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INFLUENCE OF THE HEAT OF PHASE TRANSITION ON THE ONSET OF FILTRATION CONVECTION IN MIXTURES OF LIQUIDS WITH LIMITED MUTUAL SOLUBILITY

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Consideration is given to the problem on the convective stability of mechanical equilibrium of a mixture of liquids with limited mutual solubility in a porous medium. It is assumed that in the initial state of mechanical equilibrium, the mixture fills the horizontal porous layer and is separated into two phases: the upper lighter phase and the lower heavy one. At the lower boundary of the layer, the higher temperature is maintained than that at the upper boundary. A study is made of the influence of the heat of phase transition and the ratio of phase densities on conditions for the onset of convection and on the structure of occurring flows.

Keywords: *heat of transition, filtration convection, mixtures of liquids with limited solubility.*

Introduction. A two-phase flow through permeable rock is an important mechanism of heat transfer and heat exchange in high-temperature geothermal systems. For example, we are well aware of the manifestations of thermomechanical activity in vapor-dominated Larderello systems in Italy, Geysers in California [1], etc. Despite the importance and practical significance of the problem, relatively little is known about the nature of circulation of steam and water over geometrical regions where the vapor dominates.

One characteristic of multiphase flows is that then often show instability (stability) which is absent from a single-phase flow. As investigations show, multiphase flows in porous media even with neglect of inertial forces may lose stability and pass to another stationary or even self-oscillating regime [2]. There can also be reverse situations where phase transition stabilizes equilibrium or the stationary regime [3].

On the basis of field measurements, it was assumed in [1] that two-phase convection under vapor-dominated systems consists of the rising steam and the descending water (condensate and groundwater) over the deep reservoir of circulating brine. Sondergels et al. [4] studied two-phase convection in a laboratory sander; they observed the type of countercurrent convection of the ascending steam and the descending water, that was described in [1]. In [5], a theoretical model has been developed for a one-dimensional ascending flow of boiling water in a porous medium under the assumption that steam and water are always in thermodynamic equilibrium. Schubert et al. [6], to single out and study the basic physical processes associated with convection in a porous medium with phase transition, have developed a homogeneous (one-velocity) model of steam-water convection which allowed describing this phenomenon analytically. In [7, 8], the process of filtration cooling of a heat-releasing granular bed in the presence of phase transition of the first kind has been modeled under the conditions of free convection of the heat-transfer agent. In [9], consideration has been given to the problem of mathematical modeling of flows of multicomponent mixtures with phase transitions. The free energy of the mixture was assigned in the form of a functional containing the gradients squared of the components' densities.

If we consider the configuration where the layer of a liquid is above the layer that does not mix with the indicated liquid, a gas, or a lighter liquid, such a system is always unstable [10]. However, if we speak of a liquid and its vapor, the situation radically changes. Measurements of pressure in deep bore holes together with the measured temperature distributions through the depth have made it possible to uniquely establish that in geothermal systems, the layer of water may exist over the layer of its steam [1, 11]. This is due to the fact that when the equilibrium is upset there is a phase transition playing a stabilizing role in this case. Consequently, high-temperature geothermal reservoirs exist

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where thermodynamic conditions of stabilization of the position of water over the steam are implemented. Mathematical substantiation of such a possibility was first presented in [4] on the basis of solution of the problem on the stability of the water–steam boundary when the water layer is over the steam layer. Investigations have shown the presence of the critical value of the permeability coefficient separating the regions of stable and unstable states of a geothermal system. Thus, if the permeability coefficient does not exceed the critical value, the phase-transition surface may be stable despite the fact that water is over the steam. In [12–14], a more complex example of existence of the water layer over the steam layer in a geothermal system has been proposed, which allows the motion of phases and phase transition in an undisturbed state. Linear stability has been studied and the existence of stable stationary regimes of flow has been shown which are implemented at relatively high permeabilities. A solution to the problem on linear stability has been presented in [10], which takes account of the convective transfer of energy and is applicable for any values of permeability. It has been shown that the water layer may be present over the steam layer in geothermal reservoirs whose permeability is much higher than the critical values found earlier.

Almost all the above-noted works concern the system "water–steam." As noted, such works are few in number, there are still fewer works on convection dealing with the phase transition between two phases of mixtures because of the finite mutual solubility. A distinctive feature of such a case is the possibility of both the exothermal (analogously to the steam-water mixture) and endothermal phase transitions where the slope of the equilibrium curve (dp/dT) is negative. Furthermore, for mixtures of liquids in phase transition, the density and viscosity of the phases differ little compared to the system "water–steam."

We note study [15] which is similar, in essence, although this work does not concern a a porous medium. In it, a numerical study has been made of the influence of endogenic phase transition at the boundary between the earth's upper and lower mantles (at a depth of approximately 660 km) on the structure of nonlinear convection of the entire mantle. Upon dipping into the mantle, at the indicated boundary, we have the endothermal phase transition of the lighter phase of the mantle substance to a heavier modification The indicated phase transition retards the convection, which way result in its full or partial stratification. However, in this work, the authors neglect the thermal effect, restricting themselves to taking account of the influence of the slope of the equilibrium curve (dp/dT). In the cited paper, this corresponds to the heats of phase transition M that are small in modulus, but to great slopes of the equilibrium curves Γ, so that $M\,\Gamma$ is finite.

In the present paper, we have investigated the convective stability of a mixture of liquids with limited mutual solubility in a porous medium where the light phase is over the heavy one. Consideration has been given to the influence, on the onset of convection of both the exogenic phase transition, i.e., when the slope of the curve of phase equilibrium is positive, and the endogenic phase transition when the indicated slope is negative. The obtained solutions are also applicable to the system "water–steam" through assigning relevant values of the physical and thermophysical parameters, however, this case is not discussed here.

Formulation of the Problem. Let there be a mixture of two liquids. In the range of temperatures and pressures in question, the indicated liquids mix in limited amounts so that the mixture stratifies into two phases with different relations of the concentrations of the components. The heavy phase forms the lower layer, and the ligher phase, the upper one. On passage of the mixture's particle from the lower phase to the upper one or conversely, we have phase transition to release or absorb heat. The concentrations of the components in the layers will be assumed constant, so that the phases may be considered as homogeneous liquids with effective thermophysical properties and different densities. Use is made of the Oberbeck–Boussinesq–Darcy approximation. It is required that conditions for the onset of filtration convection in such a system be investigated at an assigned temperature gradient directed downward.

In what follows, the quantities referring to the upper light phase will conventionally be denoted by the subscript v, and to the heavier phase, by the subscript w. Under the assumptions made in the problem's formulation, the system of equations in the region of the upper phase will be written as

$$\mathrm{div}\;\mathbf{v}_{\mathrm{v}} = 0\,, \quad C_{\mathrm{mv}}\frac{\partial T}{\partial t} + \rho_{\mathrm{v}0}C_{p\mathrm{v}}\mathbf{v}_{\mathrm{v}}\nabla T = \mathrm{div}\,(\lambda_{\mathrm{mv}}\;\mathrm{grad}\;T)\,,$$
$$\mathbf{v}_{\mathrm{v}} = -\frac{k}{\mu_{\mathrm{v}}}(\mathrm{grad}\;P - \rho_{\mathrm{v}}\mathbf{g})\,, \quad \rho_{\mathrm{v}} = \rho_{\mathrm{v}0}(1 - \beta_{\mathrm{v}}(T - T_0))\,. \tag{1}$$

Analogously, in the lower layer, we have the follows system:

$$\operatorname{div}\,\mathbf{v}_w = 0\,, \quad C_{mw}\frac{\partial T}{\partial t} + \rho_{w0}C_{pw}\mathbf{v}_w\nabla T = \operatorname{div}\,(\lambda_{mw}\ \operatorname{grad}\,T)\,,$$

$$\mathbf{v}_w = -\frac{k}{\mu_w}(\operatorname{grad}\,P - \rho_w\mathbf{g})\,, \quad \rho_w = \rho_{w0}(1-\beta_w(T-T_0))\,. \tag{2}$$

Boundary conditions will be written in the following manner:

the phase boundary

$$z=\xi:\quad T_w = T_v = T_*\,,\quad P_w = P_v = P_*,\ P_* = F(T_*)\,,\quad \rho_w(V - v_{wn}) = \rho_v(V - v_{vn})\,,$$

$$\rho_w h_w(V - v_{wn}) + \lambda_{mw}\frac{\partial T_w}{\partial z} = \rho_v h_v(V - v_{vn}) + \lambda_{mv}\frac{\partial T_v}{\partial z}\,; \tag{3}$$

and the lower and upper boundaries

$$z=0:\quad T = T_1\,,\quad v_{wn} = 0\,;\quad z = H:\quad T = T_2\,,\quad v_{vn} = 0\,.$$

Mechanical Equilibrium. We find fields that describe mechanical equilibrium. For this purpose, the velocity fields in (1)–(3) are set equal to zero and all the fields are assumed to be dependent on just the vertical coordinate z. The obtained problem is easily integrated; as a result we obtain the following solution describing mechanical equilibrium:

in the region of the upper phase

$$T_{vs} = T_{*s} - (T_{*s} - T_2)\frac{z-\xi_s}{H-\xi_s}\,,\quad P_{vs} = P_{*s} - \rho_{v0}g(z-\xi_s)\,,\quad \rho_{v0} = \text{const}\,; \tag{4}$$

in the region of the lower phase

$$T_{ws} = T_1 - (T_1 - T_{*s})\frac{z}{\xi_s}\,,\quad P_{ws} = P_{*s} - \rho_{w0}g(z-\xi_s)\,,\quad \rho_{w0} = \text{const}\,. \tag{5}$$

Here, the conditions

$$z=\xi_s:\quad T_{ws} = T_{vs} = T_{*s}\,,\quad P_{ws} = P_{vs} = P_{*s}\,,\quad P_{*s} = F(T_{*s})\,,$$

$$\lambda_{mw}(T_1 - T_{*s})\frac{1}{\xi_s} = \lambda_{mv}(T_{*s} - T_2)\frac{1}{H-\xi_s}\,. \tag{6}$$

are observed. Hence we obtain

$$T_{*s} = \frac{T_1 + \alpha T_2}{\alpha+1}\,,\quad T_1 - T_{*s} = \frac{\alpha(T_1 - T_2)}{\alpha+1}\,,\quad T_{*s} - T_2 = \frac{T_1 - T_2}{\alpha+1}\,,$$

where

$$\alpha \equiv \gamma\frac{\xi_s}{1-\xi_s}\,,\quad \gamma = \frac{\lambda_{mv}}{\lambda_{mw}}\,.$$

Stability of Mechanical Equilibrium. We linearize the sought fields near the position of mechanical equilibrium

$$T_v = T_{vs} + T'_v\,,\quad P = P_s + P'_v\,,\quad \mathrm{v}_v = \mathrm{v}'_v\,,$$

$$T_w = T_{ws} + T'_w\,,\quad P = P_s + P'_w\,,\quad \mathrm{v}_w = \mathrm{v}'_v\,,\quad \xi = \xi_s + \xi'\,. \tag{7}$$

Here the primed quantities are small quantities. We substitute (4)–(7) into (1)–(3). Leaving just the linear terms and omitting the primes, we have:

in the region of the upper phase

$$\operatorname{div}\,\mathbf{v}_\mathrm{v} = 0\,, \quad C_\mathrm{mv}\frac{\partial T}{\partial t} - \rho_\mathrm{v} C_{p\mathrm{v}} v_{\mathrm{v}z} A_\mathrm{v} = \operatorname{div}\,(\lambda_\mathrm{mv}\ \operatorname{grad}\,T)\,,$$

$$\mathbf{v}_\mathrm{v} = -\frac{k}{\mu_\mathrm{v}}\,(\operatorname{grad}\,P + \rho_{\mathrm{v}0}\beta_\mathrm{v} T\mathbf{g})\,, \quad A_\mathrm{v} = \frac{T_{*\mathrm{s}} - T_2}{H - \xi_\mathrm{s}}\,; \tag{8}$$

and in the region of the lower phase

$$\operatorname{div}\,\mathbf{v}_\mathrm{w} = 0\,, \quad C_\mathrm{mw}\frac{\partial T}{\partial t} - \rho_\mathrm{w} C_{p\mathrm{w}} v_{\mathrm{w}z} A_\mathrm{w} = \operatorname{div}\,(\lambda_\mathrm{mw}\ \operatorname{grad}\,T)\,,$$

$$\mathbf{v}_\mathrm{w} = -\frac{k}{\mu_\mathrm{w}}\,(\operatorname{grad}\,P + \rho_{\mathrm{w}0}\beta_\mathrm{w} T\mathbf{g})\,, \quad A_\mathrm{w} = \frac{T_1 - T_{*\mathrm{s}}}{\xi_\mathrm{s}}\,. \tag{9}$$

Boundary conditions are taken to be as follows:

$$z = 0:\quad T_\mathrm{w} = 0\,,\quad v_\mathrm{w} = 0\,;\quad z = H:\quad T_\mathrm{v} = 0\,,\quad v_v = 0\,;\quad z = \xi_\mathrm{s}:\quad T_\mathrm{w} - A_\mathrm{ws}\xi = T_\mathrm{v} - A_\mathrm{vs}\xi\,,$$

$$P_\mathrm{w} - \rho_{\mathrm{w}0} g\xi = P_\mathrm{v} - \rho_{\mathrm{v}0} g\xi\,,\quad P_\mathrm{w} - \rho_{\mathrm{w}0} g\xi = \frac{dP}{dT}\,(T_\mathrm{w} - A_\mathrm{ws}\xi)\,, \tag{10}$$

$$\rho_{\mathrm{w}0}(\dot{\xi} - \mathrm{v}_{\mathrm{w}z}) = \rho_{\mathrm{v}0}(\dot{\xi} - \mathrm{v}_{\mathrm{v}z})\,,\quad \lambda_\mathrm{mw}\frac{\partial T_\mathrm{w}}{\partial z} = \rho_{\mathrm{w}0} q(\dot{\xi} - v_{\mathrm{w}z}) + \lambda_\mathrm{mv}\frac{\partial T_\mathrm{v}}{\partial z}\,.$$

Here the point denotes the time derivative.

Next, introducing the stream function ψ

$$\mathrm{v}_z = \frac{\partial\psi}{\partial x}\,,\quad \mathrm{v}_x = -\frac{\partial\psi}{\partial z} \tag{11}$$

and the scale of the quantities H (length), $\lambda_\mathrm{mw}/(\rho_\mathrm{w} C_{p\mathrm{w}} H)$ (velocity), $\rho_\mathrm{w} C_{p\mathrm{w}} H^2/\lambda_\mathrm{mw}$ (time), $\mu_\mathrm{w}\lambda_\mathrm{mw}/k\rho_\mathrm{w} C_{p\mathrm{w}}$ (pressure), $T_1 - T_2$ (temperature), and also the Rayleigh number $\mathrm{Ra} = \dfrac{k\rho_\mathrm{w}^2 C_{p\mathrm{w}} g\beta_\mathrm{w}(T_1 - T_2)H}{\mu_\mathrm{w}\lambda_\mathrm{mw}}$, we write system (8)–(11) in dimensionless form. We have:

in the region of the upper phase

$$b_\mathrm{v}\frac{\partial T_\mathrm{v}}{\partial t} - \gamma_\mathrm{v}\frac{\partial\psi_\mathrm{v}}{\partial x} = c_\mathrm{v}\Delta T_\mathrm{v}\,,\quad \Delta\psi_\mathrm{v} = d_\mathrm{v}\mathrm{Ra}\frac{\partial T_\mathrm{v}}{\partial x}\,, \tag{12}$$

where

$$b_\mathrm{v} = \frac{C_\mathrm{mv}(T_1 - T_2)}{\rho_\mathrm{v} C_{p\mathrm{v}} A_\mathrm{v} H} = \frac{C_\mathrm{mv}}{\rho_\mathrm{v} C_{p\mathrm{v}}\gamma_\mathrm{v}}\,,\quad c_\mathrm{v} = \frac{\lambda_\mathrm{mv}\rho_\mathrm{w} C_{p\mathrm{w}} A_\mathrm{w}}{\lambda_\mathrm{mw}\rho_\mathrm{v} C_{p\mathrm{v}} A_\mathrm{v}} = \frac{\lambda_\mathrm{mv}\rho_\mathrm{w} C_{p\mathrm{w}}\gamma_\mathrm{w}}{\lambda_\mathrm{mw}\rho_\mathrm{v} C_{p\mathrm{v}}\gamma_\mathrm{v}}\,,\quad \gamma = \frac{\lambda_\mathrm{mv}}{\lambda_\mathrm{mw}}\,,$$

$$d_\mathrm{v} = \frac{\rho_\mathrm{v}\beta_\mathrm{v}}{\rho_\mathrm{w}\beta_\mathrm{w}}\,,\quad \gamma_\mathrm{v} = \frac{A_\mathrm{v} H}{T_1 - T_2} = \frac{1}{1 - (1-\gamma)\xi_\mathrm{s}}\,;$$

and in the region of the lower phase

$$b_\mathrm{w}\frac{\partial T_\mathrm{w}}{\partial t} - \gamma_\mathrm{w}\frac{\partial\psi_\mathrm{w}}{\partial x} = \Delta T_\mathrm{w}\,,\quad \Delta\psi_\mathrm{w} = \mathrm{Ra}\frac{\partial T_\mathrm{w}}{\partial x}\,, \tag{13}$$

where

$$b_\mathrm{w} = \frac{C_\mathrm{mw}(T_1 - T_2)}{\rho_\mathrm{w} C_{p\mathrm{w}} A_\mathrm{w} H}\,,\quad \gamma_\mathrm{w} = \frac{A_\mathrm{w} H}{T_1 - T_2} = \frac{\gamma}{1 - (1-\gamma)\xi_\mathrm{s}}\,,\quad \gamma = \frac{\lambda_\mathrm{mv}}{\lambda_\mathrm{mw}}\,.$$

Boundary conditions are as follows:

$$z = 0:\quad T_w = 0\,,\quad \psi_w = 0\,;\quad z = 1:\quad T_v = 0\,,\quad \psi_v = 0\,;$$

$$z = \xi_s:\quad T_v - \gamma_v\xi = T_w - \gamma_w\xi\,,\quad (1-\rho_v/\rho_w)\frac{\partial\xi}{\partial x} = \frac{\delta_w}{\mathrm{Ra}}\left(\frac{\partial\psi_w}{\partial z} - \frac{\mu_v}{\mu_w}\frac{\partial\psi_v}{\partial z}\right),$$

$$\frac{\delta_w}{\mathrm{Ra}}\frac{\partial\psi_w}{\partial z} - \frac{\partial\xi}{\partial x} = \Gamma M\left(\frac{\partial T_w}{\partial x} - \gamma_w\frac{\partial\xi}{\partial x}\right),\quad \rho_w\left(\dot\xi - \frac{\partial\psi_w}{\partial x}\right) = \rho_v\left(\dot\xi - \frac{\partial\psi_v}{\partial x}\right), \tag{14}$$

$$\frac{\partial T_w}{\partial z} = M\left(\dot\xi - \frac{\partial\psi_w}{\partial x}\right) + \gamma\frac{\partial T_v}{\partial z}\,.$$

Here we have introduced the dimensionless quantities

$$\delta_w = \beta_w\Delta T\,,\quad M = \frac{q}{C_{pw}\Delta T}\,,\quad \Gamma = \frac{\Delta T}{M\rho_w gH}\frac{dP}{dT} = \frac{C_{pw}\Delta T^2\rho_v}{gHT\Delta\rho}\,.$$

We seek the solution in the upper layer in the form

$$\psi_v = ce^{\lambda t + ik_x x}\sinh\gamma(1-z)\,,\quad T_v = de^{\lambda t + ik_x x}\sinh\lambda(1-z)\,.$$

We obtain the following system of equations for c and d:

$$ik_x\gamma_v c - [b_v\lambda - c_v(\gamma^2 - k_x^2)]d = 0\,,\quad (\gamma^2 - k_x^2)c - ik_x d_v\,\mathrm{Ra}\,d = 0\,,$$

whence

$$c_v\omega^2 - b_v\lambda\omega - k_x^2\gamma_v d_v\,\mathrm{Ra} = 0\,,\quad \omega = \gamma^2 - k_x^2\,,$$

$$\omega_{1,2} = \frac{b_v\lambda}{2c_v} \pm \sqrt{\left(\frac{b_v\lambda}{2c_v}\right)^2 + \frac{k_x^2\gamma_v d_v\,\mathrm{Ra}}{c_v}}\,,\quad \gamma_{1,2} = \sqrt{k_x^2 + \omega_{1,2}}\,.$$

The general solution in the upper layer will be obtained in the form

$$\psi_v = e^{\lambda t + ik_x x}[c_1\sinh\gamma_1(1-z) + c_2\sinh\gamma_2(1-z)]\,,$$

$$T_v = \frac{e^{\lambda t + ik_x x}}{ik_x d_v\mathrm{Ra}}[(\gamma_1^2 - k_x^2)c_1\sinh\gamma_1(1-z) + (\gamma_2^2 - k_x^2)c_2\sinh\gamma_2(1-z)]\,, \tag{15}$$

$$\omega_{1,2} = \frac{b_v\lambda}{2c_v} \pm \sqrt{\left(\frac{b_v\lambda}{2c_v}\right)^2 + \frac{k_x^2\gamma_v d_v\,\mathrm{Ra}}{c_v}}\,,\quad \gamma_{1,2} = \sqrt{k_x^2 + \omega_{1,2}}\,.$$

An analogous general solution in the lower layer will be written as

$$\psi_w = e^{\lambda t + ik_x x}[c_3\sinh\gamma_3 z + c_4\sinh\gamma_4 z]\,,$$

$$T_w = \frac{e^{\lambda t + ik_x x}}{ik_x\,\mathrm{Ra}}[(\gamma_3^2 - k_x^2)c_3\sinh\gamma_3 z + (\gamma_4^2 - k_x^2)c_4\sinh\gamma_4 z]\,, \tag{16}$$

$$\omega_{3,4} = \frac{b_w\lambda}{2} \pm \sqrt{\left(\frac{b_w\lambda}{2}\right)^2 + k_x^2\gamma_w\,\mathrm{Ra}}\,,\quad \gamma_{3,4} = \sqrt{k_x^2 + \omega_{3,4}}\,.$$

We satisfy the boundary conditions at $z = \xi_s$, assuming that

$$\xi = c_5 e^{\lambda t + ik_x x}\,. \tag{17}$$

Finally, we obtain the system of equations

$$\sum_{j=1}^{5} \alpha_{ij} c_j = 0 , \quad i = 1, ..., 5 ,$$

where the matrix α_{ij} has been assigned by the expressions

$$\alpha_{11} = \frac{\gamma_1^2 - k_x^2}{ik_x d_v \ \mathrm{Ra}} \sinh \gamma_1 (1 - \xi_s) , \quad \alpha_{12} = \frac{\gamma_2^2 - k_x^2}{ik_x d_v \ \mathrm{Ra}} \sinh \gamma_2 (1 - \xi_s) ,$$

$$\alpha_{13} = -\frac{\gamma_3^2 - k_x^2}{ik_x \ \mathrm{Ra}} \sinh \gamma_3 \xi_s , \quad \alpha_{14} = -\frac{\gamma_4^2 - k_x{}^2}{ik_x \ \mathrm{Ra}} \sinh \gamma_4 \xi_s , \quad \alpha_{15} = \frac{\gamma - 1}{1 - (1 - \gamma)\xi_s} ,$$

$$\alpha_{21} = 0 , \quad \alpha_{22} = 0 , \quad \alpha_{23} = \frac{\delta_w}{\tilde{A} \ \mathrm{Ra}} \cosh \gamma_3 \xi_s - \frac{M(\gamma_3^2 - k_x^2)}{\mathrm{Ra}} \sinh \gamma_3 \xi_s ,$$

$$\alpha_{24} = \frac{\delta_w}{\tilde{A} \ \mathrm{Ra}} \cosh \gamma_4 \xi_s - \frac{M(\gamma_4^2 - k_x^2)}{\mathrm{Ra}} \sinh \gamma_4 \xi_s , \quad \alpha_{25} = -ik_x \left(\frac{1}{N_h} - \frac{\gamma M}{1 - (1 - \gamma)\xi_s} \right) ,$$

$$\alpha_{31} = \frac{\mu_v \gamma_1}{\mu_w} \cosh \gamma_1 (1 - \xi_s) , \quad \alpha_{32} = \frac{\mu_v \gamma_2}{\mu_w} \cosh \gamma_2 (1 - \xi_s) , \quad \alpha_{33} = \gamma_3 \cosh \gamma_3 \xi_s ,$$

$$\alpha_{34} = \gamma_4 \cosh \gamma_4 \xi_s , \quad \alpha_{35} = -ik_x a_w \ \mathrm{Ra}(1 - \rho_v/\rho_w) ,$$

$$\alpha_{41} = \frac{ik_x \rho_v}{\rho_w} \sinh \gamma_1 (1 - \xi_s) , \quad \alpha_{42} = \frac{ik_x \rho_v}{\rho_w} \sinh \gamma_2 (1 - \xi_s) , \quad \alpha_{43} = -ik_x \sinh \gamma_3 \xi_s ,$$

$$\alpha_{44} = -ik_x \sinh \gamma_4 \xi_s , \quad \alpha_{45} = \lambda \left(1 - \frac{\rho_v}{\rho_w} \right) ,$$

$$\alpha_{51} = \frac{\gamma_1 (\gamma_1^2 - k_x^2)}{ik_x d_v \ \mathrm{Ra}} \frac{\lambda_{mv}}{\lambda_{mw}} \cosh \gamma_1 (1 - \xi_s) , \quad \alpha_{52} = \frac{\gamma_2 (\gamma_2^2 - k_x^2)}{ik_x d_v \ \mathrm{Ra}} \frac{\lambda_{mv}}{\lambda_{mw}} \cosh \gamma_2 (1 - \xi_s) ,$$

$$\alpha_{53} = \frac{\gamma_3 (\gamma_3^2 - k_x^2)}{ik_x \ \mathrm{Ra}} \cosh \gamma_3 \xi_s + \frac{ik_x q}{C_{pw} A_w H} \sinh \gamma_3 \xi_s , \quad \alpha_{54} = \frac{\gamma_4 (\gamma_4^2 - k_x^2)}{ik_x \ \mathrm{Ra}} \cosh \gamma_4 \xi_s + \frac{ik_x q}{C_{pw} A_w H} \sinh \gamma_4 \xi_s ,$$

$$\alpha_{55} = -\frac{\lambda q}{C_{pw} A_w H} = -\frac{\lambda M}{\gamma_w} , \quad M = \frac{q}{C_{pw} (T_1 - T_2)} .$$

Discussion of Results. The prime objective of this investigation was to study the influence of phase transition accompanied by the change in the density and viscosity of a liquid, and also of the value of specific heat of phase transition on the criterion of onset of convection and the structure of occurring motions. Consideration was given to both the case of absorption and the case of release of heat on passage of the heavier phase to a lighter one. In the work, a study was made mainly of the dependence of the critical Rayleigh number determining the threshold of the onset of convection on the value of specific heat of phase transition and on the ratio of the densities of the light and heavy phases. The basic features of the above dependences and of the structure of occurring flows were studied. We outline calculation results.

In the calculations, it was assumed for definiteness that the thickness of the layer of the lower heavy phase is equal to 0.55, and of the upper one, to 0.45. To identify the influence of the thermal effect more clearly, all the thermophysical and physical properties of the phases except densities were assumed to be identical. Figure 1 shows the characteristic neutral curves of the critical Rayleigh number Ra_{cr} as a function of the wave number k_x at different values of the dimensionless heat of phase transition M. The figure corresponds to negative M values. This means that on passage of the light phase to a heavy phase, the heat is absorbed (endothermal phase transition). As can be seen from the figure, the neutral curves have a minimum of the wave number k_x as in the classical Rayleigh problem for a single-phase one-component liquid. In this case the problem is to determine whether the onset of convection hinders or facilitates phase transition, and also to elucidate the

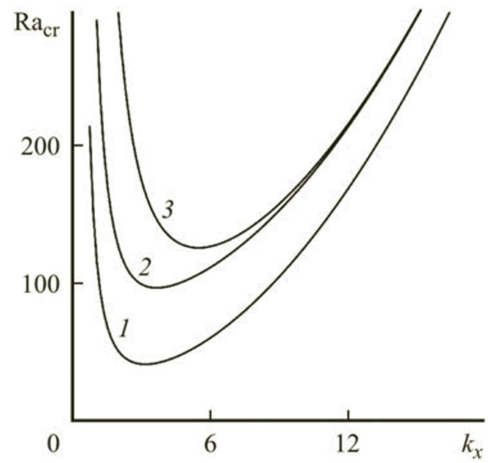

Fig. 1. Critical Rayleigh numbers vs. wave numbers at $\rho_v/\rho_w = 0.9$, $\Gamma = 2$, and different values of the heat of endogenic phase transition: $M = -0.01$ (1), -0.25 (2), and -3 (3).

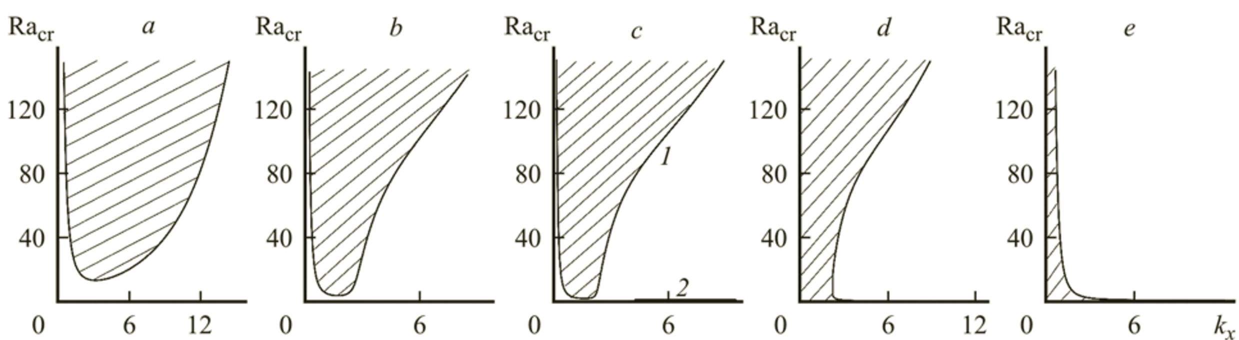

Fig. 2. Critical Rayleigh numbers vs. wave numbers at $\rho_v/\rho_w = 0.5$, $\Gamma = 2$, and different values of the heat of endogenic phase transition: $M = 0.1$ (a), 0.4 (b), 0.418 (c), 0.43 (d), and 0.5 (e). Instability regions are shown dashed.

structure of occurring flows when stability is lost. As Fig. 1 shows, a growth in the heat of endogenic phase transition at the considered values of the remaining parameters hinders the onset of convection.

In the case of positive M values, i.e., exothermal phase transition from the light phase to a heavy one, the neutral $\mathrm{Ra}_{cr}(k)$ curves on different segments of variation in M may differ qualitatively. Figure 2 shows the neutral curves corresponding to positive M values. We can see from the figure how the structure of the instability region shown dashed changes as M grows. On passage through a certain critical M number, another branch of the neutral curve appears at the bottom (Fig. 2c, 2), which corresponds to a shortwave hydrodynamic instability of the Rayleigh–Taylor type. With further growth in M, the right branch of curve 1 in Fig. 2c combines with curve 2 and the instability region takes the form shown in Fig. 2d, i.e., longwave convective disturbances become unstable, too. Finally, with further increase in M, the neutral curve acquires the form presented in Fig. 2d. Thus, when the parameter of the heat of exogenic phase transition M exceeds a certain critical value dependent on other fixed parameters mechanical equilibrium becomes impossible even at small Rayleigh numbers.

The curves in Fig. 2 have been considered for the density ratio equal to 0.5; for smaller values of this ratio, the neutral curves are qualitatively analogous. The difference is mainly in the fact that the passage from the curves in Fig. 2b to the curves in Fig. 2c and from the curves in Fig. 2c to the curves in Fig. 2d occurs at smaller M values.

Of greatest interest is the threshold Rayleigh number which is the smallest as far as the wave number is concerned, i.e., the $\mathrm{Ra}_{cr}(k_x)$ minimum in k_x. Here, it is required that the local indicated minimum, not the global one, be generally found as can be seen from Fig. 2. The indicated minimum threshold Rayleigh number will subsequently be called simply the critical Rayleigh number.

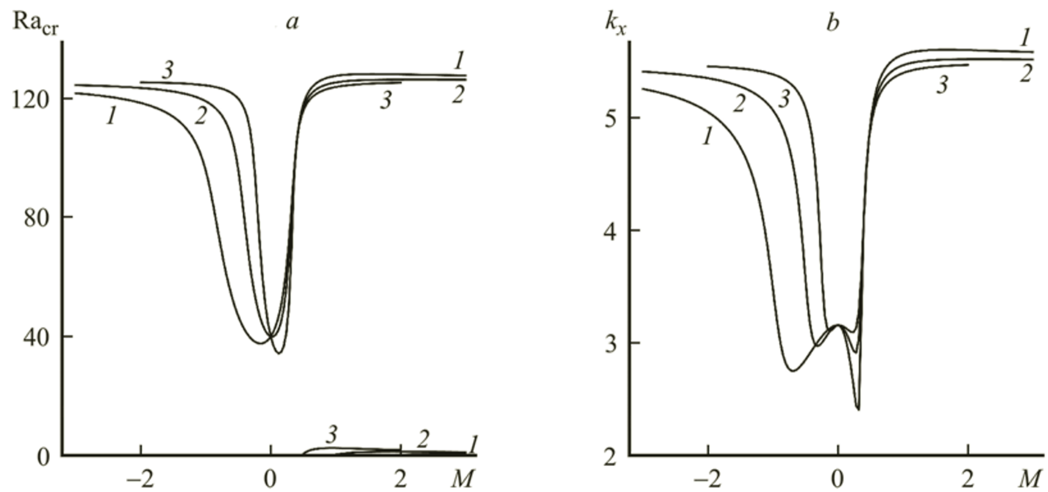

Fig. 3. Critical Rayleigh numbers (minimum in wave number) and critical wave numbers (yielding the minimum Rayleigh number) (b) vs. heat of phase transition at $\rho_v/\rho_w = 0.9$ and $\Gamma = 0.5$ (1), 1 (2), and 2 (3).

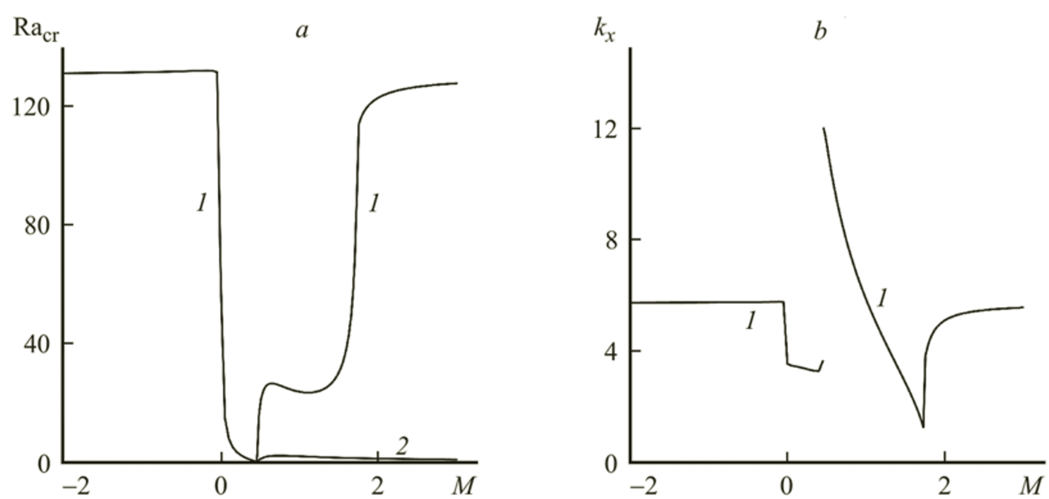

Fig. 4. Critical Rayleigh numbers (minimum in wave number) and critical wave numbers (yielding the minimum Rayleigh number) (b) vs. heat of phase transition at $\rho_v/\rho_w = 0.1$ and $\Gamma = 2$: 1) case of finite critical wave numbers; 2) short wave case.

Figure 3 shows the plots of the critical Rayleigh number versus the heat of phase transition M for a ratio of the phase densities of 0.9 and different values of the parameter Γ. The parameter Γ is proportional to the ratio of the slope of the phase-equilibrium curve (dP/dT) to the heat of phase transition. Therefore, larger Γ values at an assigned dP/dT correspond to small thermal effects. We consider first the negative values of M, i.e., endogenic phase transitions. The region above the neutral curve in Fig. 3a corresponds to the instability of mechanical equilibrium. Below the neutral curve, conversely, mechanical equilibrium is stable, at least, for fairly small perturbations. If we are to abstract from the relatively small values of $|M|$, it can be seen from the figure that the increase in the heat of phase transition, i.e., in the modulus of M at other fixed parameters, leads to an increase in the critical Rayleigh number, i.e., to a hindered onset of convection. It can be noted that at a fixed negative M value, the growth in Γ also leads to a hindered onset of convection. However, it can be seen from the figure that at relatively small values of the heat of endogenic phase transition and relatively small values of Γ, the picture may depart from the described one. Thus, at $\Gamma = 0.5$, the critical Rayleigh number on the initial small portion decreases as $|M|$ grows, i.e., the thermal effect on this small portion somewhat facilitates the onset of convection.

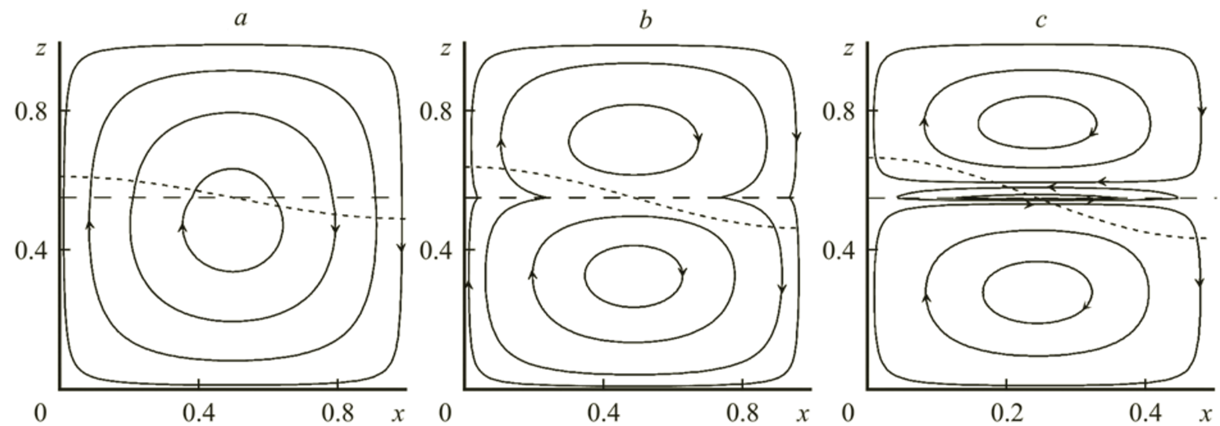

Fig. 5. Streamlines at the ratio of the phase densities $\rho_v/\rho_w = 0.9$, $\Gamma = 2$, and different values of the heat of endogenic phase transition: $M = 0.01$ (a), –0.2 (b), and –1 (c); the coarse and fine dotted lines show the position of the phase boundary in undisturbed and disturbed states respectively.

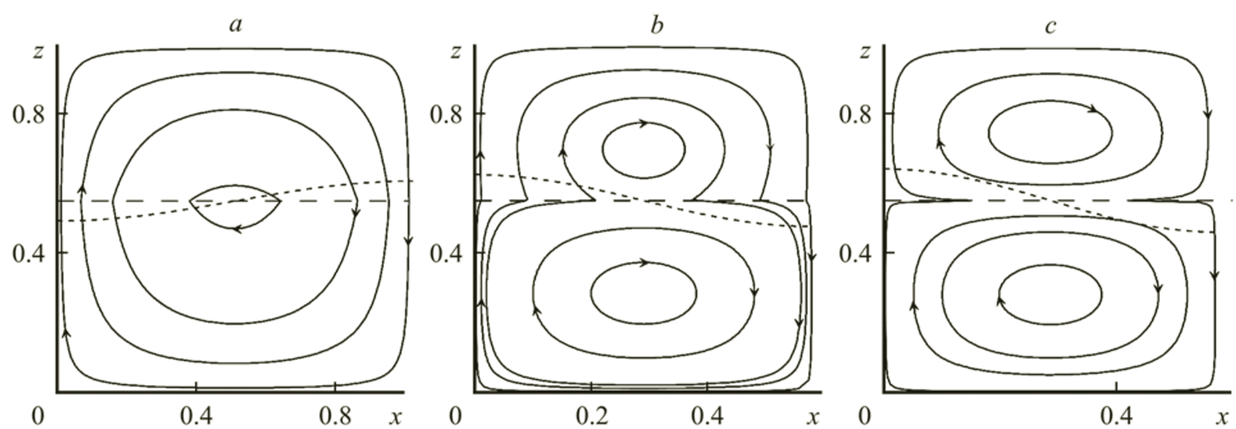

Fig. 6. Streamlines at the ratio of the phase densities $\rho_v/\rho_w = 0.9$, $\Gamma = 2$, and different values of the heat of exogenic phase transition: $M = -0.1$ (a), –0.2 (b), and –10 (c); the coarse and fine dotted lines show the position of the phase boundary in undisturbed and disturbed states respectively.

For positive M values in Fig. 3a, the picture is somewhat different. As can be seen from the figure, in addition to the upper curves, there are also the lower curves. The regions above the upper curves and below the lower curves are the regions of instability of mechanical equilibrium. Accordingly, the region between the lower and upper curves is the region of instability of mechanical equilibrium, at least for small perturbations. Figure 3b shows the critical wave numbers corresponding to the upper neutral curves in Fig. 3a. The lower curves in Fig. 3a correspond to shortwave unstable disturbances of the Rayleigh–Taylor type.

Figure 4 shows analogous dependences of the critical Rayleigh number and the corresponding wave number at small ratios of the phase densities. It can be seen that the picture remains constant qualitatively, as the ratio of the phase densities decreases, although quantitative changes may be considerable.

Finally, Figs. 5 and 6 give the characteristic streamlines for the endogenic and exogenic phase transitions respectively. At small values of the heat of phase transition, neutral flows are single-stage. With growth in $|M|$, the flow gives way to a partially two-stage one and, as $|M|$ grows further, becomes fully two-stage where the cells in the upper and lower layers may be separated by an induced adjacent vortex (roller). Figures 5 and 6 show the position of an undisturbed phase boundary as a coarse dotted line and its disturbed position, as a fine dotted line. In constructing the disturbed phase boundary, we have introduced the normalization factor for the sake of illustration. Note that for the considered parametric values in the case of endogenic phase transition, the phase boundary deviates toward the substance flow (Fig. 5). Generally speaking, this is a strict rule only for small heat-release effects, i.e., large Γ. In this case for the exogenic reaction in stable circulation of the liquid, the phase boundary must deviate to a side that is opposite to the liquid flow. In the general case, however, there can be two directions of deviation of the phase boundary as shown in Fig. 6 for the exothermal reaction, which is due to the competition of the influence of the parameters M and Γ, i.e., the values of the heat of phase transition and the slope of the phase-equilibrium curve.

Conclusions. For endogenic phase transition, the phase transition hinders, as a rule, the onset of convection, i.e., the $Ra_{cr}(-M)$ curve grows with argument. However, as can be seen from Fig. 3a, at small ratios of the phase densities and slopes of the equilibrium curves that are small in modulus, the $Ra_{cr}(M)$ curve may have a shallow local minimum in the region of small thermal effects.

For exogenic phase transition in the region of small thermal effects, the picture is asymmetric in a sense. At small Γ, the critical Rayleigh number grows with thermal effect, i.e., the thermal effect hinders the onset of convection. At large Γ, conversely, there is a shallow minimum of the $Ra_{cr}(M)$ curve. Significantly, for exogenic phase transition, there is the critical value $M_{cr} > 0$ after which we have, at small Rayleigh numbers, the Rayleigh–Taylor shortwave instability having the hydrodynamic nature (lower curve in Fig. 3a). With growth in the Rayleigh number, the equilibrium is stabilized and, on intersecting the upper curve in Fig. 3, the mixture becomes unstable again, but the instability has the convective nature now. With further growth in M, as Fig. 2c shows, it is also longwave disturbances that become unstable beginning with a certain value, and mechanical equilibrium becomes impossible.

If we speak of convective instability, for both types of phase transition, at a small thermal effect, flow is single-layer and covers the entire layer thickness. As the value of the thermal effect grows, partial stratification of the flow occurs, i.e., there are both convective cells covering the entire thickness of a porous layer and the cells located inside just the upper layer or just the lower layer. With further growth in the thermal effect, the flow tends to full stratification.

NOTATION

C_m, effective heat of a unit volume of the porous medium, J/(m^3 K); C_p, specific heat of the mixture at constant pressure, J/(kg K); F, phase-equilibrium function; g, free-fall acceleration, m/s^2; h, specific enthalpy, J/kg; k, permeability, m^2; k_x, wave number; M, dimensionless heat of phase transition; P, pressure, Pa; q, specific heat of vaporization, J/kg; T, temperature, K; V, normal component of the velocity of motion of the phase boundary, m/s; v_n, normal component of the filtration velocity, m/s; **v**, filtration-velocity vector, m/s; β, thermal expansion coefficient, K^{-1}; Γ, dimensionless value of the slope of the equilibrium curve (dp/dT); λ, increment of disturbances; λ_m, effective thermal conductivity of the porous medium, W/(m·K); μ, viscosity, N·s/m^2; ρ, density, kg/m^3; ψ, stream function. Subscripts: 0, mean value; s, mechanical equilibrium; v, quantities referring to the upper light phase; w, quantities referring to the lower heavier phase; *, parametric values at the phase boundary.

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