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ANALYSIS OF POLYMETHYLMETHACRYLATE DESTRUCTION ON HIGH-SPEED LOADING

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A system of equations is presented for describing the shock-compressed state of polymethylmethacrylate in the approximation of uniaxial deformation. With the use of the Prandtl–Reuss model, the mechanism of transition of the brittle failure of polymethylmethacrylate to the brittle-plastic one at high speeds of the striker and high times of its effect on this mechanism has been determined.

Keywords: *shock dynamic compression, shock adiabat, polymethylmethacrylate, polyethylene, high-speed impact.*

Dynamic compression of a condensed material under the action of a high-speed impact, for example, a laser pulse, may lead to the formation of a highly dense energy flux in it, origination of high temperature gradients in the material and, as a consequence, to the change of its phase state [1, 2]. Polymer materials may undergo substantial structural changes under pulse loading depending on its speed and magnitude and on the material structure. High-power pulse effect causes detwisting and reorientation of molecular chains in such a material and redistribution of molecular segments in it between its ordered and disordered parts, and on exposure to extreme effects destruction of the material may occur or its transition to a plasma state.

At the present time, the properties of many substances (metals, ion crystals) under extreme conditions have been investigated in detail. The results of such investigations are used as a basis for constructing the state diagrams of specific bodies and of their shock adiabats. Less studied are high-molecular compounds representing an important class of substances possessing unique physical properties and having complex phase diagrams. The structural formulas of polymers are complex, which makes it difficult to calculate their thermodynamic properties by the methods of quantum statistics. Such substances include polymethylmethacrylate (PMMA), which is widely used as a structural material in nanotechnologies and tests associated with explosions.

To study the processes proceeding on pulse loading of polymer materials, special devices have been developed for speeding-up polymer macroparticles and creating high dynamic pressures in it. The experiments, whose results are discussed in the present work, were carried out on a magneto-plasma accelerator of railotron type [1–3]. Using the high-speed phototraces, we calculated the dependences of the depth of a crater in the PMMA on the velocity and time of action of a polyethylene striker on it (Fig. 1). Based on the data presented in Fig. 1, with the use of Gault–Heitowit′s model [4], the dependence of the compression stress of PMMA on the time of polyethylene striker action and depth of its penetration $\sigma = \sigma(h, t)$ at the striker velocity $v = 3.0$ km/s has been calculated. The dependence obtained is presented in a three-dimensional form in Fig. 2. It is seen that at the initial moment the compression stress is distributed over the surface of a triangular pyramid, which is followed, at medium values of h and t, by the twisting of the material surface, and this stress decreases to a minimum value. At the limiting values of h and t, the PMMA surface over which the compression stress is distributed increases sharply, and the material acquires the shape of an irregular quadrilateral pyramid. An analysis of the data presented in Figs. 1 and 2 has shown that at rates of loading above $v > 3$ km/s and loading duration $t > 20$ μs the brittle destruction of PMMA passes to brittle-plastic or plastic destruction.

For describing the state of the shock-compressed PMMA and explaining the change in the mechanism of its destruction, a system of equations of the continuum mechanics was used. We consider a shock-compressed body in the approximation of its uniaxial deformation. For describing the dynamics of such elastoplastic medium we use the Prandtl–Reuss model [5]. This model ignores the relationship between the internal energy of the medium and time of its relaxation

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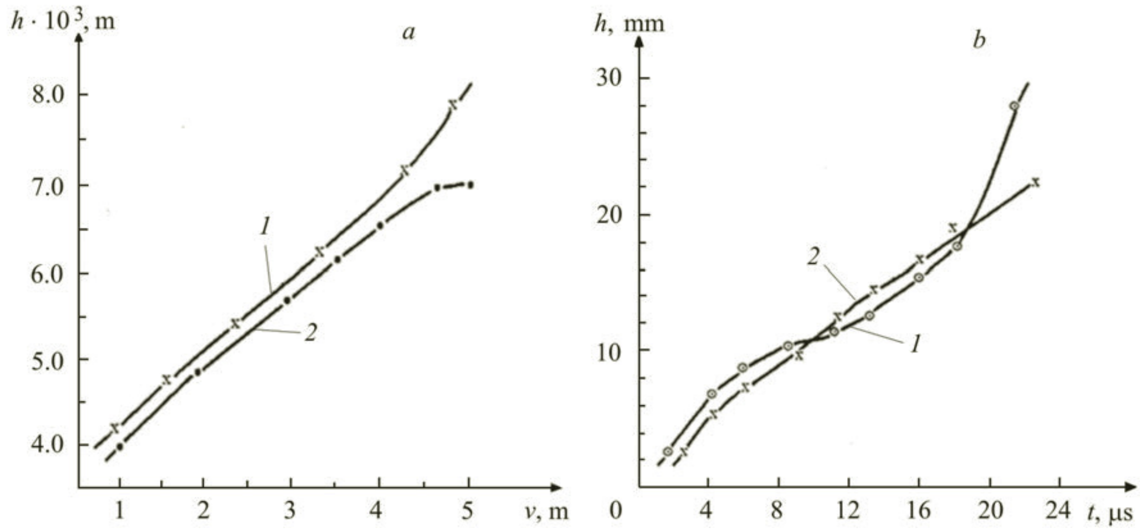

Fig. 1. Experimental (1) and theoretical (2) dependences of the crater depth in the PMMA target on the velocity (a) and time (b) of effect of the polyethylene striker on it: a) calculation by the equation $h = dAv^{2/3}$; b) calculation by the equation $t = 2.94\,\dfrac{h}{v}$.

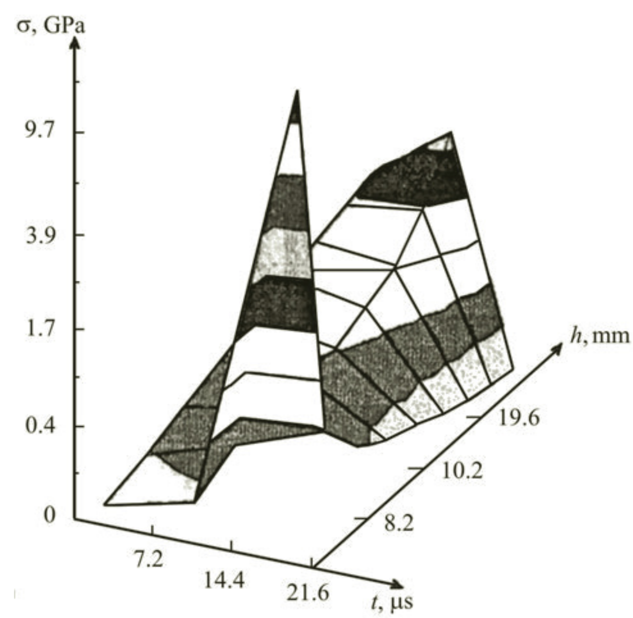

Fig. 2. Dependence of the compression stress of PMMA on the time of its loading and depth of the crater formed.

with the parameters characterizing the state of the medium, even though it is known that, for adequate description of the real properties of polymer materials, of fundamental importance is account for the relaxation processes occurring in them when they are subjected to external effects. In [6], such an account is made through the dependence of the time of equilibration of shear stresses in the material on the parameters of its state.

Let us write the system of equations of the continuum mechanics for the considered material in one-dimensional approximation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0\,, \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + p)}{\partial x} = \frac{\partial S}{\partial x}, \tag{2}$$

$$\frac{\partial(\rho\varepsilon)}{\partial t} + \frac{\partial((\rho\varepsilon + p)u)}{\partial x} = \frac{\partial(Su)}{\partial x}. \tag{3}$$

This system consists of the equations of continuity (1), motion (2), and energy (3), where S is the stress deviator, $\varepsilon = i + \frac{u^2}{2}$ is the total specific energy per unit mass, p is the pressure, ρ is the density of the medium, and i is its specific internal energy. The system of equations (1)–(3) must be supplemented with the equation for the stress deviator S. For coupling the medium stress deviator with the medium deformation, we will avail ourselves of the equation of elastoplastic model with the Mises plasticity condition [5]:

$$\frac{\partial(\rho S)}{\partial t} + \frac{\partial(\rho S u)}{\partial x} = \frac{4}{3}\mu\tilde{n}\frac{\partial u}{\partial x}. \tag{4}$$

Integrating (4) in the ranges from S_0 to S and from ρ_0 to ρ and taking into account the continuity equation (1), we obtain

$$S(\rho) = S_0 - \frac{4}{3}\mu \ln\left(\frac{\rho}{\rho_0}\right), \tag{5}$$

where

$$\rho = \rho_0 \exp\left(\frac{2L + 3S_0}{4\mu}\right). \tag{6}$$

System (1)–(6) is closed by the Mie–Grüneisen equation of state [1]:

$$\rho = \rho_0 a_0 f(x) + \rho_0 \Gamma i, \quad f(x) = \frac{(x-1)\left(x - \frac{1}{2}\Gamma_0(x-1)\right)}{[x - \varsigma(x-1)]^2}, \tag{7}$$

where $x = \rho/\rho_0$ is the degree of compression of the medium, a_0 is the speed of sound in an insufficiently deformed material, Γ_0 is the macroscopic Grüneisen function, and ς is a constant that relates the velocity of the shock wave D to the mass velocity of the medium u: $D = a_0 + \varsigma u$.

We will derive an analytical solution of these equations for a strong shock wave propagating in a material with constant velocity D. In describing an elastoplastic medium with the use of the above-given classical model, we ignored the processes associated with the appearance and accumulation of microdamages in the medium. It has been established in experiments that at striker velocities above $v = 3.0$ km/s (strong discontinuity), the relationship between the parameters of the medium on both sides of the discontinuity correspond to the Rankine–Hugoniot conditions:

$$[\bar{\mathbf{F}}] = D\bar{\boldsymbol{\theta}}, \tag{8}$$

where $\bar{\mathbf{F}}$ is the energy flux vector and $\bar{\boldsymbol{\theta}}$ is the vector of conservative variables [5]. Designating the intensity of mass rate of flow of the substance through the discontinuity surface by $\dot{M} = \rho(u - D) = \rho_0(u_0 - D)$, we will write analytical expressions for the two curves that represent the state of the medium behind the shock wave front:

$$i - i_0 = \frac{1}{2}(V - V_0)(\sigma + \sigma_0), \quad V = \frac{1}{\rho}, \tag{9}$$

$$\sigma - \sigma_0 = \dot{M}^2(V - V_0), \tag{10}$$

where $\sigma = -p + S$. Equation (9) represents the Hugoniot adiabat (HA), and Eq. (10) represents the Rayleigh–Michelson line (RM).

It is known that the relative positions of the Hugoniot adiabat and the RM line depend on the magnitude of the mass flow rate $\dot{M}$ and that their intersection point in the plane P, V corresponds to the pressure and specific volume of

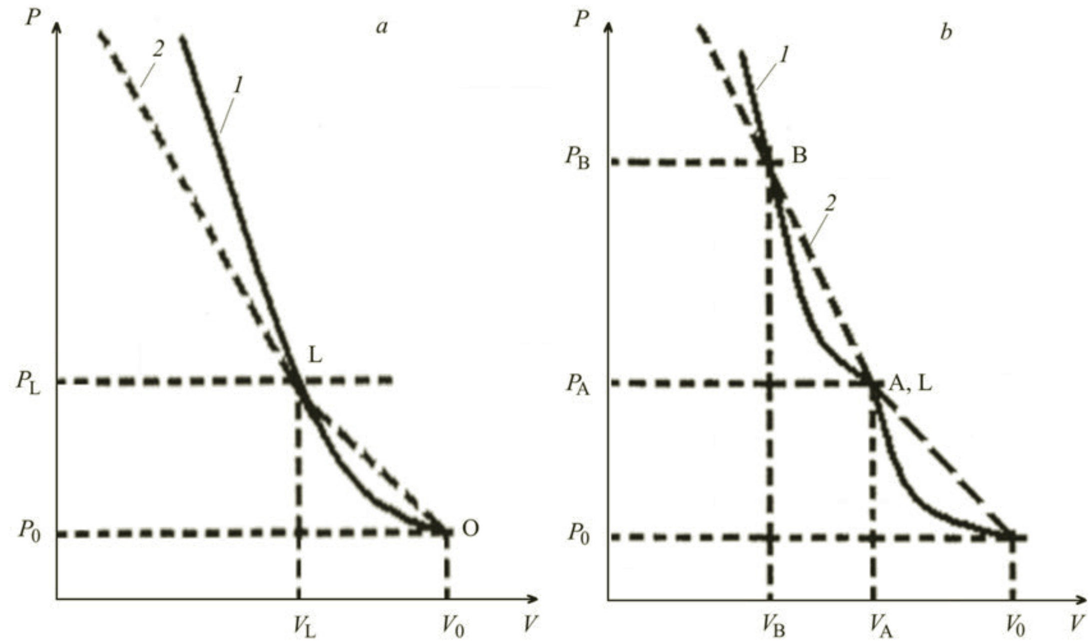

Fig. 3. Location of the Hugoniot adiabat (1) and of the Rayleigh–Michelson line (2) at $\dot{M} < \dot{M}_L$ (a) and $\dot{M} = \dot{M}_L$ (b).

the compressed medium behind the shock wave front ($\dot{M} > 0$). The determining point for the indicated two curves is the point $V = V_L$ (Fig. 3) in which the stress deviator $S = S(V)$ (5) has a discontinuity. This point on the Hugoniot adiabat with coordinates P_L and V_L corresponds to the transition of the medium to the plasticity regime. The Hugoniot adiabat and the RM line intersect if

$$\dot{M}^2 > \dot{M}_0^2 = (\rho_0 c_0)^2 + \rho_0 \left(\frac{4}{3}\mu - \Gamma_0 S_0\right), \tag{11}$$

where c_0 is the speed of sound in an undeformed medium. When the intensity of the mass flow rate $\dot{M}$ slightly exceeds the value of $\dot{M}_0$, the HA and RM line intersect at one point A located in the range V_L, V_0 (Fig. 3), which corresponds to the elastic shock wave, during the passage of which the material remains in an elastic state, and its brittle destruction is observed. When $\dot{M} = \dot{M}_L$, the HA and RM line intersect at the point L. In this case, there is another analytical solution corresponding to the point B. Thus, two shock waves appear, which will be called the weak wave A or an elastic precursor and the strong plastic compression wave B. After the passage of the elastic precursor, the material is compressed to $V = V_L$ and $P = P_L$, and its brittle destruction may occur. At the same time, behind the strong shock wave the material is compressed still more strongly: $V = V_B < V_L$ and $P = P_B < P_L$. Under these conditions, brittle-plastic destruction is being developed in the medium. This state is characterized by the intensity of the mass flow rate $\dot{M}_2$:

$$\dot{M}_2 = \rho(u - D_2) = \rho_L(u_L - D_2), \tag{12}$$

where D_2 is the velocity of the plastic compression wave propagation. The intensity $\dot{M}_2$ changes in the limits $\dot{M}_{20} \le \dot{M}_2 \le \dot{M}_L$. Here, $\dot{M}_{20}$ is the derivative of the Rayleigh–Michelson function at the point L which corresponds to the condition $V = V_L$. In the limiting case $M_2 = \dot{M}_L$, it seems that the second running plastic wave damps the elastic precursor, and a configuration is realized that corresponds to the point L. In this case, the brittle destruction of the material passes to the brittle-plastic or plastic destruction, since further a single wave plastic configuration is realized that corresponds to the point B. The parameters P_B and ρ_B correspond to a purely plastic regime originating on interaction of a polyethylene striker, moving with the velocity $v_0 > 3.0$ km/s, with a PMMA target.

The given analysis of the passage of a strong shock wave through a medium that obeys the conditions of elastic plasticity and Mie–Grüneisen equation can be used for describing the process of high-rate destruction of PMMA which is subjected to the action of a polyethylene striker. The point of intersection of HA with the RM line with the parameters P_L and

V_L corresponds to the transition of brittle destruction of PMMA to the brittle-plastic one at $v_0 = 3.0$ km/s, $u_L \sim 1.5$ km/s, and $t > 20$ m/s.

NOTATION

A, a constant; *d*, diameter of a crater; *D*, shock wave velocity; *h*, crater height; *L*, limit of plasticity or fluidity of material; *P* and *V*, pressure and specific volume of a compressed medium behind the shock wave front; *u*, speed of the medium; v_0, velocity of motion of a striker; σ, compression stress. Indices: 0, nonperturbed parameters before the shock wave front.

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