conductivity, *J. Electromagn. Waves Appl*., **34**, 330–348 (2020).
28. Anand Kumar Yadav, Reflection of plane waves from the free surface of a rotating orthotropic magneto-thermoelastic solid half-space with diffusion, *J. Therm. Stresses*, **44**, 86–106 (2021).