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SEMIEMPIRICAL MODEL OF CONVECTIVE HEAT TRANSFER OF TURBULENT GASES

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On the basis of the Prandtl semiempirical wall-turbulence hypothesis, the author has substantiated theoretically the possibility of setting boundary conditions to the equations of a mathematical model of turbulent motion and convective heat transfer of a gaseous medium on a coarse grid. It has been shown that the results of numerical simulation are in agreement with experimental data on the heat transfer of air and high-temperature gases in tubes.

Keywords: *mathematical model, heat transfer of gases, boundary layer, turbulence, semiempirical hypothesis, adequacy.*

Introduction. At the present time, to investigate engineering problems, use is made of mathematical models consisting of differential equations of turbulent motion of a gaseous medium and of convective and radiative heat transfer [1]. Their numerical solution is implemented by numerous iterations on grids containing hundreds of thousands of nodes in three-dimensional models, since fairly small dimensions of grid cells are selected to obtain reliable results.

The grid near the wall surface is refined to the greatest extent [2] so as to find, with an acceptable degree of accuracy, the derivatives of the velocity and temperature of gases, which are necessary for setting boundary conditions, upon the linear variation in the quantities. Here, the model of turbulent viscosity of the gases in the medium's wall layer is corrected by means of empirical functions [3].

Meanwhile, grid cells in the wall region can multiply be enlarged by applying, to the nonisothermal boundary layer, the Prandtl semiempirical wall-turbulence hypothesis determining the stress of a friction force which is created by turbulent vortices:

$$\sigma = \rho(l\,du/dy)^2 \ .$$

Smooth-Wall Boundary Condition to the Equations of Motion of Nonisothermal Gases. In the wall region of a turbulent boundary layer, we usually single out a thin viscous sublayer adjacent to the wall surface, in which turbulent viscosity dominates molecular one, and an equilibrium sublayer in which the generation and dissipation of kinetic turbulence energy balance each other. Between them, there is a buffer sublayer.

In a thin nonisothermal layer of gases which is adjacent to the wall, the sum of the molecular and turbulent stresses may be set equal to the shear friction stress σ_w on the wall surface:

$$\sigma_w \equiv \rho_w u_*^2 \approx \rho\nu du/dy + \rho l^2 (du/dy)^2 \ ,$$

where ρ_w is the density of the gases at the wall temperature, and u_* is the dynamic viscosity which replaces shear stress in the formulas. Determining the dimensionless variables by the relations of the quantities

$$u_+ \equiv u/u_* \ , \quad y_+ \equiv yu_*/\nu_w \ , \quad l_+ \equiv lu_*/\nu_w \ ,$$

we reduce this equality to a dimensionless form:

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$$\frac{\nu_w}{\nu}\, l_+^2 (du_+/dy_+)^2 + du_+/dy_+ - \frac{\rho_w \nu_w}{\rho \nu} = 0\ ,$$

where ν_w is the coefficient of viscosity of the gases at the wall temperature.

Solution of the quadratic equation gives an expression for the derivative of the dimensionless velocity u_+, which will be written formally as the inverse proportionality to the mixing length:

$$du_+/dy_+ = f_u \sqrt{\rho_w/\rho}/l_+\ , \tag{1}$$

where f_u is the theoretical function approaching unity away from the wall at great values of the mixing length

$$f_u = \sqrt{1 + \frac{1}{4l_+^2}\frac{\nu^2}{\nu_w^2}\frac{\rho}{\rho_w}} - \frac{1}{2l_+}\frac{\nu}{\nu_w}\sqrt{\frac{\rho}{\rho_w}}\ .$$

In the particular case of an isothermal boundary layer these formulas are transformed to simpler expressions:

$$du_+^{\text{iso}}/dy_+ = f_u^{\text{iso}}/l_+\ , \quad f_u^{\text{iso}} = \sqrt{1 + 1/(4l_+^2)} - 1/(2l_+)\ . \tag{2}$$

The superscript "iso" marks the quantities in the isothermal medium.

With account of equality (1), the Prandtl hypothesis yields a dimensionless expression for the relation of the coefficients of turbulent and molecular viscosity in the gases′ wall layer:

$$\nu_{\text{turb}}/\nu_w = l_+ f_u \sqrt{\rho_w/\rho}\ . \tag{3}$$

Formulas (1)–(3) become useful in modeling mathematically turbulent gas motion, if the dimensionless mixing length l is known. It is determined by the Van Driest exponential formula [4] which takes account of the attenuation of turbulence near the wall

$$l_+ = [1 - \exp(-\,y_+/A_+)]\,\kappa y_+\ . \tag{4}$$

The configuration factor A_+ has been found [4] from the condition that the calculated values of the dimensionless velocity near the wall, on the average, agree with experiments. However, because of the great scatter in experimental data, the value $A_+ = 26$ assigned to it turned out to be insufficiently exact for using this formula in mathematical models.

A more efficient method to determine the factor A_+, which lies in calculating its value directly from each experimental value of dimensionless velocity, was implemented in [5]. Mathematical processing of the obtained values has shown that near a smooth wall, the factor A_+ varies by a quadratic law which is expediently represented in the following generalized form:

$$A_+ \approx 30\left[1 - \left(y_+/y_+^{\log}\right)^2\right].$$

The constants $\kappa = 0.41$ and $y_+^{\log} \approx 50$ have been found from the experimental velocity distribution near the wall in the case of developed turbulence.

A refined exponential dependence (4) with quadratic variation in the factor A_+ is smoothly conjugate to the Prandtl linear law for the mixing length at the dimensionless distance from the wall $y_+ = y_+^{\log}$ where a formal transition to a logarithmic law of variation in the velocity occurs. In fact, the logarithmic law begins to act at $y_+ \approx 0.7 y_+^{\log}$. Thus, it may be assumed that at the values of the dimensionless coordinate y_+ within (0.7–1) $y_+^{\log}$, a transition region is formed which formally belongs to the buffer sublayer but possesses properties characteristic of the equilibrium sublayer of the turbulent boundary layer.

At the known mixing length l_+, it becomes possible to approximate the right-hand side of differential expression (2) by a sufficiently exact polynomial dependence and to integrate it. Thus, we have obtained an eighth-degree polynomial describing the variation in the dimensionless velocity of isothermal gases in the viscous and buffer sublayers [6]:

$$u_+^{\text{iso}} = a_0 + a_1 y_+ + a_2 y_+^2 + a_3 y_+^3 + a_4 y_+^4 + a_5 y_+^5 + a_6 y_+^6 + a_7 y_+^7 + a_8 y_+^8\ . \tag{5}$$

TABLE 1. Coefficients of Polynomial (5) to Compute Dimensionless Velocity

y_+	a_0	a_1	$10^4 a_2$	$10^4 a_3$	$10^4 a_4$	$10^6 a_5$	$10^8 a_6$	$10^{10} a_7$	$10^{12} a_8$
0–11	0	1	1	–10.6	7.18	–248	2990	–15,870	32,000
11–50	–2.58	1.986	–1296	55.4	–1.57	2.9135	–3.395	2.255	–0.6528

Its coefficients pertaining to developed turbulence are presented in Table 1. In the equilibrium sublayer at $l_+ > 50$, the dimensionless velocity of isothermal gases varies logarithmically:

$$u_+^{\text{iso}} = \frac{1}{0.41} \ln y_+ + 5.1 .$$

We take into account that at the origin of coordinates, the derivative of the dimensionless velocity $(du_+/dy_+)_\text{w}$ and the ratio of the dimensionless velocity u_+ to the coordinate y_+ are equal to unity. With this condition, it becomes possible to find a one-sided derivative of the velocity of isothermal gases on the wall from its discrete value u_P/y_P, using the rule of calculation of an unknown quantity from its known function $(u_+/y_+)_P$ at the wall node P of the grid:

$$(du/dy)_\text{w}^{\text{iso}} = \frac{u_P}{y_P} (y_+/u_+)_P^{\text{iso}} . \tag{6}$$

The same approach to formulating boundary conditions in the nonisothermal boundary layer is more difficult to implement, since in the general case the variation in the dimensionless velocity of the nonisothermal medium near the wall remains unknown. In this connection, consideration will be given to the possibility of applying the singularities of an isothermal wall layer to nonisothermal conditions of cooling of turbulent gases. Using relation (2), we introduce, into the right-hand side of formula (1), the derivative of the isothermal gases

$$\frac{du_+}{dy_+} = \frac{du_+^{\text{iso}}}{dy_+} \sqrt{\frac{\rho_\text{w}}{\rho}} \frac{f_u}{f_u^{\text{iso}}} .$$

We carry out formal integration of the resulting equality:

$$u_+ = u_+^{\text{iso}} \left\langle \sqrt{\rho_\text{w}/\rho}\ f_u \big/ f_u^{\text{iso}} \right\rangle .$$

In the angular brackets is a group of quantities averaged on the segment of integration from the wall to the nearest wall node P of the grid.

Applying the obtained results to the grid′s wall node P, we transform the dimensionless relation $(y_+/u_+)_P$ into the factor m_u changing the discrete relation u_P/y_P to a one-sided derivative of the velocity of isothermal gases on the wall

$$m_u \equiv \left(\frac{y_+}{u_+}\right)_P = \left(\frac{y_+}{u_+}\right)_P^{\text{iso}} \left\langle \sqrt{\rho/\rho_\text{w}}\ f_u^{\text{iso}} \big/ f_u \right\rangle . \tag{7}$$

Using this factor, we write a formula for calculating, on the wall, the one-sided derivative of the velocity of nonisothermal gases:

$$\left(\frac{du}{dy}\right)_\text{w} = \frac{u_P}{y_P} \left(\frac{y_+}{u_+}\right)_P = \frac{u_P}{y_P} m_u . \tag{8}$$

Expression (8) generalizes formula (6) obtained for isothermal gases.

Semiempirical Model of Convective Heat Transfer of Turbulent Gases to a Smooth Wall. In considering heat transfer in the turbulent wall layer, use is made of the notion of the medium′s dimensionless temperature [7]

$$T_+ \equiv \rho_w c_w u_* (T - T_w)/q_w ,$$

where ρ_w and c_w are the density and heat capacity of the gas at the wall temperature, T_w and T are the temperatures of the wall and of the gaseous medium near it respectively, and q_w is the density of the heat flux on the wall surface.

To find the relation of the dimensionless quantities $(T_+/y_+)_P$ at the grid node P, we take into account that in the boundary layer, convective heat transfer to the wall is usually disregarded. As a result, a differential expression for the density of the conductive heat flux near the wall is represented in the following form:

$$q_w = \rho c \left(\frac{\nu}{\mathrm{Pr}} + \frac{\nu_{\mathrm{turb}}}{\mathrm{Pr}_{\mathrm{turb}}} \right) \frac{dT}{dy} .$$

Hence, assuming the heat capacity of the gases in the wall layer to be constant, we obtain the dimensionless equality

$$1 = \frac{\rho}{\rho_w} \left(\frac{1}{\mathrm{Pr}} \frac{\nu}{\nu_w} + \frac{1}{\mathrm{Pr}_{\mathrm{turb}}} \frac{\nu_{\mathrm{turb}}}{\nu_w} \right) \frac{dT_+}{dy_+} .$$

On its right-hand side, we drop the first term whose value in the equilibrium sublayer and in most of the buffer sublayer is smaller than that of the second term. Using then formula (3) for the turbulent viscosity, we obtain the approximate expression of the derivative of the dimensionless temperature

$$dT_+/dy_+ \approx \mathrm{Pr}_{\mathrm{turb}} \sqrt{\rho_w/\rho} \Big/ (f_u l_+) .$$

A comparison of the last equation and formula (1) shows that at a certain distance from the wall, there is an approximate proportionality between the dimensionless derivatives of the temperature and velocity of nonisothermal gases:

$$\frac{dT_+}{dy_+} \approx \frac{\mathrm{Pr}_{\mathrm{turb}}}{f_u^2} \frac{du_+}{dy_+} .$$

One might expect that after the integration of the obtained equality, the proportionality will hold for the dimensionless values of temperature and velocity, at least, in the equilibrium sublayer and in the adjacent exterior part of the buffer sublayer:

$$T_+ \approx \mathrm{Pr}_{\mathrm{turb}}\, u_+ \Big/ \left\langle f_u^2 \right\rangle . \tag{9}$$

This assumption is confirmed by experimental data. Figure 1 gives, in dimensionless form, results of measuring the air temperature and velocity in a turbulent boundary layer near the plate [7, Table 12]. Curve 1 is drawn through the points of the dimensionless velocity u_+. Multiplying by 0.76 changes it to curve 2 which is coincident with the values of the dimensionless temperature T_+ in the equilibrium sublayer and in most of the buffer sublayer.

Experimental values of the dimensionless temperature T_+, which have been divided by the Prandtl number of the air correspond to the points through which curve 3 is drawn. In the region of the viscous sublayer, it is coincident, in practice, with the distribution of the dimensionless velocity. This means that the derivative of the dimensionless function T_+/Pr is equal to unity at the origin of coordinates. Consequently, the ratio of this function to the dimensionless distance y_+ may be used in calculating a one-sided gas-temperature derivative on the wall from its discrete value determined at the nearest node P of the grid:

$$\left(\frac{dT}{dy} \right)_w = \frac{T_P - T_w}{y_P} \left(\frac{y_+ \mathrm{Pr}}{T_+} \right)_P .$$

Replacing the dimensionless temperature T_+ by the dimensionless velocity u_+ according to the approximate equality (9) and using definition (7) of the factor m_u, we obtain a formula suitable for the one-sided temperature derivative with the arrangement of the wall nodes of the grid in the equilibrium sublayer and in most of the buffer sublayer alike:

$$\left(\frac{dT}{dy} \right)_w = \frac{\mathrm{Pr}}{\mathrm{Pr}_{\mathrm{turb}}} \frac{T_P - T_w}{y_P} \left\langle f_u^2 \right\rangle m_u . \tag{10}$$

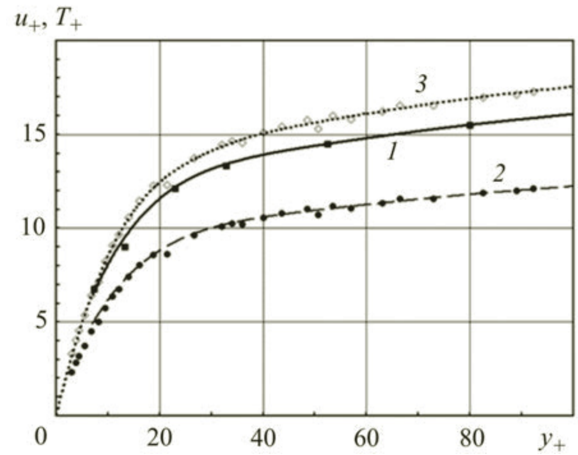

Fig. 1. Dimensionless velocity and temperature of the air near the wall: 1) velocity u_+, 2) temperature T_+, and 3) function T_+/Pr; points, experimental data [7].

The density of the convective heat flux from the turbulent medium to the wall will be determined under the conventional assumption that there is no convection directly on a solid surface and hence the transfer of heat in the thin wall layer of the heat-transfer agent is only by heat conduction according to the Fourier law:

$$q_{\mathrm{w}} = \rho_{\mathrm{w}} c_{\mathrm{w}} \nu_{\mathrm{w}} (dT/dy)_{\mathrm{w}} /\mathrm{Pr} .$$

Substitution of expression (10) into this equation leads to a formula of the semiempirical model of convective heat transfer in the turbulent wall layer of nonisothermal gases:

$$q_{\mathrm{w}} = \frac{\rho_{\mathrm{w}} c_{\mathrm{w}} \nu_{\mathrm{w}}}{\mathrm{Pr}_{\mathrm{turb}} y_P} (T_P - T_{\mathrm{w}}) \left\langle f_u^2 \right\rangle m_u . \quad (11)$$

Discussion. To simplify a computational algorithm, in the mathematical model we used the wall-transfer factor ζ determining boundary conditions on the wall

$$\zeta \equiv \frac{\nu_{\mathrm{w}}}{u_P} \left(\frac{\partial u}{\partial y}\right)_{\mathrm{w}} ,$$

where u_P is the velocity at the wall node P of the grid. Clearly, it is proportional to the dynamic velocity squared:

$$\zeta = u_*^2 / u_P . \quad (12)$$

Taking into account expression (8) for the one-sided velocity derivative, we find the discrete form of the wall-transfer factor

$$\zeta = \nu_{\mathrm{w}} m_{u,P} / y_P . \quad (13)$$

In the course of the iterations, we calculated the dynamic velocity u_* according to formula (12) from the approximate value of the factor ζ. Next, we determined the dimensionless coordinate y_+ for the wall node P of the grid, calculated the factor m_u according to formula (7), and refined the value of the wall-transfer factor ζ by relation (13).

In implementing the mathematical model numerically, we replaced the averaged physical quantities in formulas (7), (10), and (11) by their local values found at grid nodes adjacent to the wall, assuming that possible errors would be eliminated using an empirical correction.

If the factor m_u in Eq. (11) is replaced by its expression following from equality (13), the calculated formula of convective heat transfer, with account of what has been said above, takes on the following calculated form:

$$q_{\mathrm{w}} \approx \zeta \frac{\rho_{\mathrm{w}} c_{\mathrm{w}}}{\mathrm{Pr}_{\mathrm{turb}}} (T_P - T_{\mathrm{w}}) f_{u,P}^2 g_{m,P} , \quad (14)$$

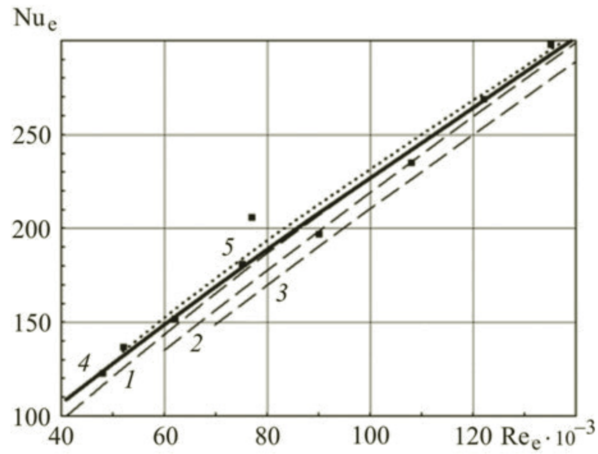

Fig. 2. Results of modeling of the convective heat transfer of gases without correcting equations (1–3) and with an empirical correction (4, 5) at different radial grid steps: 1 and 4) 2.5 mm, 2) 1.5 mm, and 3 and 5) 1 mm; points, experiment [8].

where $g_{m,P}$ is the empirical correction function which is defined at the grid node P adjacent to the wall:

$$g_{m,P} = [1 - \exp(-y_{+P}/A_m)]^{-0.5} . \qquad (15)$$

Here the factor A_m is calculated according to the formula

$$A_m = 18[1 - (y_{+P}/100)^2] .$$

The semiempirical model of connective transfer of heat was checked by comparing the results of numerical modeling and the Tamonis data [8, Table 5] on the heat transfer of a turbulent air flow at a distance of 3.52 m from the beginning of the experimental portion of a tube of diameter 150 mm. The average calculated air temperature in the cross section of the tube was 410–425 K at a wall temperature of 296 K. The turbulent Prandtl number was taken to be 0.85. For a radial step of the grid from 1 to 2.5 mm, grid nodes next the wall were beyond the viscous sublayer ($y_+ > 11$).

The conditions of motion and cooling of the air before the experimental tube have not been rigorously determined in [8], but it is noted that heat transfer in the experimental portion is close to a stabilized state in which the formation of dynamic and thermal boundary layers is known to be completed. In this connection, for a fuller agreement with experimental conditions, a topping two-meter cooled portion of the tube is provided in the mathematical model, along which a boundary layer is formed under the conditions of unstabilized motion and heat transfer.

The relation between experimental Nusselt Nu_e and Reynolds numbers Re_e determined in [8] from the values of physical quantities on the tube axis and from the heat flux to cooled walls is shown in Fig. 2 as points. The results of numerical modeling are presented by curves 1–5, with the first three of them being obtained without an empirical correction function in the equation of convective heat transfer (14).

Curve 1 calculated with a radial step of the grid of 2.5 mm and a dimensionless coordinate of wall grid nodes $25 < y_{+P} < 84$ is, on the whole, in satisfactory agreement with experimental data. Nonetheless, as might be expected, the portion of this curve that corresponds to the location of wall nodes in the buffer sublayer lies somewhat lower than the group of experimental points. Curves 2 and 3 calculated with a radial grid step equal to 1.5 mm ($16 < y_{+P} < 51$) and 1 mm ($11 < y_{+P} < 34$) respectively lie much lower than the experimental points of the Nusselt number.

The use of the empirical correction function (15) with a radial step of the grid of 2.5 mm converts curve 1 in Fig. 2 into dependence 4 between the Nusselt and Reynolds numbers which is coincident well with experimental points. Here, curve 3 obtained with a grid step of 1 mm is transformed into dashed line 5 differing little from dependence 4.

The adequacy of the semiempirical model with Eqs. (11)–(15) has also been confirmed by a comparison, in Fig. 3, of the results of numerical modeling and Tamonis′s data on radiative-convective heat transfer of high-temperature gases [8, Table 7]. The temperature of the cooled gases was measured on the axis of the experimental portion of a tube of diameter 150 mm at a temperature of its wall of 296 K. The group of experimental points along curves 1–3 corresponds to the lower-

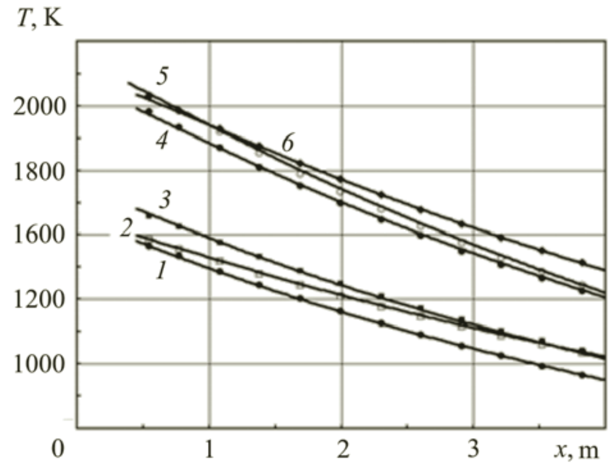

Fig. 3. Temperature on the axis of the experimental tube with a flow rate of the gases of: 1) 473, 2) 720, 3) 570, 4) 627, 5) 511, and 6) 839 kg/h; points, experiment [8], lines, numerical modeling.

than-average content of carbon dioxide (5–6%) and of steam (11–12%). Points along curves 4–6 correspond to the higher content of triatomic gases.

In this case the presence of the topping cooled portion, along which a boundary layer is formed under the conditions of unstabilized motion and heat transfer, is provided, as previously, in the mathematical model. Numerical modeling of the heat transfer to tube walls was carried out with the differential model of radiative heat transfer in selective gases [9], which, with a radial step of the grid of 1.5 mm, corresponded to the dimensionless distance to the grid nodes next to the wall $y_+ = 31–75$.

The results are presented by curves 1–6 which are in good agreement with experimental points as a rule. Of prime importance is the coincidence of the calculated and experimental dependences which demonstrates the identical rate of cooling of the gas flow along the tube length and hence the identical intensity of transfer of heat in the experimental and numerical methods of modeling.

According to Eq. (14), the heat flux of convective heat transfer is dependent on the turbulent Prandtl number whose value is known to be in the range of 0.7 to 1. Curve 2 in Fig. 3 corresponds to the mean value $\mathrm{Pr}_{\mathrm{turb}} = 0.85$. The remaining temperature curves have been obtained at a turbulent Prandtl number varying linearly from 0.7 to 0.8 in the transition region from the equilibrium sublayer to a buffer sublayer (at $50 > y_+ > 35$). This enabled us to bring them in better coincidence with the experimental points, bearing in mind that with one and the same radial step of the grid, the dimensionless distance from the wall to the grid node next to it decreases downstream with the gas flow.

CONCLUSIONS

1. The Prandtl semiempirical wall-turbulence model has been applied to a turbulent boundary layer of nonisothermal gases. It has been shown that on the basis of this model, boundary conditions to the equations of motion can be set using a coarse grid near the wall.
2. The approximate proportionality has been established between the dimensionless derivatives of the gas temperature and velocity in the equilibrium sublayer of the turbulent boundary layer and in most of the buffer sublayer. This made it possible to substantiate a semiempirical convective-heat-transfer model that requires no refinement of the grid near the wall surface.
3. The adequacy of the semiempirical model has been confirmed by the agreement between the results of numerical modeling and the experimental data on the heat transfer of turbulent flows of air and high-temperature gases to the cooled walls of the experimental tube.

NOTATION

A_+, configuration factor; c, specific mass heat at constant pressure, J/(kg·K); d, inside diameter of a tube, m; f, theoretical function; g, empirical function; l, mixing length, m; m, factor to the discrete velocity derivative; Nu, Nusselt similarity number; Pr, Prandtl similarity number; Pr_{turb}, turbulent Prandtl number; q, heat-flux density, $W/(m^2 \cdot K)$; Re, Reynolds similarity number; T, thermodynamic temperature, K; u, longitudinal velocity, m/s; u_*, dynamic velocity, m/s; x, distance along the length of the experimental portion, m; y, normal distance to the wall, m; ζ, wall-transfer factor, m/s; κ, universal constant; ν, kinematic coefficient of viscosity, m^2/s; ρ, gas density, kg/m^3; σ, shear stress, Pa. Subscripts and superscripts: iso, isothermal; e, experiment; log, logarithmic; m, correction; w, on the wall; turb, turbulent; P, at the grid node P; u, for velocity; t, at temperature; +, dimensionless quantity.

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