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MISCELLANEA

ON THE PROBLEM OF DESIGNING THE ANISOTROPIC MATERIAL OF SHELL SLEEVES WITH A FREE EDGE IN THE CORE OF A NUCLEAR ROCKET ENGINE

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The author gives numerical results on taking account of the influence of anisotropy of physicomechanical properties of free-edge cylindrical shell sleeves from orthotropic material on the stressed state in fuel assembles of the core of a nuclear rocket engine in their probable local radial interaction, which is important in designing.

Keywords: *differential equation, boundary conditions, Fourier series, stressed state, local load, physically orthotropic, transversally isotropic.*

One probable propulsion system to transport earth dwellers to distant planets, in particular, Mars, is, obviously, a nuclear rocket engine (NRE). In one of the most advanced design versions of a nuclear rocket engine, an "unprecedented decision was taken to use materials prone to brittle failure, which required a change in designing principles and in established views of strength and thermostability" [1]. On the portion of the core, the internal and external casings are protected by free-edge sleeves with a great number of individual coaxially assembled cylindrical bushings of length 50–150 mm from anisotropic materials: orthotropic porous or high-density carbide and high-density transversely isotropic pyrolytic graphite (pyrographite). Inside the heat-insulation package, there are heating sections containing fuel elements which, in the process of operation, may "produce fractures and hence fragments" (Fig. 1) by means of which local or lumped contact interactions with high co-stresses may occur in coaxially neighboring sleeves.

In this connection, the focus is placed on ensuring the intrinsic thermostability of individual elements of the package to the occurring temperature field [2, 3], on the one hand, and to their possible contact interaction because of the radial temperature gradient from the center to the periphery in the heating section of the fuel assembly, which may be as great as 3000 degrees, on the other [1]. Because of the considerable temperature difference across the thickness of the entire package of the heating section and of each separate sleeve, the sleeves can "turn inside out" at free edges, which may result in the local contact of their "turned-out" edge with the coaxially neighboring sleeve, i.e., with that more distant from the center and less heated. As a solution to the thermoelastic problem in the presence of hot spots in the shell, we can use the results in [3–6] where it has been reduced to solving partial differential equations of eighth order for the resolving function or a system of eight differential equations of first order [7].

It is of both practical and theoretical interest to investigate the stressed state of free-edge shell sleeves, the more so as this process is stochastic in many parameters [8]. However, with arbitrary boundary conditions, in particular, in the presence of the free edge, constructing analytical solutions to boundary-value problems on the basis of differential equations of eighth order faces insurmountable, in practice, obstacles. Therefore, here, we employ the method of coupling of solutions based on differential equations of fourth order in longitudinal coordinate, taking into account the stochastic character of the process of straining.

Let sleeve shells with one or two free edges be subjected, in the operating regime, to normal pressure of local character from the neighboring structural elements assembled coaxially with them. We place the origin of coordinates $x = 0$ at the free edge of the shell with thickness h and radius R and introduce the dimensionless coordinate $\alpha = x/R$. We assume that the contact normal pressure $p(\alpha, \beta)$ is transmitted to the shell by k rectangular regions in one cross section of the shell and represent it in the form of the series

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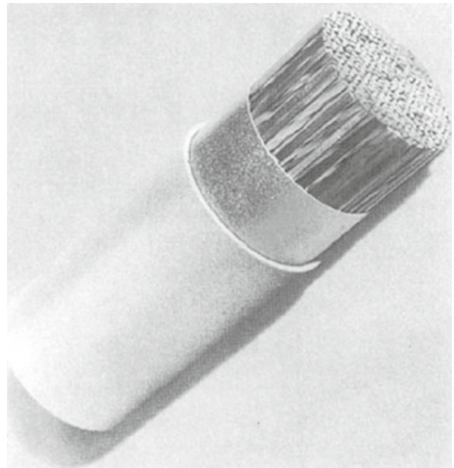


Fig. 1. Fragment of the NRE full assembly: coaxially assembled shell sleeves and fuel elements.

$$p(\alpha, \beta) = p_0\theta(\alpha)\sum_{n=0}^{\infty} \theta_n \cos kn\beta , \tag{1}$$

where $\theta(\alpha) = 1$ at $\xi_1 \leq \alpha \leq \xi_2$; $\theta(\alpha) = 0$ at $\alpha \notin (\xi_1, \xi_2)$, $\xi_1 = \xi - \alpha_0$; $\xi_2 = \xi + \alpha_0$; $\theta_n = \dfrac{k\beta_0}{\pi}$ $(n = 0)$; $\theta_n = \dfrac{2}{\pi}$ $\sin kn\beta_0$ $(n = 1, 2, 3, \ldots)$. Thus, the length of the loaded region along the generatrix is $a = 2\alpha_0 R$, and along the contour, $b = 2\beta_0 R$; the load on one region is $P = abp_0 = 4\alpha_0\beta_0 p_0 R^2$.

Let $a \approx b \approx \sqrt{Rh}$ or $\alpha_0 \approx \beta_0 \approx \sqrt{h/R}$; then in the asymptotic-synthesis method [5, 6], the stressed state with high variability is negligible, and the full stressed state may be constructed on the basis of differential equations of the ground state and the local edge effect, with each being a partial differential equation of fourth order in the longitudinal coordinate α. Here, the ground semi-momentless state plays a major role, which follows from the distinctive features of bending of the shell's middle surface in the zone adjacent to the free edge: here we mainly have bending in the circumferential direction combined with weaker bending of the generatrices of the shell as confirmed previously for isotropic shells both theoretically and experimentally [6].

The ground state is described by a modified semi-momentless theory: the resolving differential equation and the sought factors are obtained through the application of the criterion $\left|\partial^2\Phi/\partial\beta^2\right| >> \left|\partial^2\Phi/\partial\alpha^2\right|$ to the equations of the general theory of physically orthotropic shells [7]:

$$\frac{\partial^4\Phi}{\partial\alpha^4} + \frac{c^2\lambda}{1 - \nu_1\nu_2}\frac{\partial^4}{\partial\beta^4}\left(\frac{\partial^2}{\partial\beta^2} + 1\right)^2 \Phi = \frac{R^2}{E_2 h}\, p(\alpha, \beta) , \tag{2}$$

$$w = \lambda\frac{\partial^4\Phi}{\partial\beta^4} ; \quad T_1 = -\frac{E_2 h}{R}\frac{\partial^4\Phi}{\partial\alpha^2\partial\beta^2} ; \quad S = \frac{E_2 h}{R}\frac{\partial^4\Phi}{\partial\alpha^3\beta} ; \quad G_2 = -\lambda\frac{D_2}{R^2}\left(\frac{\partial^6\Phi}{\partial\beta^6} + \frac{\partial^4\Phi}{\partial\beta^4}\right); \quad G_1 = \nu_2 G_2 . \tag{3}$$

For the edge effect, we obtain the differential equation for the resolving function $w = w(\alpha, \beta)$ and expressions for the sought factors analogously, but through the application of the criterion $\left|\partial^2\Phi/\partial\alpha^2\right| >> \left|\partial^2\Phi/\partial\beta^2\right| >> |\Phi|$ [6] to the differential equations of the general theory of shells:

$$\frac{\partial^4 w}{\partial\alpha^4} + 4\eta^4 w = \frac{R^4}{D_1}\, p(\alpha, \beta) , \quad 4\eta^4 = 12(1 - \nu_1\nu_2)\lambda(R/h)^2 , \quad w = w(\alpha, \beta) ,$$
$$T_2 = -\frac{E_2 h}{R}\, w , \quad G_1 = -\frac{D_1}{R^2}\frac{\partial^2 w}{\partial\alpha^2} , \quad G_2 = -\nu_1\frac{D_2}{R^2}\frac{\partial^2 w}{\partial\alpha^2} , \quad Q_1 = \frac{D_1}{R^3}\frac{\partial^3 w}{\partial\alpha^3} . \tag{4}$$

The ground stressed state and the local edge effect are constructed separately, when the discrepancy in boundary conditions due to the separate satisfaction of the boundary conditions is eliminated through the introduction of a correcting edge effect in the edge zone [6]. The total values of normal displacement, forces, and bending moments are computed as follows:

$$\begin{gathered} w(\alpha, \beta) \approx w^{\mathrm{gr}}(\alpha, \beta), \quad T_1(\alpha, \beta) \approx T_1^{\mathrm{gr}}(\alpha, \beta), \quad T_2(\alpha, \beta) \approx T_2^{\mathrm{e}}(\alpha, \beta), \\ S(\alpha, \beta) \approx S^{\mathrm{gr}}(\alpha; \beta), \quad G_1(\alpha, \beta) \approx G_1^{\mathrm{e}}(\alpha, \beta) + G_1^{\mathrm{gr}}(\alpha, \beta), \quad G_2(\alpha, \beta) \approx G_2^{\mathrm{gr}}(\alpha, \beta) + G_2^{\mathrm{e}}(\alpha, \beta), \end{gathered} \tag{5}$$

where the superscripts "gr" and "e" refer to the ground state and the edge effect.

The solution of the resolving equation (2) of the ground state is sought in the form

$$\Phi(\alpha, \beta) = \sum_{n}^{n^*} \Phi_n(\alpha) \cos kn\beta . \tag{6}$$

Analogously we represent the sought factors for which we formulate tangential boundary conditions

$$\begin{gathered} u(\alpha, \beta) = \sum_{n}^{n^*} U_n(\alpha) \cos kn\beta, \quad \upsilon(\alpha, \beta) = \sum_{n}^{n^*} V_n(\alpha) \sin kn\beta, \\ T_1(\alpha, \beta) = \sum_{n}^{n^*} T_{1n}(\alpha) \cos kn\beta, \quad S(\alpha, \beta) = \sum_{n}^{n^*} S_n(\alpha) \sin kn\beta . \end{gathered} \tag{7}$$

Summation in the mentioned series is from $n = 1$ ($k = 2$) and $n = 2$ ($k = 1$) to the value n^* [6].

Substitution of (6) and (1) into (2) yields the ordinary differential equation of the ground state

$$\frac{d^4\Phi_n(\alpha)}{d\alpha^4} + 4\mu_n^4\Phi_n(\alpha) = \frac{p_0 R^2}{E_2 h}\,\theta_n\theta(\alpha), \quad 4\mu_n^4 = \frac{\lambda c^2}{1-\nu_1\nu_2}\, k^4 n^4 (k^2 n^2 - 1)^2 \tag{8}$$

and the relations linking the amplitude values of the resolving function and the sought factors and resulting from the substitution of (7) into (3):

$$U_n(\alpha) = \lambda n^2 \Phi_n'(\alpha), \quad V_n(\alpha) = \lambda n^3 \Phi_n(\alpha), \quad T_{1n}(\alpha) = n^2 \frac{E_2 h}{R}\,\Phi_n''(\alpha), \quad S_n(\alpha) = -n \frac{E_2 h}{R}\,\Phi_n'''(\alpha) . \tag{9}$$

The solution by the method of initial parameters under arbitrary and step pressure is obtained from the generalization of the solution written in tabular from [5] (Table 5.1) for the case of isotropic material through the replacement of the coefficient μ_n in accordance with its value in the differential equation (8) and on replacement of the relations

$$\upsilon_n(\alpha) = \frac{R}{Eh}\,\upsilon_n^*(\alpha), \quad u_n(\alpha) = \frac{R}{Eh}\,u_n^*(\alpha)$$

by the following ones:

$$V_n(\alpha) = \frac{R}{E_1 h}\,V_n^*(\alpha), \quad U_n(\alpha) = \frac{R}{E_1 h}\,U_n^*(\alpha) .$$

In the case of the edge effect, the resolving function will be represented in the form of the series

$$w(\alpha, \beta) = \sum_{n}^{n^*} W_n(\alpha) \cos kn\beta .$$

The differential equation for the nth harmonic number will be written as

$$\frac{d^4 W_n}{d\alpha^4} + 4\eta^4 W_n = \frac{R^4}{D_1}\,p_n(\alpha), \quad 4\eta^4 = 12(1-\nu_1\nu_2)\lambda(R/h)^2, \quad W_n = W_n(\alpha) .$$

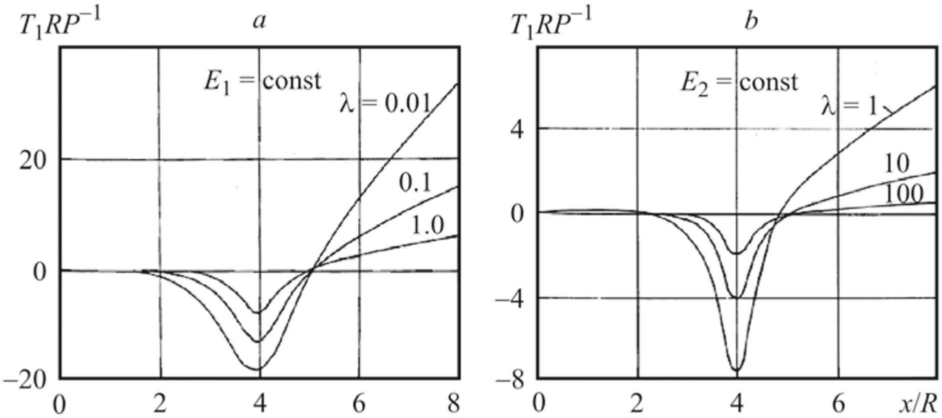


Fig. 2. Distribution of the longitudinal force along the generatrix of the shell at a constant elastic modulus in the direction of the generatrix (a) and in the circumferential direction (b).

The sought factors are represented in the form of the series

$$T_2(\alpha, \beta) = \sum_{n}^{n^*} T_{2n}(\alpha) \cos kn\beta \,, \quad G_1(\alpha, \beta) = \sum_{n}^{n^*} G_{1n}(\alpha) \cos kn\beta \,, \quad G_2(\alpha, \beta) = \sum_{n}^{n^*} G_{2n}(\alpha) \cos kn\beta \,,$$

where

$$T_{2n} = -\frac{E_2 h}{R} W_n, \; G_{1n} = -\frac{D_1}{R^2}\frac{d^2 W_n}{d\alpha^2}, \; G_{2n} = \nu_1 G_{1n}, \; Q_{1n} = -\frac{D_1}{R^3}\frac{d^3 W_n}{d\alpha^3}.$$

The stressed-strained state corresponding to the local edge effect may be constructed analogously ([5], Table 5.2) in the zone of application of a load and to the correcting edge effect at the shell's edge.

Let us dwell on consideration of a concrete shell having one free edge ($\alpha = 0$) and the other rigidly fixed ($\alpha = \alpha_1 = l/R$). Boundary conditions will be written right for the amplitude values of displacements and force factors:

$$T_{1n}(0) = S_n(0) = Q^*_{1n}(0) = G_{1n}(0) = 0 \,, \quad U_n(\alpha_1) = V_n(\alpha_1) = W_n(\alpha_1) = W'_n(\alpha_1) = 0 \,. \tag{10}$$

In the adopted method of coupling of solutions of the ground state and the edge effect, they are split into tangential

$$T_{1n}(0) = S_n(0) = 0 \,, \quad U_n(\alpha_1) = V_n(\alpha_1) = 0 \tag{11}$$

and nontangential

$$Q_{1n}(0) = G_{1n}(0) = 0 \,, \quad W_n(\alpha_1) = W'_n(\alpha_1) = 0 \,. \tag{12}$$

Here, the ground state should satisfy the tangential boundary conditions, and the edge effect, the nontangential ones [5].

Results of a parametric analysis of the influence of the versions of combination of the characteristics of a physically orthotropic material ($0.01 \le \lambda \le 100$) are given as plots of the: longitudinal force (Fig. 2), normal displacement (Fig. 3), and circumferential bending moment (Fig. 4).

It is assumed that the load P has been applied at the center of the shell over the region $a \times b = 0.5R/\alpha_0 = \beta_0 = 0.25$), $\alpha_1 = l/R = b$, and $R/h = 100$. Plots 2–4 *a* have been constructed under the assumption that $E_1 = 210$ GPa and $E_2 = \upsilon ar$ ($0.01 \le \lambda \le 1.0$), and plots 2–4 *b*, under the assumption that $E_2 = 210$ GPa and $E_1 = \upsilon ar$ ($1.0 \le \lambda \le 100$).

We can easily notice a very strong dependence of these factors on the orthotropy exponent. Therefore, such analysis may turn out to be useful in designing the shell material and optimizing the structure under assigned loads, and also for understanding the operation of the structure and creating its realistic mechanical mathematical model.

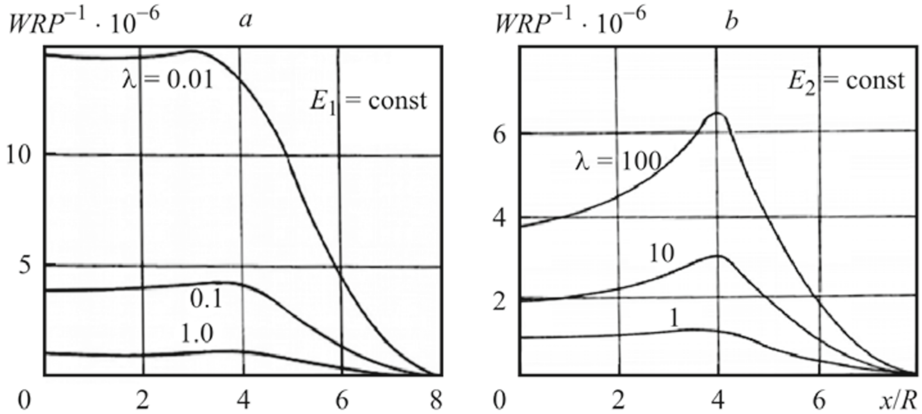


Fig. 3. Distribution of the normal displacement along the generatrix of the shell at a constant elastic modulus in the direction of the generatrix (a) and in the circumferential direction (b).

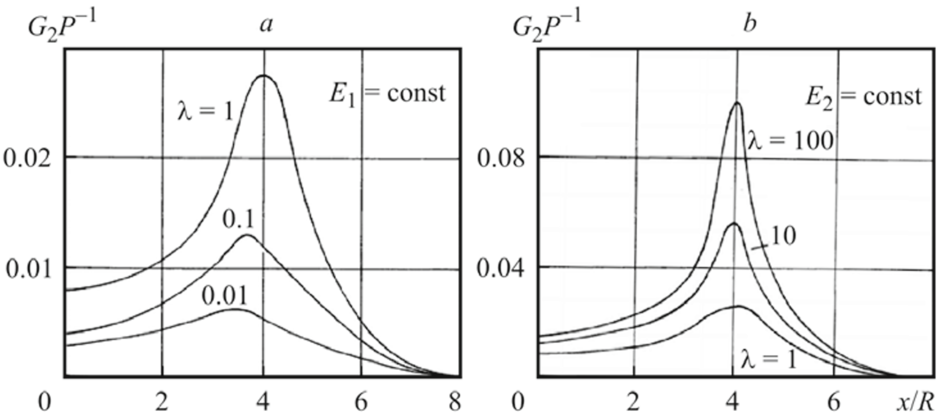


Fig. 4. Distribution of the circumferential bending moment along the generatrix of the shell at a constant elastic modulus in the direction of the generatrix (a) and in the circumferential direction (b).

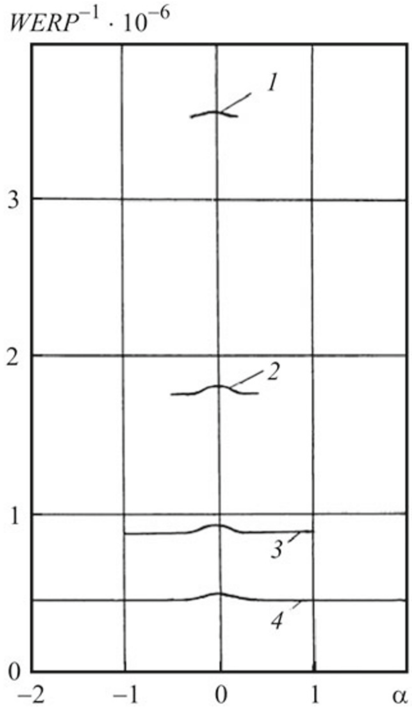
For shells with both edges free, we have constructed plots (Figs. 5 and 6) illustrating the behavior of the normal displacement and the circumferential bending moment along the zero generatrix of shells of varying length loaded at the center by two opposing radial forces P that create the pressure p_0 on each of the two regions ($k = 2$) with dimensions $a \times b = 0.25R \times 0.25R$ ($\alpha_0 = \beta_0 = 0.125$). As can be seen from the plots in Figs. 5 and 6, there is a strong dependence of the bending moment and displacement and hence of the strength and rigidity on the shell's length.

From the numerical values of normal forces and bending moments, we find the normal stresses on the exterior and interior surfaces of the shell sleeve [9]:

$$\sigma_i = \frac{T_i}{h} \pm \frac{6G_i}{h^2} \quad (i = 1, 2) . \tag{13}$$

Note that the problem on calculating transversally isotropic cylindrical shells at the pressure arbitrarily distributed over the shell's surface, without limitations on the dimensions of the shell and on variability of the load, has been considered in [7].

It is common knowledge that to evaluate the interaction force (radial pressure) of the contacting pair of coaxially assembled shell sleeves, we must primarily know the clearance between them, normal displacements from temperature fields, and compliance of each at a unit force of their interaction. Next, it is required that the condition of equality of normal



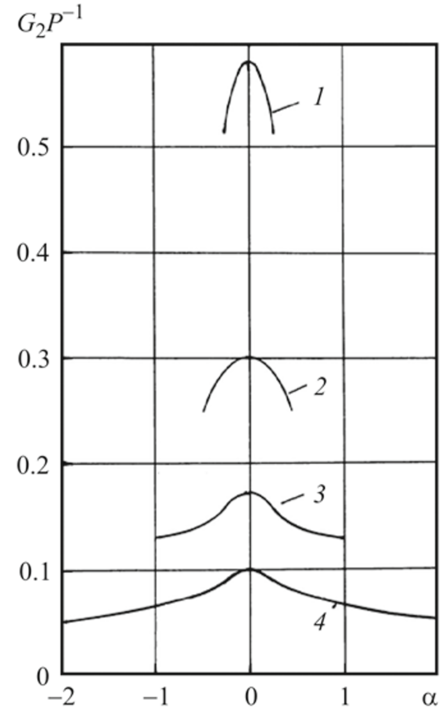


Fig. 5. Distribution of the normal displacement along the generatrix of the shell at $h/R = 1/100$ and $\lambda = 1$ and different lengths of the shell sleeve: 1) 0.5*R*, 2) 1.0*R*, 3) 2.0*R*, and 4) 4.0*R*.

Fig. 6. Distribution of the circumferential bending moment along the generatrix of the shell at $h/R = 1/100$ and $\lambda = 1$ and different lengths of the shell sleeve: 1) 0.5*R*, 2) 1.0*R*, 3) 2.0*R*, and 4) 4.0R.

displacements at the point, region, or line of their contact be written in the form of an algebraic canonical equation of the force method [9]. From the found contact force, by the solution of boundary-value problems for differential equations of anisotropic shell sleeves, we determine forces, moments, and also stresses from formula (13). At the axisymmetric temperature field, the contact force of interaction away from the edges may be obtained quite simply on the basis of momentless theory, and near the edges, from the theory of shells with account taken of the edge effect.

NOTATION

D_1 and D_2, cylindrical rigidities; E_1 and E_2, elastic moduli of the material in longitudinal (α) and circumferential (β) directions, N/m^2; $G_1(\alpha, \beta)$ and $G_2(\alpha, \beta)$, longitudinal and circumferential bending moments, N; l, R, and h, length, radius, and thickness of the shell, m; $T_1(\alpha, \beta)$ and $T_2(\alpha, \beta)$, longitudinal and circumferential forces, N/m; $w(\alpha, \beta)$, normal displacement, m; α and β, longitudinal ($\alpha = x/R$) and circumferential ($\beta = s/R$) dimensionless coordinates; α_0 and β_0, dimensionless parameters of the loaded rectangular region; θ_m and θ_n, dimensionless coefficients of Fourier series expansions of normal pressure in the α and β directions respectively; $\lambda = E_2/E_1 = \nu_2/\nu_1$, orthotropy exponent of the shell's material, dimensionless quantity; ν_1, coefficient of transverse compression in the β direction under tension in the α direction; ν_2, coefficient of transverse compression in the α direction under tension in the β direction; σ_1 and σ_2, longitudinal and circumferential stresses, N/m^2.

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