DOI 10.1007/s10891-020-02234-0
Journal of Engineering Physics and Thermophysics, Vol. 93, No. 5, September, 2020

TRANSFER PROCESSES IN RHEOLOGICAL MEDIA

INFLUENCE OF TEMPERATURE AND PRESSURE ON VISCOELASTIC FLUID FLOW IN A PLANE CHANNEL

A. V. Baranov UDC 532.135:536.242:678:065

The hydrodynamics of a steady-state nonisothermal flow of a viscoelastic polymer medium in a plane channel and heat transfer in it under boundary conditions of the first kind have been investigated. Fluid flow with a low Reynolds number and a high Péclet number was investigated, which made it possible to neglect the gravity and inertial forces, as well as the longitudinal thermal conductivity of the medium. From the rheological viewpoint, the polymer melt represents a viscoelastic fluid; therefore the Phan-Thien–Tanner fluid model was used as a rheological model of the fluid, with viscosity depending on temperature and pressure. A high-viscosity medium was considered; therefore a dissipation term was included into the equation of the energy of its flow. With the use of the indicated rheological model the velocity profile of fluid flow was obtained in an explicit form from the equation of fluid motion. It has been established that the dependence of the fluid viscosity on temperature and pressure exerts a noticeable influence on the distribution of the Nusselt number and of bulk temperature of the fluid along the channel length. It is shown that account for the temperature dependence of fluid viscosity leads to a decrease in the role of energy dissipation of its flow in the process of flow heating and that, conversely, the dependence of the fluid viscosity on pressure considerably enhances the dissipation effect. The problem has been solved numerically by the method of finite differences.

Keywords: *viscoelastic fluid, heat exchange, nonisothermal flow, dissipation.*

Introduction. Numerous publications, a partial survey of which is presented in work [1], are devoted to investigation of the processes of nonisothermal non-Newtonian media flow in various channels. In the present work, flow of high-viscosity polymer composite in a plane channel is considered (Fig. 1) on the assumption that the temperature of the medium T_0 and the temperature of the channel walls T_w do not coincide and that T_w exceeds T_0. This means that as the composite flows in the channel it will heat up from both the hot walls of the channel and due to the energy dissipation of its flow.

The flow of viscoelastic composites is accompanied by clearly manifested highly elastic effects. In the mathematical model of such flow the first difference of normal stresses becomes significant, which requires incorporation of the rheological equation of nonlinear viscoelastic medium into this model. It is considered at the present time that the best results in describing the flows of nonlinear viscoelastic media are yielded by rheological models of relaxation (velocity) type. Often, especially in the foreign literature, they are also called differential. One of the most efficient models of this type is the Phan-Thien–Tanner (PTT) model [2–15].

Among the most important theoretical and technical problems is the elucidation of the influence of pressure in polymer flows on their viscosity. In practice, in processing highly viscous polymers one deals with pressures of the order of hundreds (in extrusion) and thousands (in die casting) atmospheres that exert a substantial effect on the viscosity of polymers, which leads to the deviation of their technological parameters from the values calculated without accounting for this effect. Partial overview of the works devoted to this problem is presented in [16]. Of the publications not considered in [16] mention should be made of works [17–22] where the importance of taking into account the dependence of polymer viscosity on pressure is emphasized.

It should be noted that the presently known works devoted to investigation of heat transfer in the course of Phan-Thien–Tanner flow in channels were carried out with the use of the condition of constancy of fluid viscosity irrespective of

I. M. Gubkin Russian State University of Oil and Gas, 65 Lenin Ave., Moscow, 119296, Russia; email: alexvbaranov@ahoo.co.uk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 93, No. 5, pp. 1342–1348, September–October, 2020. Original article submitted April 4, 2019.

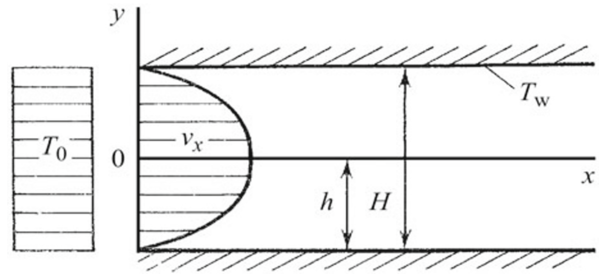


Fig. 1. Schematic diagram of flow of a viscoelastic fluid in a plane channel.

its temperature. This can also be said of the works in which other rheological models were used, for example, the presently popular Giesekus model [23, 24]. But there are no works in which the indicated rheological models could have been used with account for the dependence of the fluid viscosity on pressure. A review of the literature has shown that the quantity of works devoted to the development of simple mathematical models of nonisothermal flow of viscoelastic fluids in channels is clearly insufficient. Therefore an attempt has been made in the present work to create a model of this process that is relatively simple for engineering applications and that accounts for the key dependence of the fluid viscosity on temperature and pressure.

Mathematical Model. Consideration is given to a polymer medium having a high viscosity whose flow occurs at low Reynolds numbers. This allows one, first, to omit consideration of the hydrodynamic entry section of medium flow in the channel and consider that the flow velocity profile at the inlet to the channel is developed and, second, to neglect the inertial terms in the equation of medium motion. It is adopted that transverse (secondary) flows in the channel are absent, i.e., the consideration may be restricted only to the longitudinal component of the flow velocity v_x.

The viscoelastic rheological properties of the medium are described by the Phan-Thien–Tanner model written in a simplified form as [12]

$$f(tr\tau)\tau + \lambda\left(\frac{\partial \tau}{\partial t} + V\cdot\nabla\tau - \tau\cdot\nabla V^T - \nabla V\cdot\tau\right) = \mu(\nabla V + \nabla V^T)\,, \tag{1}$$

where r is the channel radius, λ the time of fluid flow relaxation in the channel, t the time, and τ is the extra tensor of stresses. The function $f(tr\tau)$ can be presented in both exponential and linear form [12]. To simplify the intermediate calculations we use the linear form of this function:

$$f(tr\tau) = 1 + \frac{\varepsilon\lambda}{\mu}\,tr\tau\,, \tag{2}$$

where ε is the rheological constant inversely proportional to the longitudinal viscosity of the medium. It is determined experimentally that for some grades of rubber mixtures based on various raw rubbers $\varepsilon = 1$ and $\lambda \approx 0.05$ s [15].

With account for the foregoing assumptions, the rheological equations for the considered medium take the form [12]

$$f(\tau_{xx})\tau_{xx} = 2\lambda\tau_{xy}\,\frac{\partial v_x}{\partial y}\,, \tag{3}$$

$$f(\tau_{xx})\tau_{xy} = \mu\,\frac{\partial v_x}{\partial y}\,, \tag{4}$$

where τ_{xx} and τ_{xy} are the components of the extra stress tensor. Equations (3) and (4) were written on the assumption that $\tau_{yy} = 0$. If we divide (3) by (4), we obtain

$$\tau_{xx} = \frac{2\lambda}{\mu}\,\tau_{xy}^2\,. \tag{5}$$

Taking Eq. (5) taken into account, we will write the system of rheological equations (3) and (4) in dimensionless form as

$$\sigma_{xx} = \frac{2\mathrm{Wi}}{\bar{\mu}}\,\sigma_{xy}^2\,, \tag{6}$$

$$f(\sigma_{xx})\sigma_{xy} = \bar{\mu}\,\frac{\partial V_x}{\partial Y}\,, \tag{7}$$

$$f(\sigma_{xx}) = 1 + \frac{\varepsilon \mathrm{Wi}}{\bar{\mu}}\,\sigma_{xx}\,, \tag{8}$$

$$Y = y/h\,,\quad V_x = v_x/\bar{v}_x\,,\quad \bar{\mu} = \mu/\mu_w\,,\quad \sigma_{xy} = \frac{\tau_{xy}h}{\bar{\mu}_w\bar{v}_x}\,,\quad \mathrm{Wi} = \frac{\lambda\bar{v}_x}{h}\,.$$

The rheological equations (6)–(8) are supplemented with simplified equations of motion of the medium [25]:

$$\frac{\partial\sigma_{xy}}{\partial Y} = \frac{dP}{dX} \tag{9}$$

and with the conditions of its constant flow rate:

$$\int_0^1 V_x dY = 1\,, \tag{10}$$

where $P = \dfrac{ph}{\mu_w\bar{v}_x}$ and $X = x/h$.

We supplement the given mathematical model with the boundary conditions

$$Y = 0:\quad \frac{\partial V_x}{\partial Y} = 0\,,\quad \frac{\partial\theta}{\partial Y} = 0\,, \tag{11}$$

$$Y = 1:\quad V_x = 0\,,\quad \theta = 0\,, \tag{12}$$

$$X = 0:\quad \theta = 1\,, \tag{13}$$

where $\theta = \dfrac{T_w - T}{T_w - T_0}$.

After the first integration of the equation of motion (9) we obtain

$$\sigma_{xy} = \frac{dP}{dX}\,Y\,. \tag{14}$$

Substituting (6) and (14) into (8), we obtain

$$f(\sigma_{xx}) = 1 + \frac{2\varepsilon \mathrm{Wi}^2}{\bar{\mu}^2}\left(\frac{dP}{dX}\right)^2 Y^2\,. \tag{15}$$

Substitution of (15) into (7) yields

$$\bar{\mu}\,\frac{\partial V_x}{\partial Y} = \frac{dP}{dX}\,Y + \frac{2\varepsilon \mathrm{Wi}^2}{\bar{\mu}^2}\left(\frac{dP}{dX}\right)^3 Y^3\,. \tag{16}$$

After repeated integration of (16) with account for boundary condition (12) we obtain an expression for the flow velocity profile of the medium in the channel:

$$V_x = \frac{dP}{dX}\int_1^Y \frac{Y}{\bar{\mu}}\,dY + 2\varepsilon \mathrm{Wi}^2\left(\frac{dP}{dX}\right)^3\int_1^Y \frac{Y^3}{\bar{\mu}^3}\,dY\,. \tag{17}$$

The unknown pressure gradient in the channel can be found by using the condition of constant flow rate of the medium in it (10):

$$\frac{dP}{dX}\int_0^1 F_1(X, Y)dY + 2\varepsilon Wi^2\left(\frac{dP}{dX}\right)^3\int_0^1 F_2(X, Y)dY = 1\,, \tag{18}$$

$$F_1(X, Y) = \int_1^Y \frac{Y}{\bar{\mu}}\, dY\ , \quad F_2(X, Y) = \int_1^Y \frac{Y^3}{\bar{\mu}^3}\, dY\ .$$

The posed problem is solved for a nonisothermal medium whose viscosity depends on temperature and pressure [25, 26]:

$$\bar{\mu} = \exp\,[\psi\theta + SP]\ , \tag{19}$$

where $\psi = b(T_w - T_0)$ and $S = \frac{s\mu_w \bar{v}_x}{h}$. It is apparent that in this case the mathematical model must be supplemented with an energy equation. The solution of the hydrodynamic problem is carried out simultaneously with determination of the temperature field in the flow of the medium.

Polymer melts and rubber mixtures have low thermal diffusivity; therefore the flow of such media occurs, as a rule, at high Péclet numbers (Pe > 100). This allows one in the energy equation of the flow of the medium to neglect its axial thermal conductivity in comparison with convective heat transfer in it. In this case, the energy equation of the flow of the medium with account for dissipation of this energy has the form

$$V_x \frac{\partial\theta}{\partial X} = \frac{1}{\text{Pe}} \frac{\partial^2\theta}{\partial Y^2} + \frac{\text{Br}}{\text{Pe}} \sigma_{xy} \frac{\partial V_x}{\partial Y}\ , \tag{20}$$

where $\text{Pe} = \frac{\bar{v}_x h}{a}$ and $\text{Br} = \frac{\mu_w \bar{v}_x^2}{k(T_w - T_0)}$. The bulk temperature of the flow of the medium in dimensional and dimensionless form is determined from the expressions

$$T_m = \frac{\int_0^h T v_x dy}{\int_0^h v_x dy}\ , \quad \theta_m = \int_0^1 \theta V_x dY\ . \tag{21}$$

The Nusselt number, which characterizes the local heat transfer on the channel wall, in dimensional and dimensionless form, is defined as

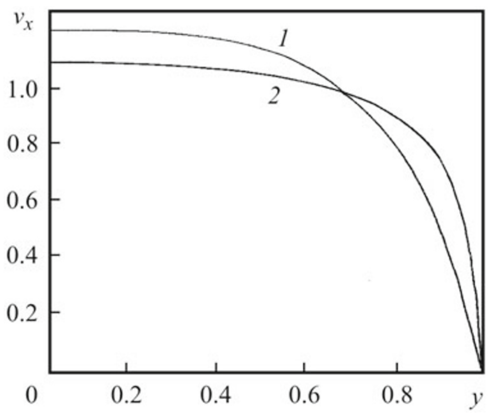
$$\text{Nu} = \frac{\alpha h}{k} = \frac{h}{|T_m - T_w|} \left(\frac{\partial T}{\partial y}\right)_{y=h}\ , \quad \text{Nu} = \frac{1}{\theta_m} \left(\frac{\partial\theta}{\partial Y}\right)_{Y=1}\ . \tag{22}$$

The posed problem was solved numerically by the iteration scheme with the use of the method of finite differences. From the onset, at the zero step of iteration, use was made of the velocity distribution of flow of the isothermal medium with a constant viscosity and of pressure gradient in it along the channel length:

$$V_x = \left(\frac{dP}{dX}\right) \frac{Y^2 - 1}{2} + \varepsilon \text{Wi}^2 \left(\frac{dP}{dX}\right)^3 \frac{Y^4 - 1}{2}\ , \tag{23}$$

$$\frac{1}{3}\left(\frac{dP}{dX}\right) + \frac{2}{5} \varepsilon \text{Wi}^2 \left(\frac{dP}{dX}\right)^3 + 1 = 0\ . \tag{24}$$

Discussion of Results. Figure 2 shows the transformation of the flow velocity profile of a viscoelastic fluid in a flat channel along its longitudinal coordinate at a fixed value of the Weissenberg number (also called the Deborah number in many works) and of the Brinkman number. It is known that the temperature dependence of the viscosity of such a liquid manifests itself most distinctly on the initial segment of its flow in the channel. At the inlet to the channel the flow velocity profile is fully developed and corresponds to a homogeneous temperature distribution, i.e., to the isothermal conditions. Thereafter the fluid flow velocity profile is transformed because of the dependence of its viscosity on temperature. The wall fluid layers are heated primarily due both to the hotter channel wall and to the fluid flow energy dissipation. This leads to a



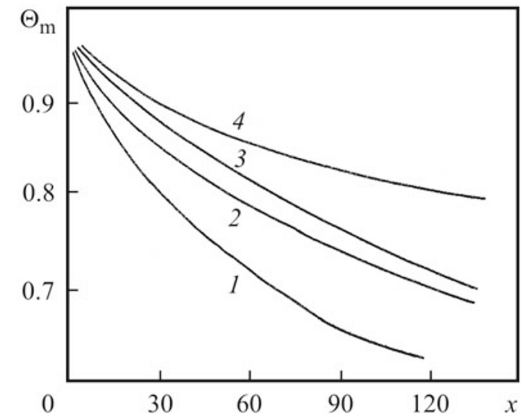


Fig. 2. Transformation of fluid flow velocity profile in the entry section of the channel at $\varepsilon Wi^2 = 0.1$ and Br = –3; 1) $x = 0$; 2) 15.

Fig. 3. Change of the mean mass fluid temperature along the channel length: 1) $b = 0.008$ deg^{-1}, $s = 0.7 \cdot 10^{-8}$ Pa^{-1}, Br = –3.0; 2) 0, 0, –3.0; 3) 0.008, 0, –3.0; 4) 0.008, 0, 0.

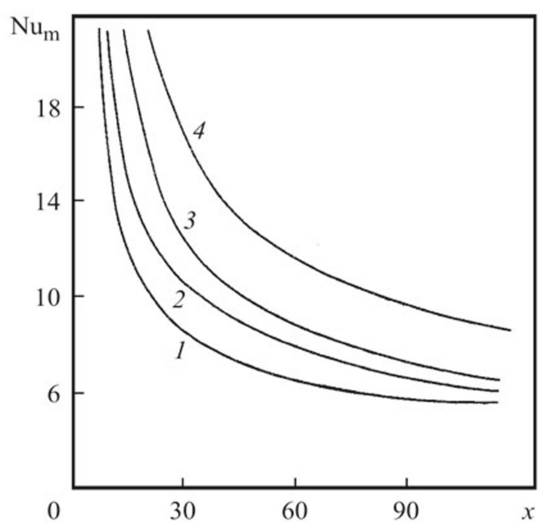


Fig. 4. Distribution of the local Nusselt number along the channel length: 1) $b = 0.008$ deg^{-1}, $s = 0.7 \cdot 10^{-8}$ Pa^{-1}, Br = –3.0; 2) 0, 0, –3.0; 3) 0.008, 0, –3.0); 4) 0.008, 0, 0.

decrease in the fluid viscosity in this region. As a result, the fluid flow velocity near the channel wall increases, whereas the fluid flow velocity in the middle of the channel decreases, and the flow velocity profile becomes planer. Thereafter, as the fluid moves along the channel, it is heated over the entire section of its flow, and the velocity profile of this flow is elongated again. As a result, at a great enough distance from the inlet into the channel the fluid velocity profile approaches the form typical of isothermal flow.

An analysis of the results presented in Figs. 3 and 4 allows evaluation of the effect of various factors on the bulk temperature of flow of a viscoelastic fluid in a plane channel and the local Nusselt number of this flow. In particular, it is seen that disregard of the dissipation of the fluid flow energy leads to an error that increases with the length of the channel. The graphs presented illustrate also the influence of the dependence of fluid viscosity on its temperature and pressure in it on the main characteristics of heat transfer during fluid flow in a channel. An increase in the fluid viscosity on pressure increase enhances the dissipative effect, which leads to its rapid heating.

Figure 5 shows the fluid temperature profiles at a certain distance from the inlet to the channel at fixed values of Br and Wi. A comparison of the curves presented in this figure allows one to determine the influence of the dependence of

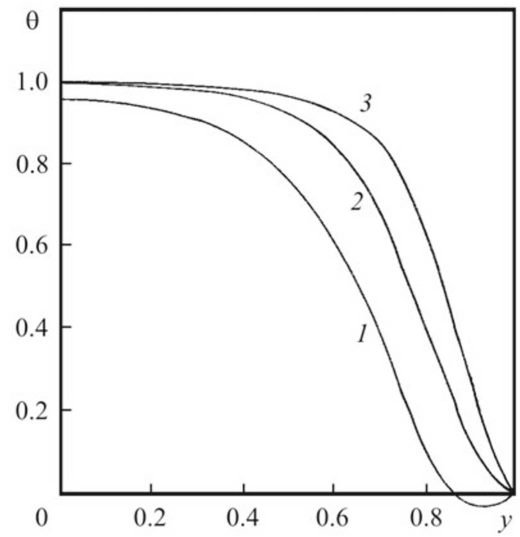


Fig. 5. Temperature profiles at a certain distance from the inlet into the channel at $x = 20$, $\varepsilon \mathrm{Wi}^2 = 0.1$, and Br = –3: 1) $b = 0$, $s = 0.7 \cdot 10^{-8}$ Pa–1; 2) 0, 0; 3) 0.008, 0.

the fluid viscosity on its temperature and pressure in it on the temperature field during its flow in the channel. It is seen that disregard of any of these factors leads to a noticeable error in the calculation of the fluid flow parameters. Account for the temperature dependence of the fluid viscosity leads to a reduction of the role of the flow energy dissipation in the process of fluid heating, whereas account for the dependence of the fluid viscosity on pressure, conversely, considerably enhances the dissipation effect.

Conclusions. The problem on nonisothermal flow of a viscoelastic polymer medium in a plane channel has been solved with the use of the Phan-Thien–Tanner rheological model, which allowed obtaining an expression for flow velocity in explicit form with account for the dissipative heat releases in the channel, as well as the dependence of the fluid viscosity on its temperature and pressure in it. The profiles of the fluid temperature and of the velocity of its flow have been calculated. A substantial effect of fluid flow energy dissipation, as well as of the fluid temperature and pressure in it, on the profile of its temperature and on the distribution of bulk fluid temperature and of the local Nusselt number of its flow along the channel is shown.

NOTATION

a, thermal diffusivity of fluid; b and s, empirical rheological constants; h, half-height of the channel; k, thermal conductivity of the fluid; p, pressure; T, fluid temperature; T_{m}, bulk fluid temperature in the given section of the channel; T_{w}, temperature of the channel wall; T_0, fluid temperature at the inlet to the channel; v_x and $\overline{v}_x$, axial flow velocity component and its mean value; x and y, longitudinal and transverse coordinates; α, coefficient of heat transfer on the channel wall; μ, fluid viscosity; μ_{w}, fluid viscosity at temperature T_{w}; Pe, Nu, Br, and Wi, Péclet, Nusselt, Brinkman, and Weissenberg numbers, respectively.

REFERENCES

1. A. V. Baranov, Nonisothermal flow of rheologically complex media under conditions of chemical transformations, *Mekh. Kompoz. Mater. Konstr.*, **16**, No. 3, 384–399 (2010).
2. P. J. Oliveira and F. T. Pinho, Plane contraction flow of upper convected Maxwell and Phan-Thien–Tanner fluids as predicted by a finite-volume method, *J. Non-Newtonian Fluid Mech.*, **88**, Nos. 1–2, 63–88 (1999).
3. M. Aboubacar, H. Matallah, and M. F. Webster, Highly elastic solutions for Oldroyd-B and Phan-Thien–Tanner fluids with a finite volume/element method: planar contraction flows, *J. Non-Newtonian Fluid Mech.*, **103**, No. 1, 65–103 (2002).
4. S. H. Hashemabadi, S. G. Etemad, and J. Thibault, Analytical solution for dynamic pressurization of viscoelastic fluids, *Int. J. Heat Fluid Flow*, **24**, 137–144 (2003).

5. M. S. Alves, P. J. Oliveira, and F. T. Pinho, Benchmark solutions for the flow of Oldroyd-B and PTT fluids in planar contractions, *J. Non-Newtonian Fluid Mech*., **110**, No. 1, 45–75 (2003).
6. D. O. Cruz and F. T. Pinho, Skewed Poiseuille–Couette flows of sPTT fluids in concentric annuli and channels, *J. Non-Newtonian Fluid Mech*., **121**, No. 1, 1–14 (2004).
7. S. H. Hashemabadi and S. M. Mirnajafizadeh, Analytical solution of simplified Than-Thien–Tanner fluid between nearly parallel plates of a small inclination, *J. Appl. Sci.*, **7**, No. 9, 1271–1278 (2007).
8. Cao Wei, Shen Changyu, and Zhang Chujie. Computing flow-induced stresses of injection molding based on the Phan-Thien–Tanner model, *Arch. Appl. Mech*., **78**, No. 5, 363–377 (2008).
9. R. J. Poole and M. A. Alves, Velocity overshoots in gradual contraction flows, *J. Non-Newtonian Fluid Mech.*, **160**, No. 1, 47–54 (2009).
10. T. Chinyoka, Poiseuille flow of reactive Phan-Thien–Tanner liquids in 1D channel flow, *Trans. ASME, J. Heat Transf*., **132**, No. 11, 111701/1–111701/7 (2010).
11. K. Yapici, B. Karasozen, and Yu. Uludag, Numerical analysis of viscoelastic fluids in steady pressure-driven channel flow, *Trans. ASME, J. Fluids Eng.*, **134**, No. 5, 051206/1–051206/9 (2012).
12. Luis L. Ferras, Joao M. Nobrega, and T. Pinho Fernando, Analytical solutions for channels flows of Phan-Thien–Tanner and Giesekus fluids slip, *J. Non-Newtonian Fluid Mech.*, **171–172**, 97–105 (2012).
13. D. V. Anan'ev and A. A. Kainova, Heat transfer and hydrodynamics in the Phan-Thien–Tanner laminar viscoelastic fluid flow in a convergent channel, *Izv. Ross. Akad. Nauk, Énergetika*, No. 5, 51–64 (2014).
14. D. V. Anan'ev, G. R. Khalitova, and U. K. Vachagina, Hydrodynamics and heat transfer in laminar viscoelastic fluid flow in a plane-slotted channel, *Teplofiz. Aéromekh*., **22**, No. 1, 49–60 (2015).
15. E. K. Vachagina, A. I. Kaidyrov, A. A. Kainova, and G. R. Khalitova, Viscoelastic fluid flow in a prismatic channel of square cross section considering rubber mixtures as an example, *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 1, 9–17 (2016).
16. A. V. Baranov and N. A. Anisimova, Rheological characteristic features of polymer compositions in the processing processes, *Mekh. Kompoz. Mater. Konstr*., **14**, No. 2, 188–206 (2008).
17. D. M. Binding, M. A. Couch, and K. Walters, The pressure dependence of the shear and elongational of polymer melts, *J. Non-Newtonian Fluid Mech.*, **79**, Nos. 2–3, 137–155 (1998).
18. Mehrdad Massoudi and Tran X. Phuoc, Unsteady shear flow of fluids with pressure-dependent viscosity, *Int. J. Eng. Sci.*, **44**, Nos. 13–14, 915–926 (2006).
19. R. R. Huilgol and Z. You, On the importance of the pressure dependence of viscosity in steady non-isothermal shearing flows, *J. Non-Newtonian Fluid Mech.*, **136**, Nos. 2–3, 106–117 (2006).
20. A. Kalogirou, S. Poyiadji, and G. C. Georgiou, Incompressible Poiseuille flows of Newtonian liquids with a pressure-dependent viscosity, *J. Non-Newtonian Fluid Mech.*, **166**, Nos. 7–8, 413–419 (2011).
21. A. Hirn, M. Lanzendorfer, and J. Stebel, Finite element approximation of flow of fluids with shear-rate-and pressure-dependent viscosity, *IMA J. Numer. Anal.*, **32**, No. 4, 1604–1634 (2012).
22. Kostas D. Housiadas, An exact analytical solution for viscoelastic fluids with pressure-dependent viscosity, *J. Non-Newtonian Fluid Mech.*, 223, 147–156 (2015).
23. A. Mahdavi Khatibi, M. Mirzazadeh, and F. Rashidi, Forced convection heat transfer of Giesekus viscoelastic fluid in pipes and channels, *Heat Mass Transf.*, **46**, 405–412 (2010).
24. Hadi Mokarizadeh, Maedeh Asgharian, and Ahmadreza Raisi, Heat transfer in Couette–Poiseuille flow between parallel plates of the Giesekus viscoelastic fluid, *J. Non-Newtonian Fluid Mech.*, **196**, 95–101 (2013).
25. N. V. Tyabin, O. Kh. Dakhin, and A. V. Baranov, Effect of temperature and pressure on rheological complex media flow in a plane channel, *Teplofiz. Vys. Temp*., **20**, No. 1, 81–86 (1982).
25. N. V. Tyabin, O. Kh. Dakhin, and A. V. Baranov, Conjugated heat transfer in the flow of a non-Newtonian fluid with variable properties in a flat duct, *J. Eng. Phys. Thermophys*., **45**, No. 3, 979–983 (1985).